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Parity nonconserving asymmetry in neutron-deuteron and proton-deuteron scattering

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Parity nonconservation in nucleon (N)-deuteron(d) scattering is examined at low energies (≤ 40 MeV), particularly at 15 MeV. A Faddeev treatment is employed. For the strong N-N force a separable interaction, which fits scattering cross sections up to 60—100 MeV, is used; for the weak parity nonconserving NN force, an isoscalar short range term due to ρ and ω exchanges and an isovector pion exchange contribution are included. Comparisons with parity nonconserving experimental asymmetries in NN scattering are made. For the parity nonconserving asymmetry in N-d scattering the contributions of various terms are separated, so that the model dependence of our results can be discussed. It is found that multiple scattering effects are important. The energy dependence of the parity nonconserving asymmetry in N-d scattering is found to differ qualitatively from that in NN scattering.

[NUCLEAR REACTIONS $d(N,N)d$, calculated total cross section parity nonconserving asymmetry, $E=14$—40 MeV.]

I. INTRODUCTION

The nature of the nonleptonic weak interaction remains to be elucidated. In addition to the strangeness-changing decays of strange baryons and mesons, the interaction can be studied through the observation of parity nonconserving (PNC) observables in hadronic reactions.[1–3] These effects are assumed to arise from the interference of a strong and a nonleptonic weak amplitude. Indeed, the observations of circular polarizations of photons emitted in nuclear transitions and of other parity nonconserving asymmetries are of the order of magnitude expected from such an interference. However, to date, a quantitative understanding of the parity nonconserving observables has not been obtained, so that it has not yet been possible to deduce the parity nonconserving interaction between nucleons. Detailed nuclear structure analyses are required to make certain that a consistent weak nucleon-nucleon potential is obtained.[4,5] Although considerable progress has occurred in the past several years,[6] much work remains to be done. Analyses presently underway may clarify the interpretation of past measurements.

The PNC nucleon-nucleon force arises from the product of vector and axial-vector nonleptonic weak currents.[1,2,7] However, the connection between the measured (or deduced) force and the basic theory of weak interactions depends upon a model for the structure of the hadrons. Again, considerable progress has been made recently by using quark models and SU(6) symmetry.[7] The lack of knowledge of the structure of the hadrons (e.g., confinement mechanisms) only allows one to place limits on the expected weak PNC force between nucleons. Further progress may occur through the use of QCD (quantum chromodynamics) and scaling (renormalization techniques).[7]

Among the hadrons, the proton-proton system is the simplest one which can be studied experimentally in detail.[8] Polarized proton beams permit one to measure parity nonconserving asymmetries as a function of energy and for various final states. Such experiments have been or are being carried out[8–12] at 15, 45, 800, and 6000 MeV. On the other hand, polarized neutron-proton scattering experiments are more difficult and have not yet been undertaken. The weak n-p force has been investigated through very low energy neutron capture on hydrogen.[13] The experimental results found to date only provide upper limits.

The next simplest system to study in order to obtain information on the weak force between neutrons and protons is the three nucleon system. Experimental investigations of neutron capture by deuterium and of proton scattering on deuterium have been and are being undertaken.[4] An early analysis of the 15 MeV scattering experiments used a DWBA approximation,[15] with antisymmetry taken into account only in an approximate manner. Later analyses used Faddeev-type treatments.[16,17] One of the advantages of the three-body system is that an exact scattering analysis can be made in terms of the Faddeev treatment. If the p-p weak force is known and three-body forces are negligible, then p-d scattering should permit one to deduce the PNC p-n force.

In this paper, we present a detailed analysis of 15 MeV polarized proton-deuteron scattering utilizing a Faddeev treatment. Preliminary results were presented earlier but contained an error due to an incorrect relative normalization factor.[16] Meanwhile, a second such study of this scattering process has been carried out by Desplanques, Benayoun, and Gignoux.[17] In our analysis, we use a separable hadronic NN force which fits s-wave scattering up to 60–100 MeV; for simplicity we neglect the hadronic tensor force. We examine the effect of these approximations in the NN PNC problem by comparing results with those of similar model calculations using more realistic local potentials. For the PNC force, we also make simplifying assumptions: We consider an isoscalar force of short range and an isovector force due to pion exchange. The other components of the weak force are neglected. In part this restriction was imposed to permit us to compare the Faddeev treatment with the DWBA and similar approximations made by Henley in his analysis of $\vec{p}d$ scattering. In addition, based upon analyses of $\vec{N}N$ PNC experiments as well as nuclear transitions in the light nuclei, there is only ambiguous support for other components of a more general PNC potential.[7] In Sec. II we describe the weak PNC force used in our model calculations. In Sec. III we discuss the results of our model for PNC asymmetry in NN scattering. Our three-body formalism for calculating $\vec{N}d$ scattering is described in detail in Sec. IV. In Sec. V we present results for $\vec{n}d$ and $\vec{p}d$ scattering at 14.4 MeV and the predicted energy dependence of A_{nd}. Model dependence of the calculation is discussed in Sec. VI. Section VII contains our summary and conclusions.

II. THE WEAK PNC FORCE

Effects of parity nonconservation have been observed in several nonleptonic nuclear reactions. It is generally believed that the observed asymmetries arise from manifestations of the first order $\Delta S = 0$ nonleptonic weak interactions, the dominant effect in nuclear systems being a PNC contribution to the NN interaction.[1,7] However, there is no definitive theory of the weak interactions of physical hadrons. The desire to remedy this situation is the primary motivation for this and related studies. It is expected that for low energy processes ($E \leq 300$ MeV) one can use a weak (NN) potential form arising from the exchange of mesons. Invariance arguments[1–3] suggest that these exchanges are dominated by single pseudoscalar and vector mesons (except for possibly important contributions from two pion exchange). Here we adopt the philosophy of utilizing a PNC potential based upon meson exchanges to fix the range and spin-isospin structure of the weak force. A general diagram representing terms in such a potential is shown in Fig. 1. As stated earlier, simplicity, general invariance arguments, and octet dominance[1–3] lead us to include only isoscalar ρ and ω

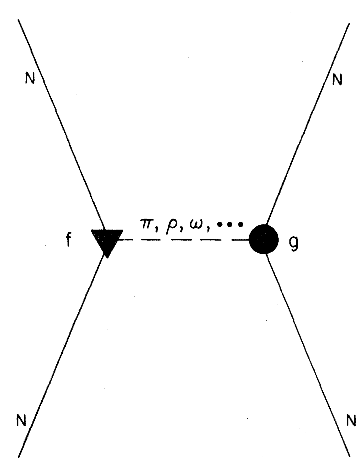

FIG. 1. One boson exchange contributions to the weak PNC potential; the weak vertex is indicated as a triangle, the strong vertex as a circle.

terms and an isovector π term in the PNC potential. It should be noted by the interested reader that arguments have been presented which indicate that octet dominance in strange particle decays may be dynamical in origin and may not apply to the NN problem.[18] An isotensor term in the PNC NN potential cannot contribute to N-d scattering because the total isospin is $\frac{1}{2}$. However, the omission of the isotensor term in NN scattering would have implications for the N-d problem if the PNC potential were adjusted to fit the NN experimental data.

Parity nonconserving potentials due to one boson exchange have been reviewed numerous times, and we will not dwell upon their derivation.[1–3] The ρ and ω exchange isoscalar potentials resulting from such considerations are of the form

$$V^{\rho}_{\rm PNC}(r_{ij})=-\frac{1}{M}f_\rho g_\rho \vec{\tau}_i\cdot\vec{\tau}_j((1+\mu_v)i\sigma_i\times\vec{\sigma}_j\cdot[\vec{p}_{ij},v(r_{ij})]+(\vec{\sigma}_i-\vec{\sigma}_j)\cdot\{\vec{p}_{ij},v(r_{ij})\})\ , \tag{1}$$

$$V^{\omega}_{\rm PNC}(r_{ij})=-\frac{1}{M}f_\omega g_\omega((1+\mu_s)i\vec{\sigma}_i\times\vec{\sigma}_j\cdot[\vec{p}_{ij},v(r_{ij})]+(\vec{\sigma}_i-\vec{\sigma}_j)\cdot\{\vec{p}_{ij},v(r_{ij})\})\ , \tag{2}$$

where M is the nucleon mass (939 MeV); f_ρ and f_ω are the weak vertex coupling constants; g_ρ and g_ω are the strong vertex coupling constants; $\vec{\sigma}_i$ and $\vec{\tau}_i$ are the nucleon spin and isospin operators; and $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$ and $\vec{p}_{ij}=\frac{1}{2}(\vec{p}_i-\vec{p}_j)$ are the relative position and momentum of the two nucleons. The $[a,b]$ and $\{a,b\}$ are the standard commutator and anticommutator operations. We assume for the isovector anomalous magnetic moment μ_V a value of 3.70 but neglect the small and perhaps controversial isoscalar magnetic coupling of the ω to the nucleon (i.e., we assume $\mu_S=0$). We also neglect the momentum transfer dependence of the weak vertices and utilize the factorization approximation[19] in estimating the *magnitude* of f_ρ (Ref. 20):

$$|f_\rho|=\frac{Gm^2}{2g_\rho}g_A\cos^2\theta$$

$$\cong 1.4\times10^{-6}\ . \tag{3}$$

This value is in agreement with that derived in Ref. 17, whose sign we shall adopt since it is in agreement with that of A_{pp} in Refs. 9 and 10 and with the theoretical considerations based on quark models.[7] Here G is the weak coupling constant; m is the average mass of the ρ and ω (755 MeV); g_ρ is given by ${g_\rho}^2/4\pi=0.62$; g_A is the axial vector renormalization constant (1.24); and $\cos^2\theta\cong0.98$. Although we use $f_\rho=-1.4\times10^{-6}$ in our numerical work, it is trivial to scale our results for other values of f_ρ. The contribution of ρ exchange to the PNC asymmetry is computed separately, and the isoscalar ρ-exchange contribution to the asymmetry is linear in f_ρ.

There exists a large range of possible values for f_ω (see, for example, McKellar and Pick[21]); for simplicity we have assumed $f_\omega=f_\rho$ in our numerical calculations. The constant g_ω is also not well determined, and we adopt the value of $\sqrt{2}g_\rho$. Again, our numerical results for the contribution of ω exchange to the asymmetry are proportional to $f_\omega g_\omega$, and they can therefore readily be scaled for other values of this product of coupling constants. The function

$$v(r)=\exp(-mr)/(4\pi r)$$

defines the range of the PNC potential in terms of the mass of the exchanged meson.

As mentioned above, we keep only the longest range part of the isovector PNC potential, namely that generated by the exchange of a single charged pion. (CP conservation forbids π^0 exchange.) This potential connects $I=0$ and 1 states of the NN system. There is also a short-range isovector interaction which connects $I=1$ states but which we neglect herein. For the one pion exchange interaction one obtains the PNC potential

$$V^{\pi}_{\rm PNC}(r_{ij})=\frac{if_\pi g_\pi}{2\sqrt{2}M}(\vec{\tau}_i\times\vec{\tau}_j)_z(\vec{\sigma}_i+\vec{\sigma}_j)\cdot[\vec{p}_{ij},v_\pi(r_{ij})]\ , \tag{4}$$

where f_π is the weak coupling constant and g_π is the strong one given by ${g_\pi}^2/4\pi=14.4$ The function

$$v_\pi(r)=\exp(-m_\pi r)/(4\pi r)\ ,$$

where m_π is the mass of the charged pion (139 MeV). In the absence of neutral currents, early estimates based upon SU(3) considerations and current algebra gave, for the Cabibbo value of f_π, $|f_\pi^{\rm Cab}|=4.3\times10^{-8}$; we assume $f_\pi^{\rm Cab}$ to be positive. Although the early work on $f_\pi^{\rm Cab}$ has been superseded, we use it in part of our study for reference purposes. If neutral currents are postulated, then $f_\pi^{\rm nc}$ is expected to be positive and some 4—10 times larger than the Cabibbo value. We will assume a factor of 10 and positive sign in our numerical results to follow. Once again, the contribution of π exchange to the asymmetry scales linearly for other values of f_π.

Such a model of the PNC force is certainly in-complete. Although it contains the important features of the isoscalar and long-range $(I=0\leftrightarrow I=1)$ isovector interaction, it lacks the short-range isovector $(I=1\leftrightarrow I=1)$ and the isotensor parts. However, analyses of PNC experiments in nuclei which require isoscalar, π exchange, and isotensor components are ambiguous in the need for any (sizable) short range isovector $(I=1\leftrightarrow I=1)$ potential. The isotensor potential plays no role in elastic and total cross sections for Nd scattering. We believe that our model should permit us to estimate the order of magnitude of the asymmetry in $\vec{n}d$ and $\vec{p}d$ scattering due to PNC effects as well as to study significant aspects of the PNC experiment in the trinucleon system, such as the energy dependence of the asymmetry.

III. PNC ASYMMETRY IN *NN* SCATTERING

The most basic, but already very difficult, scattering experiment to look for PNC effects is the measurement of an asymmetry in the total cross section of longitudinally polarized nucleons scattered from a hydrogen target. The asymmetry is given by

$$A=(\sigma_+-\sigma_-)/(\sigma_++\sigma_-)\ , \tag{5}$$

where σ_+ (σ_-) is the total cross section for a $+$ $(-)$ helicity of the incident nucleon. Assuming a $V_{\rm PNC}$, one can calculate such an asymmetry for $\vec{p}p$ and $\vec{n}p$ scattering. However, measurements of the asymmetry A for $\vec{n}p$ scattering are much more difficult than for $\vec{p}p$ scattering. The calculation in terms of total cross sections is expedited by use of the optical theorem to relate the total cross section to the imaginary part of the forward elastic scattering amplitude. In particular,

$$\sigma_\pm=-\frac{8\pi^3}{k}\,{\rm Im}\sum_m\{\langle\vec{p},\pm,m\,|\,M^S+M^W\,|\,\vec{p},\pm,m\rangle\}\ , \tag{6}$$

where m is the spin projection of the target. The M^S and M^W are the strong and weak scattering amplitudes, respectively. We utilize standard two potential theory in terms of t matrices in order to calculate M^W within a specific model for the parity conserving strong interaction.

Because of the complexity of the $\vec{N}d$ problem, we limit our consideration of NN parity conserving potential models to those without tensor terms. We examine the Malfliet-Tjon I-III (MT I-III) potential[22] results for the $\vec{N}N$ problem in order to compare with the previous results of Brown *et al.*[20] for the Hamada-Johnston (HJ) (Ref. 23) and Bryan-Gersten (BG) (Ref. 24) potentials, both of which do have tensor force combinations in the triplet channel. We then carry out the same calculation using a rank-one, separable potential representation of the NN strong interaction in order to study the effect of not including short-range repulsion in the calculation of A_{pp} and A_{np}. Our separable potentials are of the form

$$V_i(k,k')=-\frac{\lambda_i}{M}v_i(k)v_i(k')\ , \tag{7a}$$

$$v_i(k)=(k^2+\beta_i{}^2)^{-1}\ , \tag{7b}$$

where $\lambda_t=0.3819$ fm^{-3}, $\beta_t=1.406$ fm^{-1} and $\lambda_s=0.1533$ fm^{-3}, $\beta_s=1.183$ fm^{-1}. The choice of such a simple interaction is open to criticism, especially the neglect of the NN p-wave interaction, the long range tensor force, and the short range repulsion. However, the complexity of the three-body calculation is enormous. In addition, the simple separable potential model reproduces the singlet phases well up to 100 MeV, the triplet phases up to 55 MeV, and the deutron pole. We shall see that the $\vec{N}N$ PNC asymmetry predictions do not differ by large factors from those of the more realistic models, so that such a potential model appears to be adequate for our stated purposes.

In the study of the PNC asymmetry in the two-body problem it is possible to mock up the effect of hadronic repulsion at short distances by reducing the strengths of the weak potential at short distances. We do so by means of a cut off or regularization of the weak $V_{\rm PNC}$:

$$V(r)=\exp(-mr)/(4\pi r)$$

is replaced by

$$V^{\rm reg}(r)=[\exp(-mr)-\exp(-\Lambda r)]/(4\pi r)\ .$$

However, we remark that such a procedure is of limited validity in the three-body calculation because it cannot properly account for the effects of repulsion in all strongly interacting pairs. That is, it mocks up the hadronic repulsion between the pair of nucleons which interact via the weak PNC potential but not between those pairs which interact only via the strong potential. We will return to this point later in the paper.

In Tables I and II we present results for $\vec{N}N$ asymmetries calculated at 14.4 MeV. We include the work of Brown *et al.* for the HJ and BG potentials as well as our own results for the MT I-III potential and the separable potential defined above. We note that our NN elastic total cross sections are $\sigma_{pp}\cong 450$ mb and $\sigma_{np}\cong 630$ mb, in very reasonable agreement with the experimental values of 460 and 640 mb, respectively. In addition to the above

TABLE I. $\vec{N}N$ asymmetries ($\times 10^7$) calculated at 14.4 MeV incident energy using the strong interaction models of Hamada-Johnston, Bryan-Gersten, Reid, Malfliet-Tjon, and the separable model of this paper (with and without regularization of the weak PNC interaction).

Model	$A_{pp}^{\rho+\omega}$	$A_{np}^{\rho+\omega}$	$(A_{np}^{\pi})^{\rm Cab}$	$(A_{np}^{\pi})^{\rm nc}$
HJ	−1.0	−0.22	0.05	
BG	−1.1	−0.40	0.05	
RSC	−1.29	−0.46	0.08	
MT I-III	−1.49	−0.78	0.10	1.0
Sep	−3.30	−1.47	0.12	1.15
Sep (Reg)	−1.00	−0.41	0.10	1.0

model results, we also quote a value for A_{pp} using the singlet potential of the Reid soft core model. Finally, we include results for a separable potential calculation in which $V_{\rm PNC}^{\rho}$, $V_{\rm PNC}^{\omega}$, and $V_{\rm PNC}^{\pi}$ are regularized by means of an $\exp(-\Lambda r)/(4\pi r)$ subtraction with a mass Λ of 950 MeV. This value was chosen to yield agreement with more complete calculations of the asymmetries.

We wish to call the following points to the attention of the reader. For A_{pp}, where only the singlet NN force acts, all models give essentially the same result, except for the separable model. It makes little difference whether the short-range repulsion comes from a true hard core as in the case of the HJ potential or from a relatively soft core as in the case of the MT I potential. On the other hand, the nonregularized separable potential result is clearly a factor of 2—2.5 too large. For A_{np} the ρ-ω contribution depends crucially upon the nature of the hadronic NN model, whether a tensor component is included as well as the type of short-range repulsion used. The HJ result is a factor of 2 smaller than the BG result, which indicates the sensitivity of the asymmetry to the exact form of the short-range repulsion. The MT III result with soft repulsion but no tensor force is almost two times larger than the BG result, where there is short-range repulsion and a tensor force. Again, the separable potential model (without short-range repulsion) gives an asymmetry about a factor of 2 larger than the more realistic MT III potential model. If one regularizes the PNC potentials as described above and recalculates the asymmetries using the separable potential model for the strong interaction, the $A_{pp}^{\rho+\omega}$ and $A_{np}^{\rho+\omega}$ are reduced to approximately the correct size. Apparently, the tensor force plays an important role in the np triplet state, as one can see by comparing results for A_{np}^{π} in the MT and BG models. One can reduce the value of A_{np}^{π} in the separable potential model to a very reasonable one by means of the same regularization procedure using the same cutoff mass of 950 MeV. Such a prescription may be considered doubtful if one believes that the smaller values obtained for the BG and HJ strong interaction models arise from the tensor nature of the triplet force in those models. Based upon the considerations of the NN asymmetries discussed here, one might expect estimates of the asymmetry in $\vec{N}d$ scattering in a simple separable potential (without regularization) to be roughly a factor of 2 larger than those resulting from a calculation employing a more realistic potential.

TABLE II. $\vec{N}N$ asymmetries ($\times 10^7$) at 14.4 MeV broken into electric and magnetic contributions of the ρ, contributions of the ω, and contributions of the π.

	RSC	MT I-III	Sep	Sep (Reg)
A_{pp}-$\rho^{\rm elec}$	−0.91	−1.03	−0.80	−0.24
A_{pp}-$\rho^{\rm mag}$			−1.37	−0.41
A_{pp}-ω	−0.38	−0.46	−1.13	−0.35
A_{np}-$\rho^{\rm elec}$	−0.34	−0.71	−0.09	−0.02
A_{np}-$\rho^{\rm mag}$			−1.15	−0.32
A_{np}-ω	−0.12	−0.07	−0.23	−0.07
A_{np}-π	0.08	0.10	0.115	0.10

We close this section by noting that the experimental value[9] for the asymmetry A_{pp} at 15 MeV is $-(1.7\pm0.8)\times 10^7$. The theoretical asymmetries for all of the models in Table I must be considered to be in reasonable agreement with this value except for the simple separable potential calculation, where the weak potential is not regularized. We also include in Fig. 2 a comparison of the predicted energy dependence of A_{pp} for various models; the data are those reported for 15 MeV (Ref. 9) and 45 MeV (Ref. 10) $\vec{p}p$ scattering. The HJ results are from Ref. 20 (after changing the sign of f_ρ) and include all partial waves in the parity conserving denominator $(\sigma_+ + \sigma_-)$ of Eq. (5); note that the denominator in Ref. 20 is $\frac{1}{2}(\sigma_+ + \sigma_-)$. All other results are s wave only, which accounts for the lack of curvature. Examination of this figure shows that, except for the simple separable potential model, there is essential agreement among the results of the various strong NN models at 14.4 MeV which remains as one goes to higher energies.

IV. THREE-BODY ASYMMETRY FORMALISM

We next consider the asymmetry in the total cross section for the scattering of longitudinally polarized nucleons (omitting Coulomb forces) but from a deuterium target. As in the case of two-body scattering,

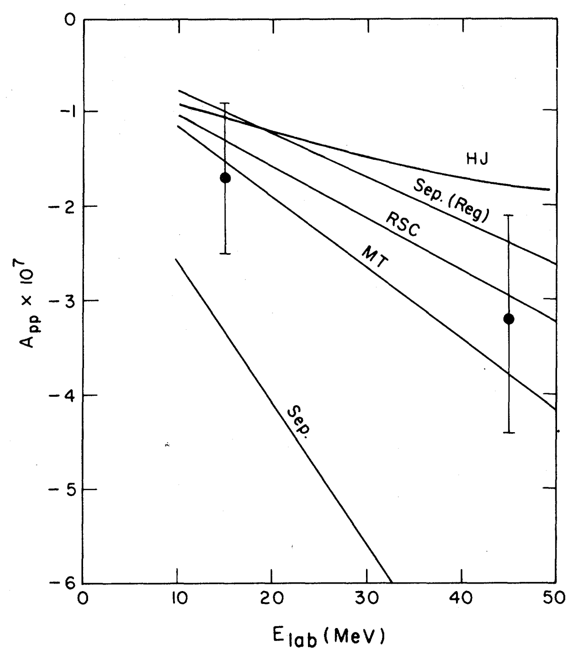

FIG. 2. Energy dependence of A_{pp} for various NN strong potential models; data are from Refs. 9 and 10.

we relate the total cross sections required in the expression for the asymmetry to the imaginary part of the forward elastic scattering amplitude by means of the optical theorem.

Let us define $\sigma(m_1,m_2)$ as the total cross section for particle 1 (nucleon), with spin projection m_1, to be scattered from particle 2 (the deuteron), with spin projection m_2. Both spin projections are with respect to the incident beam direction. The experimentally measured asymmetry A_{nd} is given by

$$A=\frac{\sum_{m_2}[\sigma(m_1=+\frac{1}{2},m_2)-\sigma(m_1=-\frac{1}{2},m_2)]}{\sum_{m_2}[\sigma(m_1=+\frac{1}{2},m_2)+\sigma(m_1=-\frac{1}{2},m_2)]}\ . \tag{8}$$

The spin-dependent, total cross section is given in terms of the forward scattering amplitude by

$$\sigma(m_1,m_2)=-\frac{8\pi^3}{k}\mathrm{Im}\langle\vec{k},m_1,m_2\,|\,T\,|\,\vec{k},m_1,m_2\rangle\ . \tag{9}$$

The scattering amplitude is expressed in terms of the familiar channel transition amplitudes. For the convenience of the reader we summarize the relevant three-body formulae.

Given a Hamiltonian $H=H_0+V_1+V_2+V_3$, where V_1 is the two-body potential between particles 2 and 3, etc., the scattering matrix $U_{\beta\alpha}(z)$ is described by the equations

$$U_{\beta\alpha}(z)=(1-\delta_{\beta\alpha})V_\alpha+W_{\beta\alpha}(z)\ , \tag{10a}$$

$$W_{\beta\alpha}(z)=\sum_{\substack{\gamma\neq\alpha\\ \gamma\neq\beta}}V_\gamma-\sum_{\substack{\gamma\neq\alpha\\ \gamma\neq\beta}}V_\gamma G(z)V_\delta\ , \tag{10b}$$

where $G(z)=(H-z)^{-1}$. The scattering amplitude relating the incoming state in which particle α is free (and the remaining two are bound) to the outgoing state in which particle β is free (and the remaining two are bound) is given by

$$_\beta\langle p_f\,|\,T(z)\,|\,p_i\rangle_\alpha={}_\beta\langle p_f\,|\,U_{\beta\alpha}(z)\,|\,p_i\rangle_\alpha\ . \tag{11}$$

Since the total cross section depends only upon the imaginary part of the matrix element, we can replace the operator $U_{\beta\alpha}(z)$ by $W_{\beta\alpha}(z)$ and obtain

$$\sigma(m_1,m_2)=-\frac{8\pi^3}{k}\times\mathrm{Im}_\beta\langle\vec{k}m_1m_2\,|\,W_{\beta\alpha}(z)\,|\,\vec{k}m_1m_2\rangle_\alpha\ . \tag{12}$$

The operator $W_{\beta\alpha}(z)$ satisfies the Faddeev equations

$$W_{\beta\alpha}(z)=\sum_{\substack{\gamma\neq\beta\\ \gamma\neq\alpha}}T_\gamma-\sum_{\gamma\neq\beta}T_\gamma G_0(z)W_{\gamma\alpha}(z)\ , \tag{13}$$

where $G_0(z)=(H_0-z)^{-1}$. The input for the Faddeev equations is the two-body t matrix T_γ. This T_γ is the solution of the two-body Lippmann-Schwinger equation driven by V_γ

$$T_\gamma=V_\gamma-V_\gamma G_0T_\gamma\ . \tag{14}$$

At this point we have all of the ingredients necessary to calculate exactly the asymmetry A_{nd} for any given two-body interaction composed of a parity conserving strong potential and a parity nonconserving weak potential

$$V_\gamma=V_\gamma^S+V_\gamma^W\ . \tag{15}$$

Of course, the antisymmetrization procedure (because of the identical particle nature of the projectile and target constituents) reduces the number of relevant (distinct) amplitudes. Though an exact calculation could be done given sufficient computer size and time, the weak strength of the PNC V_γ^W compared to the strong V_γ^S makes it natural to treat V_γ^W as a perturbation. In theory, three-nucleon calculations with only a strong two-body interaction have been carried out, and the resulting scattering wave functions are available. These can serve as the unperturbed input for a perturbation calculation.

Recall that for the case of two-body scattering where $V=V^S+V^W$ and $T^S=V^S-V^SG_0T^S$, the exact solution of

$$T=V-VG_0T \tag{16}$$

is

$$T=T^S+(1-TG_0)V^W(1-G_0T^S)\ . \tag{17}$$

When expanded to first order in V^W, Eq. (17) becomes

$$T\cong T^S+(1-T^SG_0)V^W(1-G_0T^S)\ , \tag{18}$$

which we will write as

$$T\equiv T^S+T^W\ . \tag{19}$$

For the three-body perturbation theory we follow Sloan.[25] Let $W^S_{\beta\alpha}$ be the solution of Eq. (13) for $T_\gamma=T^S_\gamma$. Then for $T_\gamma=T^S_\gamma+T^W_\gamma$, the exact solution of Eq. (13) satisfies the following equation:

$$W_{\beta\alpha}=W^S_{\beta\alpha}+\sum_\gamma [1-\delta_{\beta\gamma}+W^S_{\beta\gamma}G_0] \times T^W_\gamma[1-\delta_{\gamma\alpha}+G_0W_{\gamma\alpha}]\ . \tag{20}$$

To first order in the weak interaction, we can write this equation as

$$W_{\beta\alpha}\cong W^S_{\beta\alpha}+\sum_\gamma [1-\delta_{\beta\gamma}+W^S_{\beta\gamma}G_0] \times T^W_\gamma[1-\delta_{\gamma\alpha}+G_0W^S_{\gamma\alpha}]\ , \tag{21}$$

which we define to be

$$W_{\beta\alpha}\equiv W^S_{\beta\alpha}+W^W_{\beta\alpha}\ . \tag{22}$$

Explicitly, one has

$$W^W_{\beta\alpha}=\sum_{\substack{\gamma\neq\beta\\ \gamma\neq\alpha}} T^W_\gamma+\sum_{\gamma\neq\beta} T^W_\gamma G_0W^S_{\gamma\alpha}+\sum_{\gamma\neq\alpha} W^S_{\beta\gamma}G_0T^W_\gamma + \sum_\gamma W^S_{\beta\gamma}G_0T^W_\gamma G_0W^S_{\gamma\alpha}\ , \tag{23}$$

but for future reference we introduce the shorthand notation

$$W^W=T^W+T^WGW^S+W^SGT^W+W^SGT^WGW^S \tag{24}$$

and

$$W=W^S+W^W\ . \tag{25}$$

Typical diagrams contributing to Eq. (24) are shown in Fig. 3(a).

If we now take $W_{\beta\alpha}$ of Eq. (22) and evaluate its matrix element $\langle nd\mid W_{\beta\alpha}\mid nd\rangle$ between states of a free nucleon and a normal parity deuteron, we neglect the contribution of the odd parity admixtures of the deuteron. One must therefore add these components to the wave function illustrated in Fig. 3(b). We refer to these contributions to the asymmetry below as the "deuteron distortion effect."

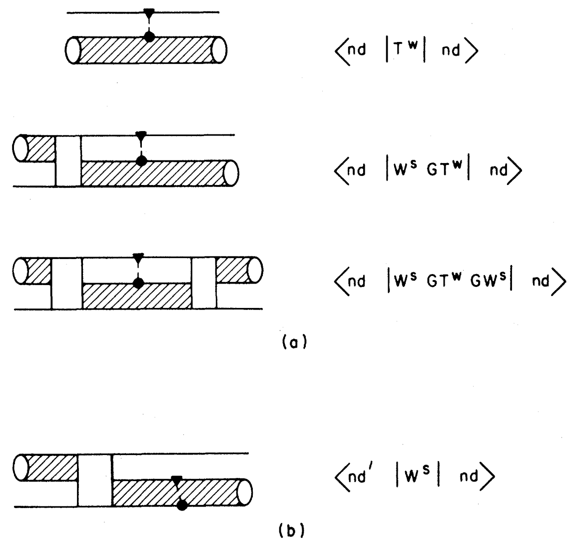

FIG. 3. (a) Diagrammatic representation of three types of scattering terms contributing to the Nd PNC component of the total cross section. (b) Diagrammatic representation of the "deuteron distortion" contribution to the Nd PNC component of the total cross section.

The form of the matrix elements $\langle M^S\rangle$ and $\langle M^W\rangle$ used to determine the total cross section

$$\sigma(m_1,m_2)=-\frac{8\pi^3}{k} \times \mathrm{Im}\langle \vec{k},m_1,m_2\mid M^S+M^W\mid \vec{k},m_1,m_2\rangle \tag{26}$$

then reads

$$\langle M^S\rangle=\langle Nd\mid W^S\mid Nd\rangle\ , \tag{27}$$

$$\langle M^W\rangle=\langle Nd\mid W^W\mid Nd\rangle+\langle Nd'\mid W^S\mid Nd\rangle + \langle Nd\mid W^S\mid Nd'\rangle\ . \tag{28}$$

Here d labels the normal even-parity deuteron and d' the additional odd-parity part of the deuteron. We obtain the wave function of the odd parity deuteron in a standard way from Eq. (18) for the two-body t matrix.

The major part of the calculational effort is the generation of $\langle Nd\mid W^W\mid Nd\rangle$ from Eq. (23). Although the initial and final states are two-body nucleon-deuteron states, it is practical to choose as intermediate states explicit three-nucleon states in the $|j_1 j_{23} J\rangle$ representation. The reason is that we have to evaluate two-body matrix elements of the weak interaction which depend upon $|l_{23}s_{23}j_{23}\rangle$.

We generate the necessary unperturbed amplitudes with a code written by Larson.[26] In that code the three-nucleon scattering problem is solved for an s-wave, spin-dependent separable potential, although there exists the possibility of including p-wave interactions. Because of time limitations we have restricted our considerations to wave functions generated by the s-wave strong interaction. With these wave functions describing the matrix elements of W^S, we calculate the matrix elements of W^W using Eq. (24), being careful to take into account the proper antisymmetry and recoupling requirements. The singularities encountered in the integrations are treated with standard procedures of regularization and subtraction; see Ref. 27. The same matrix elements of W^S are used to determine the effect of the deuteron odd-parity admixture given by $\langle Nd' | W^S | Nd\rangle + \langle Nd | W^S | Nd'\rangle$.

V. NUMERICAL RESULTS

A. $\vec{n}d$ and $\vec{p}d$ scattering at 14.4 MeV

Using the separable potential representation of the NN strong interaction and the PNC model discussed in Sec. II, we have studied A_{nd} in detail at 14.4 MeV. In Table III we have broken A_{nd} into components: the electric contribution of ρ exchange, the magnetic contribution of ρ exchange (that proportional to μ_V), the contribution of ω exchange (recall that we assume the magnetic coupling to be zero), and the contribution of π exchange both with and without an assumed enhancement of the Cabibbo (or charged current) coupling by a factor of 10. We have made a further subdivision of $\langle M^W\rangle$ [see Eq. (28)]: D is the component due to the deuteron odd parity admixture and the remaining $\langle nd | W^W | nd\rangle$ is decomposed according to Eq. (24) into T, $TGW+WGT$, and $WGTGW$. It is clear that ρ and ω exchanges provide the dominant contribution to the asymmetry in our version of the Cabibbo model. However, if there is an order of magnitude enhancement of $V^{\pi}_{\rm PNC}$ due to neutral currents, then the π exchange contribution becomes comparable to that of $V^{\rho}_{\rm PNC}$. We find a total asymmetry of longitudinally polarized neutrons from deuterons in our application of the Cabibbo model of

$$A_{nd}^{\rm Cab} = -1.52\times 10^{-7} \tag{29}$$

and in the neutral current model of

$$A_{nd}^{\rm nc} = -0.05\times 10^{-7} . \tag{30}$$

The asymmetry for polarized protons (in the absence of Coulomb effects) scattered from deuterons follows immediately by inverting the sign of the π-exchange contribution and is

$$A_{pd}^{\rm Cab} = -1.84\times 10^{-7} , \tag{31}$$

and in the neutral current model is

$$A_{pd}^{\rm nc} = -3.31\times 10^{-7} . \tag{32}$$

These estimates are to be compared with the reported experimental value for the scattering of protons by deuterons[9,14]

$$A_{pd} = (-0.35\pm 0.85)\times 10^{-7} . \tag{33}$$

We emphasize that the theoretical estimates above must be considered to have at least a factor of 2 uncertainty due to the absence of short-range repulsion in our separable potential representation of the strong NN force. However, the magnitude of A_{pd} does not seem to be in serious disagreement with the present experimental value for the $\vec{p}d$ asymmetry.

As mentioned above, the various asymmetry components in Table III have been further broken down into their Born (T), linear in GW ($TGW+WGT$), and quadratic in GW ($WGTGW$) contributions. Except for $T_{\rho\,\rm mag}$, the Born contribution is generally small compared to the total asymmetry contribution for a specific meson exchange. This feature is expected, since the off-shell scattering due to the parity conserving strong interaction is important. It is perhaps surprising that the quadratic ($WGTGW$) contributions, which dominate, have a sign opposite that of the Born and linear terms. The size of the quadratic contribution emphasizes the importance of the off-shell Nd scattering in A_{Nd}. The cancellation that occurs for each of the meson exchanges indicates that any theoretical estimate of A_{Nd} is sensi-

TABLE III. Neutron-deuteron asymmetries calculated at 14.4 MeV incident nucleon energy in units of 10^{-7}, including the decomposition into contributions from ρ, ω, and π exchange according to Eq. (24) and the contribution (D) from the odd parity admixtures of the deuteron.

Component	$A_{nd}^{\rho\,\rm el}$	$A_{nd}^{\rho\,\rm mag}$	A_{nd}^{ω}	$(A_{nd}^{\pi})^{\rm Cab}$	$(A_{nd}^{\pi})^{\rm nc}$
Total	−0.64	−0.22	−0.82	0.16	1.63
D	0.002	0.027	0.001	0.034	
T	0.018	1.35	0.160	−0.013	
$TGW+WGT$	0.158	0.115	0.205	0.002	
$WGTGW$	−0.822	−1.71	−1.19	0.137	

TABLE IV. Numerical values of individual transitions contributing to A_{nd} as a function of the L of the strong amplitude W for the linear $(TGW+WGT)$ terms.

L		$A_{nd}^{\rho\,\mathrm{el}}$	$A_{nd}^{\rho\,\mathrm{mag}}$	A_{nd}^{ω}	$(A_{nd}^{\pi})^{\mathrm{Cab}}$
0	$s\leftrightarrow p$	-6.05×10^{-9}	-4.69×10^{-8}	-1.23×10^{-8}	-1.23×10^{-9}
	$p\leftrightarrow d$	-9.73×10^{-13}	-7.46×10^{-11}	3.38×10^{-13}	-1.96×10^{-10}
	$d\leftrightarrow f$	6.00×10^{-13}	1.21×10^{-13}	4.52×10^{-13}	-9.88×10^{-12}
	$f\leftrightarrow g$	-1.11×10^{-13}	-1.74×10^{-13}	-4.07×10^{-13}	-1.66×10^{-12}
1	$s\leftrightarrow p$	2.24×10^{-8}	6.11×10^{-8}	3.40×10^{-8}	1.78×10^{-9}
	$p\leftrightarrow d$	3.57×10^{-11}	-2.90×10^{-10}	-4.76×10^{-12}	-2.02×10^{-10}
	$d\leftrightarrow f$	1.51×10^{-12}	5.41×10^{-13}	1.14×10^{-12}	-5.52×10^{-11}
	$f\leftrightarrow g$	-8.38×10^{-14}	1.08×10^{-13}	-2.16×10^{-15}	5.34×10^{-13}
	$g\leftrightarrow h$	-5.08×10^{-14}	-1.10×10^{-13}	-2.59×10^{-13}	-3.36×10^{-13}
2	$s\leftrightarrow p$	-7.57×10^{-10}	-2.03×10^{-9}	-1.15×10^{-9}	-1.04×10^{-10}
	$p\leftrightarrow d$	5.95×10^{-11}	2.16×10^{-11}	-2.67×10^{-11}	3.14×10^{-10}
	$d\leftrightarrow f$	1.20×10^{-12}	-1.19×10^{-13}	1.74×10^{-12}	-6.97×10^{-11}
	$f\leftrightarrow g$	1.60×10^{-14}	1.36×10^{-13}	-3.74×10^{-14}	-1.30×10^{-12}
	$g\leftrightarrow h$	-1.10×10^{-13}	-4.24×10^{-13}	-1.90×10^{-13}	-5.16×10^{-13}

tive to details of the calculation. It is clear that the small value of A_{nd} in Eq. (3) must be assumed to have a large uncertainty associated with it.

The effect of deuteron distortion through V_{PNC} upon A_{Nd} (i.e., the transition from an odd parity component of the deuteron to one of normal parity due to the interaction with the projectile) is apparently small in our model. Although the pion exchange distortion contribution D_π to the $\vec{N}d$ asymmetry is similar in magnitude to the T, $TWG+WGT$, and $WGTGW$ terms, that is not the case for deuteron distortion effects stemming from the exchange of the heavier vector mesons. The reason is an almost complete cancellation between the contributions of the $L=0$ and 1 neutron-deuteron scattering wave functions.

Let us now turn to the question of convergence of the calculation with respect to partial waves. Because of the NN central potential assumption, the various amplitudes composing the $\vec{N}d$ asymmetry separate into spin doublet and spin quartet contributions for each total orbital angular momentum L. In the numbers quoted above, we have summed $L=0,1,2,3$. Computations involving $L=2,3$ were very time consuming; for the most part they showed that results obtained by summing only $L=0,1$ were good to better than 10%, and that the asymmetry calculations do converge rapidly with respect to the total L at this energy. An exception is the result for D_ρ and D_ω where $L=0$ and 1 practically cancel

TABLE V. Numerical values of individual transitions contributing to A_{nd} at 14.4 MeV as a function of the $L-L'$ of the strong amplitudes W entering the quadratic $WGTGW$ terms. The $(L=1)-(L'=0)$ matrix elements are the same as the $(L=0)-(L'=1)$ matrix elements, etc.

$L-L'$		$A_{nd}^{\rho\,\mathrm{el}}$	$A_{nd}^{\rho\,\mathrm{mag}}$	A_{nd}^{ω}	$(A_{nd}^{\pi})^{\mathrm{Cab}}$
0–1	$s\leftrightarrow p$	-4.15×10^{-8}	-8.49×10^{-8}	-6.00×10^{-8}	6.35×10^{-9}
	$p\leftrightarrow d$	6.76×10^{-11}	-3.02×10^{-10}	2.70×10^{-10}	3.30×10^{-10}
	$d\leftrightarrow f$	4.27×10^{-11}	7.19×10^{-12}	4.15×10^{-11}	-2.25×10^{-11}
	$f\leftrightarrow g$	1.34×10^{-12}	-6.16×10^{-14}	3.54×10^{-13}	1.18×10^{-11}
1–2	$s\leftrightarrow p$	2.14×10^{-10}	-1.14×10^{-10}	2.20×10^{-10}	4.90×10^{-11}
	$p\leftrightarrow d$	1.72×10^{-10}	-2.34×10^{-10}	1.26×10^{-10}	1.07×10^{-10}
	$d\leftrightarrow f$	2.76×10^{-12}	-1.87×10^{-12}	8.32×10^{-13}	-2.20×10^{-11}
	$f\leftrightarrow g$	6.54×10^{-13}	-4.22×10^{-14}	6.43×10^{-13}	3.21×10^{-12}
	$g\leftrightarrow h$	1.44×10^{-13}	1.67×10^{-13}	2.40×10^{-13}	-8.57×10^{-13}

each other. We expect here that summing up to $L=3$ is sufficient for D_ρ and D_ω. They remain small and have very little effect on the total asymmetry. The $L=2$ contribution is also important in D_π. We see, therefore, an overall slower convergence in the distortion terms. For the pion exchange terms in general, the convergence is somewhat slower. This is to be anticipated since the deuteron has a large radius and the π-exchange potential has a long range. In all cases, summing to $L=3$ appears adequate.

In Tables IV and V we present values for the largest transitions in the $TGW+WGT$ and $WGTGW$ components of A_{nd} for a given L or $L-L'$ labeling the strong amplitude(s) W involved. It is clear from Table IV that the largest contributions to the part of A_{nd} linear in W are the two-body weak sp transitions. (In the Born-type terms, there are only $s\leftrightarrow p$ transition contributions.) However, there are sizable pd and df transitions, especially in the $L=1$ and 2 amplitudes for π exchange. The largest $L=0$ and 1 ρ and ω transitions outside of the $s\leftrightarrow p$ family are reduced in strength by a factor of about 100. One can see from Table V that the quadratic $WGTGW$ π exchange term is not dominated by the $s\leftrightarrow p$ transitions as is the case for the terms linear in W. However, the $s\leftrightarrow p$ transitions are the most important ones for the heavier meson exchanges involving $L-L'$ of 0—1; for 1—2 transitions the $s\leftrightarrow p$ transitions are of comparable magnitude but are both considerably smaller than the dominant 0—1 $s\leftrightarrow p$ transitions. Careful study of the numerical results shown in these two tables leads one to conclude that the convergence of A_{Nd} is rapid both with respect to total L and with respect to the two-body, weak transitions.

We have remarked above that we believe our A_{Nd} results may be large due to the lack of repulsion in our separable hadronic NN potentials. This conclusion was inferred from the A_{pp} and A_{np} results quoted in Table I. However, because of the numerous cancellations apparent in the decomposition of A_{nd} in Tables III and IV, it is evident that such intuitive statements may not be valid. For this reason we have calculated A_{nd} with the regularized $V_{\rm PNC}$ in an effort to simulate the effects of short range repulsion in the PNC transitions. We emphasize that such a procedure is not likely to be valid for terms quadratic in W. Because the $WGTGW$ terms dominate our theoretical estimates, the following should not be taken as a complete elucidation of the effect of short range repulsion. However, it should be clear from this exercise that the above results cannot be simply scaled, since the simulated repulsive effects do differ from term to term.

In Table VI we have collected the results using simulated repulsion which should be compared with those in Table IV for the $TGW+WGT$ contribution to A_{nd}. The transitions involving ρ and ω exchanges

TABLE VI. Numerical values of individual transitions contributing to A_{nd} as a function of the L value of the strong amplitude W for the linear terms $(TGW+WGT)$. These results computed using the regularized PNC weak potentials are to be compared with the numbers of Table IV.

L		$A_{nd}^{\rho\,\rm el}$	$A_{nd}^{\rho\,\rm mag}$	A_{nd}^{ω}	$(A_{nd}^{\pi})^{\rm Cab}$
0	$s\leftrightarrow p$	-1.72×10^{-9}	-1.35×10^{-8}	-3.97×10^{-9}	-1.05×10^{-9}
	$p\leftrightarrow d$	-1.11×10^{-12}	-4.20×10^{-11}	5.46×10^{-13}	-1.96×10^{-10}
	$d\leftrightarrow f$	5.78×10^{-13}	8.22×10^{-13}	1.70×10^{-13}	-9.88×10^{-12}
	$f\leftrightarrow g$	-6.65×10^{-14}	-1.77×10^{-13}	-2.64×10^{-13}	-1.66×10^{-12}
1	$s\leftrightarrow p$	6.49×10^{-9}	1.70×10^{-8}	1.02×10^{-8}	1.54×10^{-9}
	$p\leftrightarrow d$	1.92×10^{-11}	-1.55×10^{-10}	1.24×10^{-12}	-1.98×10^{-10}
	$d\leftrightarrow f$	1.07×10^{-12}	6.54×10^{-12}	6.54×10^{-12}	-5.52×10^{-11}
	$f\leftrightarrow g$	-1.52×10^{-13}	-1.13×10^{-13}	-1.09×10^{-13}	-5.30×10^{-13}
	$g\leftrightarrow h$	-2.65×10^{-14}	-4.02×10^{-14}	-2.19×10^{-13}	-3.36×10^{-13}
2	$s\leftrightarrow p$	-2.30×10^{-10}	-6.00×10^{-10}	-3.58×10^{-10}	-1.00×10^{-10}
	$p\leftrightarrow d$	3.24×10^{-11}	1.20×10^{-11}	-1.44×10^{-11}	3.11×10^{-10}
	$d\leftrightarrow f$	7.68×10^{-13}	-1.14×10^{-13}	1.20×10^{-12}	-6.97×10^{-11}
	$f\leftrightarrow g$	6.32×10^{-14}	3.00×10^{-13}	-4.92×10^{-14}	-1.30×10^{-12}
	$g\leftrightarrow h$	-1.35×10^{-13}	-4.55×10^{-13}	-2.21×10^{-13}	-5.16×10^{-13}

TABLE VII. Numerical values of individual transitions contributing to A_{nd} as a function of $L-L'$ of the strong amplitudes W in the quadratic term $WGTGW$. These results computed using the regularized PNC weak potentials are to be compared with the numbers of Table V.

$L-L'$		$A_{nd}^{\rho\,\text{el}}$	$A_{nd}^{\rho\,\text{mag}}$	A_{nd}^{ω}	$(A_{nd}^{\pi})^{\text{Cab}}$
0—1	$s\leftrightarrow p$	-1.35×10^{-8}	-2.54×10^{-8}	-1.94×10^{-8}	5.83×10^{-9}
	$p\leftrightarrow d$	9.03×10^{-11}	6.11×10^{-11}	4.08×10^{-11}	3.75×10^{-10}
	$d\leftrightarrow f$	9.62×10^{-12}	-3.87×10^{-12}	6.92×10^{-12}	-2.20×10^{-11}
	$f\leftrightarrow g$	1.25×10^{-12}	1.13×10^{-12}	3.92×10^{-13}	1.18×10^{-11}
1—2	$s\leftrightarrow p$	3.28×10^{-11}	-5.62×10^{11}	2.41×10^{-11}	4.86×10^{-11}
	$p\leftrightarrow d$	5.95×10^{-11}	-4.78×10^{-11}	$3.0\ \times10^{-11}$	1.02×10^{-10}
	$d\leftrightarrow f$	8.32×10^{-14}	-3.79×10^{-13}	4.15×10^{-13}	-2.19×10^{-11}
	$f\leftrightarrow g$	2.95×10^{-13}	-5.95×10^{-14}	2.85×10^{-13}	3.21×10^{-12}
	$g\leftrightarrow h$	9.95×10^{-14}	4.72×10^{-14}	1.49×10^{-13}	-8.49×10^{-13}

calculated with the regularized V_{PNC} are for the most part approximately a factor of 2 smaller, as was true in the case of A_{pp} and A_{np}, and as one might expect. This is not always so, as one can see by comparing df and fg transitions for ρ magnetic. For long range π exchange only the sp transitions are noticeably reduced in magnitude when the regularized form of V_{PNC}^{π} is used. This is also in accord with our NN experience and intuition. In Table VII we quote values for the transitions composing $WGTGW$, which are to be compared with those in Table V for the angular momenta $L-L'$ equal to 0—1 and 1—2. The dominant ρ and ω $s\leftrightarrow p$ transitions essentially scale; the smaller, higher order transitions do not. The π exchange $s\leftrightarrow p$ and $p\leftrightarrow d$ transitions are slightly reduced in magnitude. A similar comparison can be made for the decomposition of A_{nd} in Table III and is shown in Table VIII. It is quite clear that the effect of simulated repulsion is not primarily a factor of 2 scaling. In particular, one should compare the ρ-electric terms in $WGTGW$ and $TGW+WGT$ or the π terms in $WGTGW$. The total asymmetries for the case with simulated repulsion are also given in Table VIII. In spite of the disparate modifications of the various transitions contributing to the values of A_{nd}, the reductions in the final results do not disagree with the intuitive result based upon the NN PNC calculations with and without repulsion. The value of $A_{nd}^{\text{Cab}}=-0.44\times10^{7}$ is about a factor of 4 smaller than the results discussed above with no simulated short-range repulsion; the corresponding value of $A_{pd}^{\text{Cab}}=-0.74\times10^{-7}$ is about a factor of 2 smaller than the nonregularized separable potential result and is in agreement with the experimental asymmetry, Eq. (33). However, we caution, once more, against assuming that short-range repulsion merely scales A_{Nd}.

TABLE VIII. Neutron-deuteron asymmetries calculated at 14.4 MeV incident nucleon energy in units of 10^{-7}, including the decomposition into contributions from ρ, ω, and π exchange according to Eq. (24) and the contribution (D) from the odd parity admixtures of the deuteron. These results are computed using the regularized PNC weak potential and should be compared with those of Table III.

Components	$A_{nd}^{\rho\,\text{el}}$	$A_{nd}^{\rho\,\text{mag}}$	A_{nd}^{ω}	$(A_{nd}^{\pi})^{\text{Cab}}$
Total	-0.224	-0.094	-0.276	0.149
D	-0.001	0.007	0.0003	0.030
T	-0.004	0.380	0.053	-0.011
$TGW+WGT$	0.046	0.027	0.058	0.003
$WGTGW$	-0.266	-0.508	-0.387	0.127

B. Energy dependence of A_{nd} and A_{pd}

The energy dependence of the theoretical estimate of A_{pp} is such that it appears that the optimal energy for making such a measurement is around 50 MeV, where A_{pp} is a maximum. With this in mind we have asked about the energy dependence of A_{nd} and A_{pd}. In Table IX we quote values for 14.4, 25, and 40 MEV incident neutrons. We point out that we retain all of the three-body partial waves necessary to reproduce the experimental total cross section. The energy dependence of the asymmetry A_{nd} as well as A_{pd} is strong, and very different from that for A_{pp}. This is again a result of the many cancellations among the various components, since there is markedly different energy dependence in the individual terms comprising the asymmetry. The quadratic terms and the Born term dominate the asym-

TABLE IX. Energy dependence of the $\vec{n}d$ and $\vec{p}d$ asymmetry in units of 10^{-7}. The various contributions are decomposed according to heavy meson or pion exchange. The energies are the laboratory energies of the incident neutron or proton.

E (MeV)	14.4	25	40
D_π	0.034	0.037	0.033
$D_{\rho\omega}$	0.030	0.044	-0.074
T_π	-0.013	-0.021	-0.008
$T_{\rho\omega}$	1.53	2.64	3.43
$(TGW+WGT)_\pi$	0.002	0.007	0.004
$(TGW+WGT)_{\rho\omega}$	0.478	0.592	0.547
$(WGTGW)_\pi$	0.137	0.163	0.126
$(WGTGW)_{\rho\omega}$	-3.72	-3.92	-2.37
A_{nd}^{Cab}	-1.52	-0.47	1.68
A_{pd}^{Cab}	-1.84	-0.84	1.39

metry; whereas the magnitude of the quadratic terms decreases with energy, the Born term increases in strength with energy.

This study of the energy dependence of A_{nd} was carried out with the separable s-wave potential model, which has no strong short-range repulsion. Because of this approximation, the magnitudes of A_{nd} are likely to be too large. However, we do believe that the qualitative features of the energy dependence are realistic. The $\vec{n}d$ and $\vec{p}d$ asymmetries change sign as the projectile energy ranges from 14 to 40 MeV, while the absolute values at both energies are roughly of equal magnitude.

VI. MODEL DEPENDECE

There is another Faddeev calculation of the parity nonconservation in Nd scattering of which we are aware, that of Desplanques, Benayoun, and Gignoux.[17] In that paper the strong interaction potential was local; both the Malfliet-Tjon s-wave potential model and the Reid-soft-core potential model, truncated to the $^1S_0$ and $^3S_1$-$^3D_1$ partial waves, were used. In order to compare their results with ours, we have used exactly the same weak interaction as was employed in Ref. 17 to generate the results for A_{pd} quoted in their Table II. (Note that their weak coupling constant f_π is a factor of 10 larger than our f_π^{Cab} and their $f_{\rho,\omega}$ is the negative of the one which we use here but corresponds to that of previous work.[16,20]) Such a comparison of the results obtained in the two independent investigations is of interest in understanding the model dependence of the calculation of parity nonconservation in this simple nuclear system, where exact calculations are possible.

In Table X results for A_{pd} are summarized for four different model calculations at an energy of approximately 14 MeV. They are separated into ρ, ω, and π contributions. The column labeled MT contains the results of Ref. 17 for the strong interaction model of Malfliet and Tjon. The column labeled Brown OM contains the results quoted in Ref. 17 for an optical model approximation to pd scattering combined with the weak interaction defined by Brown *et al.*[20] The column labeled Sep (Reg) contains the results from our separable potential calculation where we have utilized the regularized PNC weak potential. The last column labeled Sep contains the results of the separable potential calculation with *no* regularization of the weak interaction.

The MT results of Desplanques *et al.* differ in detail with our Sep and Sep (Reg) results, although they agree in sign and order of magnitude. For the ρ and ω contributions, there is no reason to expect any close agreement because of the many cancellations which occur among the various terms. The equality of the ρ and ω terms in the separable models is the result of a coincidental cancellation among the T, TGW, and $WGTGW$ terms, terms which differ substantially in their separate ρ and ω contributions. Furthermore, it is clear that the effect of repulsion on the short-range ρ and ω components is large, and one should anticipate that repulsion in the MT and Sep (Reg) models would lead to different effects. One could argue that the model dependence for the longer range π contribution should be less; i.e., the pion contribution to parity nonconservation in the scattering problem should be less sensitive to the short range properties of the strong interaction. However, one should also be cautious in this case. In the Sep (Reg) and Sep models, where we have a decomposition of the π contribution to A_{Nd}, the $WGTGW$ term dominates. Thus, the effect of off-shell (and on-shell) rescattering is large, and the dependence of the π contribution upon the short range behavior of the strong in-

TABLE X. Comparison of contributions to A_{pd} ($\times 10^7$) near 14 MeV arising from ρ, ω, and π (our neutral current assumption) exchange for the four models described in Sec. VI of the text.

	MT	Brown OM	Sep (Reg)	Sep
A_{pd}^{ρ}	0.85	0.61	0.32	0.86
A_{pd}^{ω}	0.13	0.11	0.28	0.82
A_{pd}^{π}	-0.90	-1.13	-1.49	-1.63
Total	0.08	-0.41	-0.89	0.05

teraction is significant. We view the results in Table X as an indication that extracting useful information concerning parity nonconservation in the nucleon-nucleon interaction from experiments on even simple nuclear systems ($A > 2$) will require a sophisticated treatment of all aspects of the nucleon-nucleon force and the non-negligible multiple scattering effects.

VII. SUMMARY AND CONCLUSIONS

The study of parity nonconservation in $\vec{N}d$ scattering offers a real possibility of obtaining information about the weak np amplitude. It is unfortunately the case that multiple scattering effects, such as those embodied in the quadratic ($WGTGW$) terms, tend to dominate the process, making simple approximate analysis schemes invalid. The cancellations that occur among the separate terms comprising the quadratic ($WGTGW$) and linear ($WGT + TGW$) contributions to A_{Nd} are quite complex. Controlled approximations to an exact formalism appear to be required. At 14 MeV incident energy, the lowest partial wave contributions dominate, as one would expect. However, the many cancellations between various terms in the calculation imply that higher partial waves should be retained in order to make definitive statements. The energy dependence of A_{Nd} differs significantly from that measured for A_{pp}. For A_{Nd} we obtain a sign change as the incident energy is increased from 14 to 40 MeV. Although our model is not suffficiently sophisticated to permit one to make quantitative statements concerning PNC effects in the $\vec{N}d$ system, comparison of our results at 14 MeV with those for local potential models of the strong interaction in that energy range indicates that the qualitative features of our calculation are reasonable. Thus, the strong energy dependence which we find for A_{Nd} is an interesting prediction. However, in conclusion it seems clear from this work that the extraction of useful information concerning the properties of the weak interaction from the study of PNC effects in the scattering of protons from few-nucleon (^{2}H, ^{3}He, and ^{4}He) systems will require a sophisiticated treatment of all aspects of the scattering problem in order to obtain quantitative results; simple approximations are not warranted.

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