# High-spin states in $^{28}$Si from variation after angular-momentum projection

W. Bauhoff*
I. Institut für Experimentalphysik, Universität Hamburg, D-2000 Hamburg 50, West Germany

H. Schultheis and R. Schultheis
Institut für Theoretische Physik, Universität Tübingen, D-7400 Tübingen, West Germany
(Received 18 October 1982)

Variations after angular momentum projection for the $0^+, \ldots, 10^+$ $(K=0)$ levels of the ground-state band and the excited prolate band in $^{28}$Si yield almost rotational extensions of both experimental bands, and a crossing of the oblate and prolate bands that should occur between the $8^+$ and $10^+$ levels. The band assignments of some recently observed levels are discussed.

[NUCLEAR STRUCTURE $^{28}$Si; variation after $0^+, \ldots, 10^+$ projection for oblate and prolate bands.]

A number of new experimental data on high-spin states in $^{28}$Si have become available very recently, viz., the observation of an excited prolate $0^+, \ldots, 6^+$ rotational band,[1] the extension[2] to $8^+$ of the well-known $0^+, \ldots, 6^+$ rotational ground-state band, further $8^+$ determinations without definite band assignments,[3,4] and a suggested backbending of the ground-state band associated with a newly observed low lying $10^+$ level.[4] The properties of excited bands in the *sd* shell have also received attention as a possible explanation[5] of "anomalons" observed in high-energy cosmic-ray tracks and in heavy-ion collisions at 2 GeV/nucleon. This has motivated us to study the structure of the oblate and prolate band in a variational calculation.

In $^{28}$Si a number of nuclear-structure calculations with various nucleon-nucleon forces have confirmed the existence of two minima at prolate and oblate deformation of the potential energy in the intrinsic frame. With few exceptions,[6–8] many-body studies of the bands associated with the prolate and oblate minimum have generally been restricted to projecting the state, which minimizes the energy of the intrinsic calculation, onto angular momenta $0^+, 2^+, 4^+, \ldots$. This implies two assumptions the validity of which is difficult to verify: Firstly, the band is taken to be strictly rotational without allowing the parameters of the states of the band to shift with increasing angular momentum. Secondly, this common intrinsic state is assumed to coincide with the minimum energy state of the unprojected variation. While such a procedure may yield reasonable approximations to the excitation energies (since the energy of the exact $0^+, 2^+, 4^+, \ldots$ solutions is stationary) they do not allow one to investigate any changes in the structure of the intrinsic wave function that may occur within the band with increasing angular momentum. In fact, the experimental prolate and oblate bands reveal considerable deviations from a strictly rotational $J(J+1)$ pattern. Therefore, and in order to determine possible extensions of the observed bands to higher angular momenta, we have studied both bands in the present note by allowing each angular-momentum projected state to vary independently.

In many calculations in $^{28}$Si the prolate minimum tends to have an excitation that is much below the experimental 6.691 MeV $0^+$ level to which it should correspond. In some cases the calculated prolate and oblate minima are almost degenerate or occur even in reversed order such that the calculated ground-state deformation is prolate. This problem is avoided in the present investigation by selecting an appropriate effective nucleon-nucleon interaction. We use the Brink-Boeker $C_1$ force[9] because it has previously been shown[10] to reproduce the level spacing between the prolate and oblate $0^+$ state rather accurately. The Hamiltonian also includes the exact Coulomb energy and the full center of mass term.

The method of this calculation is similar to that of Refs. 10 and 11 except that the Peierls-Yoccoz projection

$$P^J_{KM}=\frac{2J+1}{8\pi^2}\int d\Omega\,\langle JK|\mathcal{R}^+(\Omega)|JM\rangle\mathcal{R}(\Omega)$$

and the consecutive variation has also been performed for higher angular momenta $(K=0)$ up to the $10^+$ level in both the prolate and oblate band. For the intrinsic variational states Brink's microscopic alpha cluster model[12] has been utilized, viz., a Slater determinant of 1*s* harmonic-oscillator single-nucleon orbitals with quartet coupling, that are centered around given positions $\vec{R}$, and have a common oscillator width $b$. The parameters $\vec{R}$ and $b$ determine the wave function of the 28 particle system

$$\Phi(\vec{x}_1, \ldots, \vec{x}_{28}; \vec{R}_1, \ldots, \vec{R}_7, b) \ .$$

In $^{28}$Si this amounts to a total of 16 independent variational parameters. In order to reduce the number of parameters in such a calculation it is customary to perform the variation only under certain symmetry restrictions. Rather than using an *ad hoc* symmetry constraint we have first carried out an unprojected variation of all the 16 parameters, and then restricted the variation after projection to the point symmetry of the unprojected result, i.e., $D_{5h}$ symmetry for the oblate ground-state band, and $D_{3h}$ for the excited prolate band.

Figure 1 shows the calculated variation-after-projection bands, and the spectrum that follows from projecting the intrinsic state with minimum $0^+$ energy onto $2^+, 4^+, \ldots, 10^+$ without further variation. For a perfectly rotational band all intrinsic states would coincide, and both types of calculation should lead to identical results. The figure indicates, however, that increasing deviations from a strictly rotational pattern occur in the higher levels of both bands. In the ground-state band such deviations set in at lower angular momentum than in the excited band. In all cases the deviations are rather small, and the structure of the $2^+, \ldots, 10^+$ intrinsic states remains close to that of the $0^+$ band head. The calculated oblate ground-state band has quadrupole moments and rms radii between $Q_0 = -73$ fm$^2$, $R_{\rm rms} = 3.21$ fm for the $0^+$ state, and $Q_0 = -65$ fm$^2$, $R_{\rm rms} = 3.19$ fm for the $10^+$ state. The corresponding values for the excited prolate band are $Q_0(0^+) = 100$ fm$^2$, $R_{\rm rms}(0^+) = 3.32$ fm and $Q_0(10^+) = 120$ fm$^2$, $R_{\rm rms}(10^+) = 3.46$ fm.

We note that the cluster structure does not change with increasing angular momentum within each band, and remains close to that of the band head. The oblate ground-state band has sizable alpha clustering (consisting of a ring of five alphas and two more alphas on the axis; for a density plot, see Ref. 10). In the prolate band the alpha clusters form two parallel

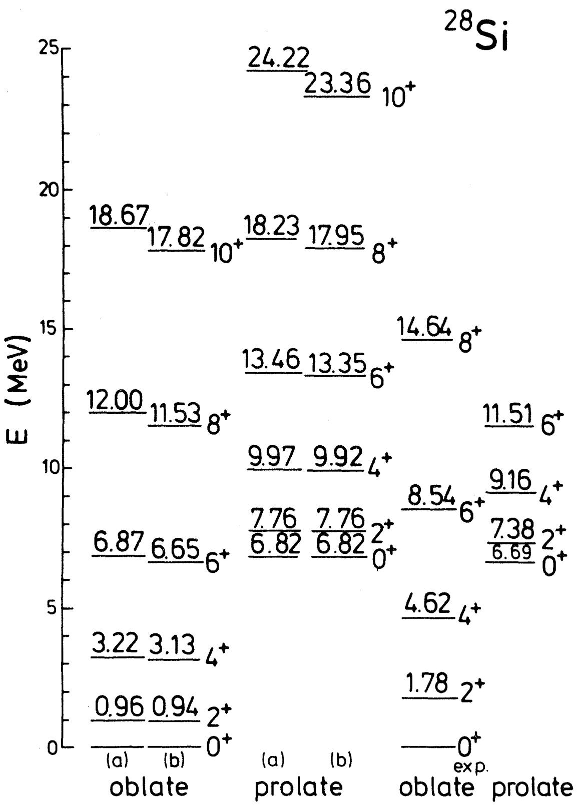


FIG. 1. Calculated prolate and oblate bands in $^{28}$Si for the $C_1$ force in comparison with the experimental data. The calculated levels result from (a) $J^\pi$ projection of the minimum-energy $0^+$ state, and (b) additional independent variations for each $J^\pi = 2^+, \ldots, 10^+$. All experimental levels (Refs. 1 and 2) plotted have definite band assignments.

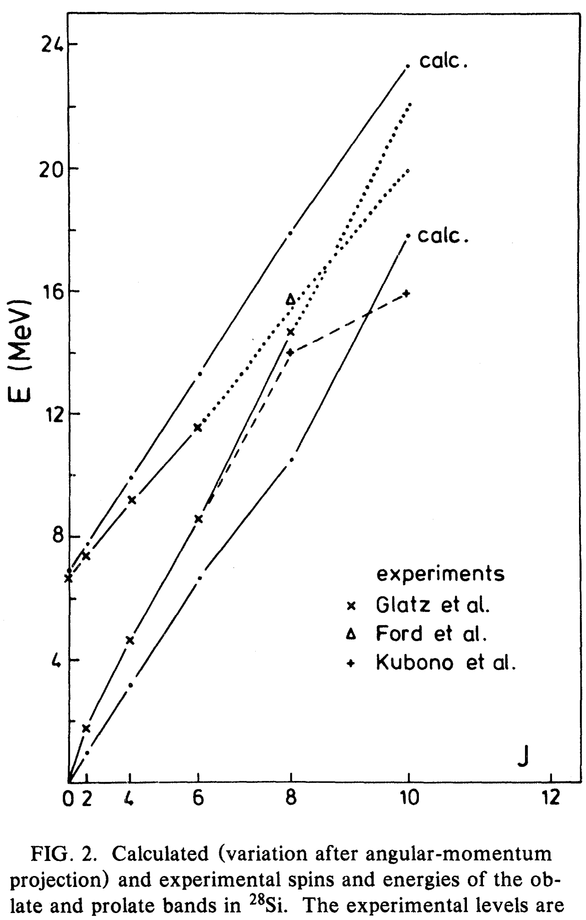


FIG. 2. Calculated (variation after angular-momentum projection) and experimental spins and energies of the oblate and prolate bands in $^{28}$Si. The experimental levels are connected by solid lines where the gamma transitions are known. For the dotted extrapolations the calculated moments of inertia are readjusted to the known experimental values. Data of Ford *et al.* (Ref. 3), Glatz *et al.* (Refs. 1 and 2), and Kubono *et al.* (Ref. 4) are plotted.

equilateral triangles with one alpha inbetween ($D_{3h}$ point symmetry). The prolate density distribution, however, has less pronounced clustering than the ground-state band. This is different from the predicted shape-isomeric band in $^{32}S$, where the calculations yield substantial $^{16}O + ^{16}O$ clustering[13] for the intrinsic state.

The calculated rotational constants deviate from the experimental ones: for the ground-state band it is too small (a feature common to all microscopic calculations in $^{28}Si$), whereas it is too large for the excited prolate band. This has to be corrected if the experimental position of new high-spin states are to be extrapolated. From a readjustment of the calculated energies to the known experimental levels, the following new levels can be predicted: The ground-state band is expected to have a $10^+$ level at about 22 MeV, and the prolate band should continue with an $8^+$ and a $10^+$ level at about 15.5 and 20 MeV, respectively. So the expected prolate $8^+$ level is very close to the experimental $8^+$ level at 15.8 MeV which has been attributed[3] to the ground-state band. This assignment, however, was made before the excited $0^+, \ldots, 6^+$ band[1] had been observed. It was based mainly on the fact that the $8^+$ level was the lowest one known in $^{28}Si$ at that time, and not on enhanced electromagnetic transition rates within the band. From the levels known now it is tempting to consider the $8^+$ level rather as a member of the prolate band. It should be noted that the two bands are expected to cross between $J = 8$ and $J = 10$. It would be very interesting to confirm this crossing experimentally.

Recently, Kubono *et al.*[4] have observed a 15.97 MeV level with a tentative $10^+$ assignment. Because of its low energy they have attributed the level to the ground-state band, which then would show a marked backbending (see Fig. 2). We note, that our calculation gives no indication of such a sizable change in the moment of inertia between the oblate $8^+$ and $10^+$ states. The experimental $10^+$ level may, of course, belong to a third band that crosses the ground-state band. This has been suggested in Refs. 14 and 15, where an attempt has been made to utilize a combination of the liquid-drop, Strutinsky, and cranking model in spite of the small particle number of $^{28}Si$. However, the experimental spin assignment of the 15.97 MeV level is not quite definite, and an $8^+$ rather than $10^+$ spin has not been ruled out in Ref. 4. A spin-8 level at 15.97 MeV is close to the $8^+$ level at 15.8 MeV observed earlier by Ford *et al.*[3] and would then also fit into our predicted extension of the oblate band. However, a definite band assignment cannot be made on the basis of the existing data, and gamma transition measurements would be highly desirable for a resolution of the ambiguity.

The work was supported in part by the Deutsche Forschungsgemeinschaft.

*Now at School of Physics, University of Melbourne, Parkville, Victoria, Australia 30S 2.

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