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# Low-spin states in even Po and Rn isotopes and the interplay between collective and quasiparticle configurations

A. Zemel
*Kernfysisch Versneller Instituut, Rijksuniversiteit Groningen, 9747 AA Groningen, The Netherlands*

J. Dobes
*Nuclear Physics Institute of the Czechoslovak Academy of Science, 25068 Rez, Czechoslovakia*
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Low-spin states in even Po and Rn isotopes are studied within the framework of the interacting-boson-approximation + two-quasiparticle model. The irregular behavior of the $4^+$ levels is shown to result from an interplay between collective, quasiproton, and quasineutron states. Some predictions are made for the corresponding Ra isotopes.

[NUCLEAR STRUCTURE $^{200-210}$Po, $^{202-212}$Rn, $^{204-214}$Ra; calculated levels, $B(E2)$. IBA + two-quasiparticle model.]

## I. INTRODUCTION

Most of the experimental and theoretical studies of Po and Rn isotopes below shell closure at $N=126$ have been devoted to the discovery and understanding of the high-spin isomers that are abundant in these nuclei. In particular, the $(h_{9/2})^n$ proton configuration assignment to the lowest $8^+$ isomers was confirmed by precise magnetic moment measurements both for $^{204-210}$Po (Refs. 1 and 2) and for $^{206-214}$Rn.[3] Now states with much higher spins have been discovered and discussed.[4–12]

The nature of the lower spin states in these nuclei is, however, much less understood. Nagamiya and Inamura[13] noticed the similarity between the excitation energies of the $2^+$ states in $^{200-208}$Po and those of the corresponding Pb isotopes, and remarked that these states do not have a pure $(\pi h_{9/2})^n$ structure, in contrast to the yrast $6^+$ and $8^+$ states. Recently, Poletti *et al.*[12] stressed the importance of both proton and neutron configurations in the structure of the $4^+$ states of $^{206-210}$Rn and attributed the interesting trend in their feeding properties to the interplay between these configurations. A similar mixing was proposed to determine the structure of the $2^+$ states in $^{208}$Po.[14] A collective interpretation has been attempted by Ritchie *et al.*,[15] who compared the experimental level schemes of $^{204-208}$Rn to an interacting-boson-model (IBA-1) calculation. These approaches have, however, some difficulties in describing the detailed features of the states. Thus, the excitation energies of the $4^+$ levels assigned to proton configurations in Ref. 12 increase with neutron number, in contrast to the behavior of the corresponding $6^+$ and $8^+$ states. On the other hand, the IBA alone cannot reproduce the irregular behavior of the excitation energies of the $4^+$ levels (see Figs. 1 and 2), or the observed peculiarities in their feeding and decay.[12,15]

It appears, therefore, that both quasiparticle and collective aspects should be considered in order to describe the low-spin states properly. In fact, the weak coupling method, in which states from the $(\pi h_{9/2})^n$ configuration are coupled to a phonon space, has been applied to $^{206,208}$Po,[16,17] but the results fail to describe the configuration mixing in the $4^+$ states[16] and the correct $8^+$-$6^+$ energy differences.[13] Similarly, a quadrupole interaction between the quasiproton states and the core has been invoked to calculate the excitation energies of the high-spin isomers in these nuclei.[7–9]

A detailed and consistent interpretation of the systematics of low-spin states in the even Po and Rn isotopes is not yet available. In the following, an attempt to provide such an interpretation within the framework of the IBA + two quasiparticles is described.

## II. IBA + TWO QUASIPARTICLES

The interacting boson model is successful in describing phenomenologically low-lying collective states. In order to extend the description to higher spins and excitation energies, a coupling of the collective core to a two quasiparticle band has been suggested.[18] This idea has been further developed and applied to Hg,[19] Ba, and Ce isotopes.[20] Since the quasiparticle features seem to play an important role

 

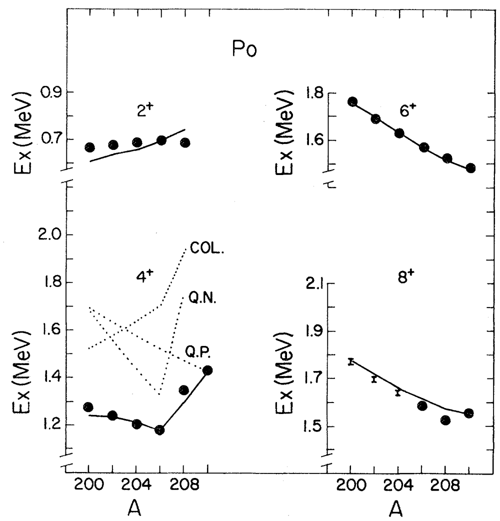


FIG. 1. Experimental (full points) and calculated (solid lines) yrast states in the even Po isotopes. The error bars denote unobserved transitions, assumed to have $E_\gamma < 14$ keV (Ref. 13). The dotted lines display the positions of the collective (COL), quasineutron (Q.N.), and quasiproton (Q.P.) $4^+$ states, when the mixing parameter $\beta_\rho$ is set to zero. Note the zero suppression in the energy scales.

in the low-spin states of the even Po and Rn isotopes, it may be of interest to test whether this new extension of the IBA model might provide a good description of these states. In this work we follow

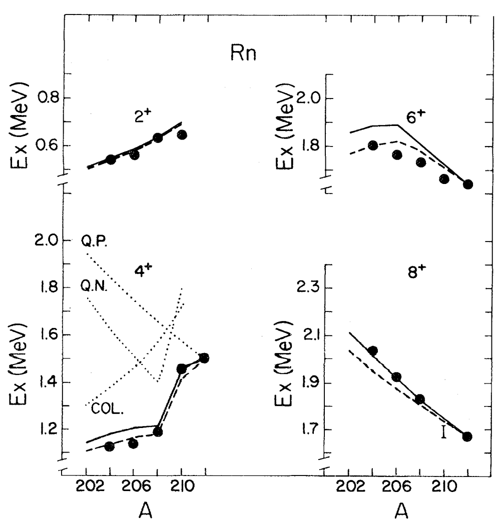


FIG. 2. Experimental (full points) and calculated yrast states in the even Rn isotopes. The solid and dashed lines represent the calculations within cases (a) and (b), respectively (see text). The dotted lines display the positions of the collective (COL), quasineutron (Q.N.), and quasiproton (Q.P.) $4^+$ states when the mixing parameter $\beta_\rho$ is set to zero. Note the zero suppression in the energy scales.

the approach and methods of Ref. 20, which treats the collective states within the proton-neutron IBA (IBA-2) formalism. We repeat here briefly the salient features of the calculation.

The model space includes states with $N_\pi$ proton bosons and $N_\nu$ neutron bosons (the usual IBA-2 space) and states in which either one proton boson or one neutron boson is broken to form a quasiparticle pair. The quasiparticles are assigned to an orbital with spin $j$ and allowed to couple to a total quasiparticle spin $J=4,6,\ldots,2j-1$. The mixing between states with different numbers of bosons is introduced through a boson-number-changing term in the quadrupole operator:

$$Q_\rho = Q_\rho^B + \alpha_\rho[a_\rho^\dagger \tilde{a}_\rho]^{(2)} + \beta_\rho[[a_\rho^\dagger a_\rho^\dagger]^{(4)}\tilde{d}_\rho]^{(2)} - \beta_\rho[d_\rho^\dagger[\tilde{a}_\rho\tilde{a}_\rho]^{(4)}]^{(2)}, \quad (\rho=\pi,\nu)$$

where

$$Q_\rho^B = (d_\rho^\dagger s_\rho + s_\rho^\dagger \tilde{d}_\rho) + \chi_\rho[d_\rho^\dagger \tilde{d}_\rho]^{(2)} ,$$

$a_\rho^\dagger$ is the quasiparticle creation operator, and $s_\rho^\dagger$ and $d_\rho^\dagger$ are $s$-boson and $d$-boson creation operators. The total Hamiltonian of the complex system is written as

$$H = \epsilon(d_\pi^\dagger \tilde{d}_\pi + d_\nu^\dagger \tilde{d}_\nu) + \kappa Q_\pi \cdot Q_\nu + H_F ,$$

where the fermion part, $H_F$, consists of single-particle energies ($\epsilon_\rho$) and two body interactions ($V_\rho^F$):

$$H_F = \epsilon_\pi \hat{n}_\pi + \epsilon_\nu \hat{n}_\nu + V_\pi^F + V_\nu^F ,$$

where $\hat{n}_\pi$ ($\hat{n}_\nu$) are the number operators for protons (neutrons). As described in Ref. 20, the boson part of the Hamiltonian is diagonalized first within the spaces generated by $(N_\pi,N_\nu)$, $(N_\pi-1,N_\nu)$, and $(N_\pi,N_\nu-1)$ bosons using the standard IBA-2 program NPBOS. The lowest states obtained for the last two systems are then coupled to the two quasiprotons (neutrons). Finally, the full Hamiltonian is diagonalized for the complex system.

Evidently, the number of parameters involved in the calculation is large, rendering any attempt to obtain them through a fitting procedure meaningless. Fortunately, most of the parameters can be directly related to experimental data or obtained from previous work. The number of free parameters can thus be reduced to a minimum.

The parameters of the fermion Hamiltonian $H_F$ for the $(\pi h_{9/2})^2$ configuration were taken from the excitation energies of the $4^+$, $6^+$, and $8^+$ states in the semimagic nuclei $^{210}$Po and $^{212}$Rn. From the measured magnetic moments, the structure of these states is known to be rather pure. The situation for the $(\nu f_{5/2})^{-n}$ configuration is much more complex. The $4^+$ states in $^{198-206}$Pb isotopes are known to have appreciable mixing with other configurations as is evident from the measured magnetic moments,[21] or from previous shell-model calculations.[22] Assuming that the structure of the quasineutron states in the corresponding Po and Rn isotopes is not very different, we obtained the parameters of the neutron part of $H_F$ for each neutron number from the excitation energy of the $4^+$ state in the appropriate Pb isotone. A reference to these states as belonging to the $(\nu f_{5/2})^{-n}$ configuration should therefore be considered as only a nomenclature rather than a real physical assignment. Similarly, the $d$ boson energy $\epsilon$ for each Pb isotope [the space with $(N_\pi-1,N_\nu)$ bosons in the Po calculation] was derived from the experimental $2^+$ energies.

The parameter $\alpha_\pi$ (or rather $\kappa\alpha_\pi$) which determines the strength of the boson-quasiproton quadrupole interaction was estimated from Ref. 23, where the states built on an $h_{9/2}$ proton in odd Au isotopes were considered. Based on microscopic calculations,[24] the $Z$ dependence of this parameter was assumed to be proportional to the occupation number $(\Omega-2N)/\Omega$, where $\Omega=2j+1$, and $N$ is the number of protons occupying the orbital $j$ (1, 2, and 4 for Au, Po, and Rn, respectively). $\alpha_\nu$, which has only a minor effect on the calculated level schemes, was kept constant for all the isotopes considered and was taken to be equal to the $\alpha_\pi$ value in Po. The values for the $Q^B$ parameter, $\chi_p$, were extracted from a fit to the levels of the even Os and Pt isotopes,[25] using extrapolated values for the heavier isotopes.

Only three parameters are left to be determined, namely the $d$ boson energy $\epsilon$, the quadrupole coupling constant $\kappa$, and the coefficient of the boson-number-changing term $\beta_p$ (assumed to be equal for protons and neutrons). These parameters were obtained by a fit to the $2^+$, $4^+$, and $6^+$ states in $^{206}$Po and used in the calculations for all the other Po and Rn isotopes. Using the techniques of Ref. 24, the $Z$ dependence of $\beta_\pi$ is found to be the same as that of $\alpha_\pi$, namely $\beta_\pi$ is proportional to $(\Omega-2N)/\Omega$. The value of $\beta_\pi$ for the Rn isotopes was modified accordingly. To test the sensitivity of the model to this parametrization we repeated the calculations assuming that the variation is a factor of 2 smaller (i.e., taking for $\alpha_\pi$ and $\beta_\pi$ the average of the Po and the previous Rn values). The other parameters were not varied as functions of $N_\pi$ or $N_\nu$. They are summarized in Table I. We note that the ratio between the derived values of $\alpha_\pi$ and $\beta_\pi$ is quite close to the microscopic theory[24] expectations:

$$\beta_\pi=\sqrt{10\cdot 9}\begin{Bmatrix} \frac{9}{2} & \frac{9}{2} & 4 \\ 2 & 2 & \frac{9}{2} \end{Bmatrix}\alpha_\pi=-0.64\alpha_\pi\ .$$

(The sign of $\beta_\pi$ could not be determined since both energies and transition probabilities depend only on its absolute value.)

## III. RESULTS

The results of the calculations are compared with the experimental Po and Rn levels in Figs. 1 and 2. Note that the zero is considerably suppressed in all scales, to make the comparison more transparent. For the Rn isotopes, two calculated curves are displayed corresponding to the two values of the parameters $\alpha_\pi$ and $\beta_\pi$, which are taken to be (a) modified relative to the value derived for $^{206}$Po according to the $(\Omega-2N)/\Omega$ dependence (solid line), and (b) equal to the averages of the Po values and the Rn values of case (a) (dashed line). The two cases will be compared below. The agreement between the calculated and observed levels is good, taking into account the crudeness of the approximations and the fact that the parameters were not allowed to vary between the different isotopes. Of particular interest is the agreement obtained for the $4^+$ states, which show a markedly different behavior as compared to the $2^+$, $6^+$, and $8^+$ states. This behavior results from the interplay and mixing between the three configurations, as is evident from the dotted lines in the figures, which show the positions of the states corresponding to each configuration when the mixing parameter $\beta_p$ is set to zero. Clearly, no configuration can alone account for the

TABLE I. Values of parameters in the Hamiltonian.

| Element | $\epsilon$ (MeV) | $\kappa$ (MeV) | $\alpha_\nu$ | $\beta_\nu$ | $\alpha_\pi$ | $\beta_\pi$ | $\chi_\pi$ |
|---|---|---|---|---|---|---|---|
| Po | 0.9 | −0.187 | 1.28 | −1.06 | 1.28 | −1.06 | −1.0 |
| Rn [case (a)] | 0.9 | −0.187 | 1.28 | −1.06 | 0.43 | −0.35 | −1.0 |
| Rn [case (b)] | 0.9 | −0.187 | 1.28 | −1.06 | 0.855 | −0.705 | −1.0 |
| Ra | 0.9 | −0.187 | 1.28 | −1.06 | −0.43 | 0.35 | −1.0 |
| Neutron number | 116 | 118 | 120 | 122 | 124 | | |
| $\chi_\nu$ | 0.95 | 1.05 | 1.15 | 1.22 | 1.30 | | |

observed trend.

In contrast, the structure of the $2^+$ (collective) and $8^+$ (quasiprotons) states is found to have a pure nature. This is also true for the $6^+$ states in the Po isotopes, and is consistent with the near constancy of the measured magnetic moments.[1–3]

Consider now those states for which some deviations occur. The calculated slope for the $2^+$ states in the Po isotopes is a clear example. The deviations may be due to the simplified boson Hamiltonian employed and to the fact that only one proton boson is available for these nuclei. The better fit obtained for the $2^+$ states in the Rn isotopes (where $N_\pi=2$), except for $^{210}$Rn (where $N_\nu=1$) supports this view. For the semimagic nuclei (with $N_\nu=0$) the calculation of the collective states is, of course, even less valid. The higher lying $2_1^+$ states found in these nuclei are believed to have a quasiparticle nature and cannot therefore be described in the present model which considers pairs with $J=4,6,\ldots,2j-1$ only.

The $6^+$ states in the Rn isotopes show some deviations too. It appears that the $Z$ dependence assumed for the parameters $\alpha_\pi$ and $\beta_\pi$ [case (a)], might be too strong, underestimating the amount of mixing necessary to describe these states. Alternatively, the lowering of the experimental energies might be due to protons in the $(f_{7/2})$ shell, which are claimed to play an important role away from the closed neutron shell.[11] In $^{204}$Rn the corresponding $8^+$ state has also been observed.[11] This configuration is not included in our model space. On the other hand, the dashed line [case (b)] is in better agreement with the experimental $4^+$ and $6^+$ states, but somewhat underestimates the $8^+$ energies. A more complete calculation, in which the quasiparticles are allowed to occupy more than one shell, and a detailed microscopic theory which might provide for slightly different behaviors of $\alpha_\pi$ and $\beta_\pi$ may be required in order to clarify this point.

In the framework of this model, transition probabilities can also be calculated and compared with experiment. $B(E2)$ values were obtained along the lines described in Ref. 20, including the (small) contribution of the boson-number-changing part of the quadrupole operator. The proton effective charge $e_\pi^F=26\ e\,\mathrm{fm}^2$ was derived from the measured half-life of the $8^+$ isomer in $^{210}$Po, which is in agreement with the value deduced from the quadrupole moment of $^{209}$Bi.[2] The same value was assumed for the neutron effective charge, but the results for all transitions of interest are insensitive to this choice. The boson effective charge $e_\pi^B=e_\nu^B=18.2\ e\,\mathrm{fm}^2$ was taken from the fit to $E2$ transitions in Th isotopes.[26]

The results for the $8^+$-$6^+$ transition in the Po isotopes are compared with the experimental values in Table II. In most cases, the transition energies are not yet determined. However, as has been discussed by Yamazaki,[17] the conversion process dominates the decay below 100 keV, and the conversion emission probability is insensitive to the transition energy. The relation between $B(E2)$ values and half-lives behaves like a step function, and given that the transition energy is within certain limits, $B(E2)$ can be deduced from the measured $T_{1/2}$. The experimental values in Table II were obtained assuming 4 keV $<E_\gamma<$ 14 keV (region II of Ref. 17). $^{200}$Po may be an exception, as the apparent reduction of its $B(E2)$ (210 $e^2\mathrm{fm}^4$ as compared to 400 $e^2\mathrm{fm}^4$ in $^{202}$Po) is unlikely to be correct. The transition energy may be below the binding energy of the next $(3p_{1/2})$ atomic level [$E(M_2)=3.85$ keV], or the half-life value of Ref. 13 is much too large (as was found to be the case for $^{202}$Po, Refs. 2 and 6). One can see that the gradual increase of the $B(E2)$ values with boson number is reproduced by the model. A similar comparison for the Rn isotopes is hard to perform because many data are lacking or conflicting. Both calculation and experiment (e.g., Ref. 11) agree with the trend of increasing $B(E2)$ with increasing boson number.

TABLE II. $B(E2)$ values for $8^+\to6^+$ transitions in Po isotopes.

| $A$ | $B(E2)$ Experiment | ($e^2$fm$^4$) Theory | Ref. |
|---|---|---|---|
| 200 | 210[a] | 343 | 13 |
| | 370[b] | | |
| 202 | 400[a] | 287 | 2,6 |
| 204 | 280[a] | 235 | 1 |
| 206 | 190[a] | 179 | 1 |
| 208 | 108 | 131 | 27 |

[a]Assuming $E(M_2)<E_\gamma<E(L_3)$, where $E(M_2)$ and $E(L_3)$ are the binding energies of the $3p_{1/2}$ and $2p_{3/2}$ atomic levels, respectively (see text).
[b]Assuming $E(M_2)>E_\gamma$.

It is also interesting to see whether our model can reproduce the surprising trend observed for the branching ratio of the $E2$ transitions from the $6^+$ state to the two $4^+$ states in $^{206-210}$Rn.[12] In $^{210}$Rn the $6^+$ level decays to both $4^+$ states with equal $B(E2)$ values, whereas in $^{208}$Rn only the second $4^+$ level is fed, in spite of the large energy factor. Such a hindrance of the transition to the lower $4^+$ state does not occur in $^{206}$Rn. In fact, the corresponding $\gamma$ ray is the only decay mode observed for the $6^+$ level in this nucleus.

In Table III we present the $B(E2)$ ratios for these transitions as calculated using the parameters of cases (a) and (b). The results of case (a) agree well with the observed trend. It is interesting to note

TABLE III. Branching ratios for $6^+ \to 4^+$ transitions in Rn isotopes.

| $A$ | $\frac{B(E2)(6_1^+ \to 4_1^+)}{B(E2)(6_1^+ \to 4_2^+)}$ Theory (a) | Theory (b) | Experiment (Ref. 12) |
|---|---|---|---|
| 210 | 0.74 | 1.16 | 1.0 |
| 208 | 0.1 | 1.0 | $<0.1$[a] |
| 206 | 2.9 | 12.8 | $>0.01$[a] |

[a]Only one branch has been observed. Limit quoted refers to the $E_\gamma{}^5$ factor.

that the anomaly in $^{208}$Rn is not due to cancellation of contributions with oppostie phases or to a pure quasineutron nature of the $4_1^+$ state. This state has an appreciable collective component, but the hindrance of the $B(E2)$ value is caused by the smallness of the quasiproton component (the $6^+$ level has a nearly pure quasiproton structure). In $^{210}$Rn both $4^+$ states have comparable two quasiproton components, yielding $B(E2)$ values (125 and 169 $e^2$fm$^4$) that agree in absolute values with the experimental results [115(14) and 114(12) $e^2$fm$^4$, respectively). The results of calculation (b) fail to reproduce the $^{208}$Rn value, reflecting the sensitivity of this branching ratio to the different amounts of mixing introduced in the two cases.

The Ra isotopes (with $N_\pi = 3$) could provide a further test of the model. Again, the quasiproton parameters can be obtained from the experimental levels of $^{214}$Ra. The results of the calculations [within case (a)] are displayed in Fig. 3. There is a strong mixing in the $6^+$ states, but not in the $8^+$ states. The rapid increase in the $8^+$-$6^+$ transition energies is counterbalanced by a large reduction of the $B(E2)$ values, due to the small quasiproton components calculated for the $6^+$ states and to mutual cancellation of quasiparticle and collective contributions of opposite phases. Thus, long half-lives are still expected. The large $8^+$-$6^+$ energy differences also suggest that states outside our model space (such as states with $f_{7/2}$ protons[11]) may become the yrast $8^+$ states in the lighter isotopes. Unfortunately, the lack of experimental data prevents a test of the above predictions.

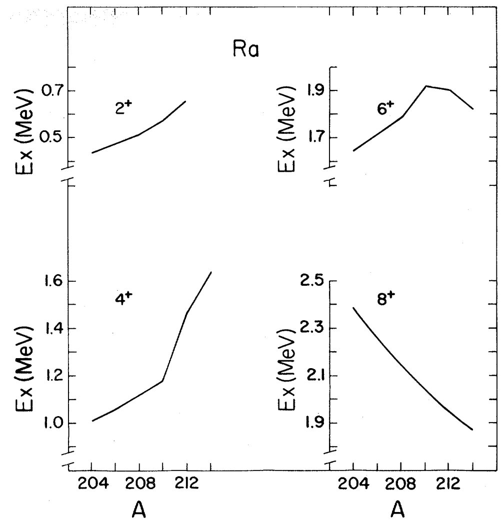


FIG. 3. Calculated levels in the even Ra isotopes within case (a). Note the zero suppression in the energy scales.

In summary, the IBA + two quasiparticle model has been shown to account well for the properties of the lower spin states in the even Po and Rn isotopes. Collective as well as quasiproton and quasineutron configurations were found to be important in the structure of the $4^+$ states in these nuclei. A similar interplay is believed to take place at higher spins in many backbending nuclei. It is hoped that calculations of the type described above may contribute to our understanding of the roles of the different quasiparticle bands in the backbending phenomenom.

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[1]S. Nagamiya, Y. Yamazaki, O. Hashimoto, T. Nomura, K. Nakai, and T. Yamazaki, Nucl. Phys. A211, 381 (1973).
[2]O. Hausser, T. K. Alexander, J. R. Beene, E. D. Earle, A. B. McDonald, F. C. Khanna, and I. S. Towner, Nucl. Phys. A273, 253 (1976).
[3]K. H. Maier, D. J. Decman, H. Grawe, H. Haas, and W.-D. Zeitz, Hyp. Int. 9, 87 (1981), and references therein.
[4]B. Fant, Phys. Scr. 4, 175 (1971).

[5]K. Wikstrom, I. Bergstrom, J. Blomqvist, C. J. Herrlander, B. Fant, and V. Rahkonen, Phys. Scr. 10, 292 (1974).

[6]H. Beuscher, D. R. Zolnowksi, D. R. Haenni, and T. T. Sugihara, Phys. Rev. Lett. 36, 1128 (1976).

[7]J. Blomqvist, I. Bergstrom, C. J. Herrlander, C. G. Linden, and K. Wikstrom, Phys. Rev. Lett. 38, 534 (1977).

[8]I. Bergstrom, J. Blomqvist, C. J. Herrlander, J. Hattula, O. Knuuttila, E. Luikkonen, and V. Rahkonen, Z. Phys. A 287, 219 (1978).

[9]V. Rahkonen, B. Fant, C. J. Herrlander, and K. Honkanen, University of Jyväskylä (JYFL) Annual Report, 1981, p. 79.

[10]D. Horn, O. Hausser, T. Faestermann, A. B. McDonald, T. K. Alexander, J. R. Beene, and C. J. Herrlander, Phys. Rev. Lett. 39, 389 (1977).

[11]D. Horn, C. Baktash, and C. J. Lister, Phys. Rev. C 24, 2136 (1981).

[12]A. R. Poletti, G. D. Dracoulis, and C. Fahlander, Phys. Rev. Lett. 45, 1475 (1980), A. R. Poletti, G. D. Dracoulis, C. Fahlander, and I. Morrison, Nucl. Phys. A380, 335 (1982).

[13]S. Nagamiya and T. Inamura, Nucl. Phys. A182, 84 (1972).

[14]T. S. Bhatia, T. R. Canada, P. D. Barnes, R. Eisenstein, C. Ellegaard, and E. Romberg, Phys. Rev. Lett. 30, 496 (1973).

[15]B. G. Ritchie, F. T. Avignone, H. K. Carter, R. L. Mlekodaj, and E. H. Spejewski, Phys. Rev. C 23, 1717 (1981).

[16]W. J. Treytl, E. K. Hyde, and T. Yamazaki, Nucl. Phys. A117, 481 (1968).

[17]T. Yamazaki, Phys. Rev. C 1, 290 (1970).

[18]A. Gelberg and A. Zemel, Phys. Rev. C 22, 937 (1980).

[19]I. Morrison, A. Faessler, and C. Lima, Nucl. Phys. A372, 13 (1981).

[20]N. Yoshida, A. Arima, and T. Otsuka, Phys. Lett. 114B, 86 (1982), and references therein.

[21]W. Thomas, K. Kroth, and H. R. Zilligen, Z. Phys. A 280, 371 (1977).

[22]C. Roulet, G. Albouy, G. Auger, J. M. Lagrange, M. Pautrat, K. G. Rensfelt, H. Richel, H. Sergolle, and J. Vanhorenbeeck, Nucl. Phys. A323, 495 (1979).

[23]R. Bijker and A. E. L. Dieperink, Nucl. Phys. A379, 221 (1982).

[24]O. Scholten, Ph.D. thesis, University of Groningen, 1980 (unpublished); O. Scholten and A. E. L. Dieperink, in *Interacting Bose-Fermi Systems in Nuclei,* edited by F. Iachello (Plenum, New York, 1981), p. 343.

[25]R. Bijker, A. E. L. Dieperink, O. Scholten, and R. Spanhoff, Nucl. Phys. A344, 207 (1980).

[26]O. Scholten, in *Interacting Bosons in Nuclear Physics,* edited by F. Iachello (Plenum, New York, 1979), p. 17.

[27]O. Dragoun, V. Brabec, A. Mastalka, A. Kovalik, M. Rysavy, M. Ya. Kuznetsova, and Yu.V. Norseev, Nucl. Phys. A391, 29 (1982).