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Surface alpha clustering

I. Rotter
Zentralinstitut für Kernforschung, Rossendorf, DDR-8051 Dresden, German Democratic Republic
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The problem of alpha decay is discussed on the basis of a theory which describes discrete and continuous states in a unified manner. A formula for numerical calculations is given in which configurational mixing as well as channel coupling is taken into account. The R-matrix approximation is shown to be justified if the width is spread over a small number of decay channels. Generally, renormalization of the wave function is necessary if a factorization of the width is assumed. The importance of channel coupling for the case of a small reduced width is discussed.

[RADIOACTIVITY Continuum shell model, spectroscopic factors, influence of channel coupling on absolute values of partial widths.]

I. INTRODUCTION

The problem of understanding alpha decay of nuclei has stood since the early days of nuclear physics. A great number of experimental investigations, not only of the very alpha decay but also of reactions with transfer of alpha particles between two nuclei or with knockout of alpha particles out of nuclei, have been performed. Theoretical investigations on alpha decay are based on the formula for the decay constant or lifetime of the parent nucleus. The problem concerns the fact that a discrete state of the parent nucleus needs to be considered, the coupling of which to the surrounding continuous states, formed by the residual nucleus and the alpha particle, determines its lifetime. Therefore, both discrete and continuous states need consideration with comparable accuracy although they show a very different mathematical behavior.

Most calculations of alpha widths were performed on the basis of the R-matrix theory. The formula for the width is approximated, in these calculations, by the product of a spectroscopic factor and a penetration factor. While the spectroscopic factor is defined as the overlap of the wave function of the parent nucleus with the wave function of the two residual nuclei in a bound state, the penetration factor describes the penetration of the performed alpha particle through the barrier. The spectroscopic factor describes the nuclear structure properties of the nuclei and can be calculated on the basis of the existing nuclear structure models with high accuracy. The penetration factor is calculated by means of the Coulomb wave functions in a comparably much worse approximation.

Numerical calculations have shown that the accuracy in the description of the nuclear structure wave functions has a large influence on the alpha widths calculated. Configuration mixing has been shown to play an important role, especially for the alpha widths of heavy nuclei since they are small in absolute value; see Refs. 1–6. Despite much effort to improve the nuclear structure wave functions of the parent and residual nuclei, the calculated alpha widths of heavy nuclei are generally still too small in absolute value (see Refs. 7 and 8).

For this reason, the alpha widths are attempted to be calculated without using R-matrix theory, i.e., directly from the matrix elements connecting initial and final states of the decay.[9–13] The disadvantage of these calculations consists in the fact that, with the exception of the method,[13] the nuclear structure wave functions are calculated with less accuracy than in the R-matrix theory calculations in which the proved wave functions of the nuclear structure models are used. Moreover, in the numerical non-R-matrix calculations channel coupling is not taken into account although the theory allows its inclusion. The results obtained did not solve the problem of small theoretical alpha widths.

Some years ago Fliessbach[14] proposed to calculate the alpha widths on the basis of the R-matrix theory, but carefully to normalize the final state wave function. In further investigations Fliessbach[15] stated, however, that the reduced width amplitude (or spectroscopic factor) defined by him depends on the reaction by means of which the decaying state is studied.

Recently, a method was proposed[16] for calculating the alpha widths on the basis of a continuum shell model. The results obtained numerically for several polonium isotopes are on the order of magnitude of

the experimental widths in absolute value, although configuration mixing is not taken into account. However, the calculations are defective, as will be shown in Sec. II.

Watt *et al.*[17] pointed out that the Pauli principle, in most shell-model calculations of spectroscopic factors, is violated. They renormalized the shell model wave function and found agreement with the spectroscopic factor obtained from the experimental data by means of the antisymmetrized calculation of Jackson and Rhoades-Brown[13,18] for the decay of ^{212}Po. The problem of renormalization and factorization will be discussed in detail in Sec. III. It shows once more that the problem of small theoretical alpha widths has not yet been solved. Recent experimental investigations of alpha widths of heavy nuclei confirmed only the old values.[19]

Although configuration mixing of the nuclear wave functions has been shown to enlarge the calculated alpha widths, coupling of the decay channels is not taken into account in any numerical calculation. High shell model configurations treated as bound states are shown to enhance the calculated alpha decay width by several orders of magnitude.[4–6] But the high shell model states are indeed unbound. Their mixing should be taken into account by the method of channel coupling in order to correctly describe the wave functions at the nuclear surface.

In order to characterize the present situation in describing alpha widths of heavy nuclei, the following can be stated. While the wave functions of the bound states are treated with a high degree of accuracy, the properties of the unbound states are usually approximated with lesser accuracy. It is therefore meaningful to look for a theory in which the same Hamiltonian operator describes the coupling of the discrete states (configuration mixing), the coupling of the continuous states (channel coupling), as well as the coupling of the discrete to the continuous states (decay width). Such a theory can be formulated (the continuum shell model).

It is the aim of the present paper to provide the formalism for calculating decay widths (see Secs. II and III). Using the numerical results for nucleon widths, the influence of channel coupling on the alpha widths of heavy nuclei, as well as some aspects of surface alpha clustering, will be discussed in Sec. IV. Conclusions are drawn in Sec. V.

II. THE DECAY WIDTHS

The decay widths will be calculated here on the basis of a theory which describes the coupling of the bound states and the coupling of the unbound states, as well as the coupling between the bound and unbound states by the same Hamiltonian operator. The widths of the decaying states follow from the imaginary part of the eigenvalues of the operator (see Barz *et al.*[20])

$$H_{QQ}^{\text{eff}}=H_{QQ}+H_{QP}G_P^{(+)}H_{PQ}\ , \tag{1}$$

which appears effectively in the subspace of discrete states (Q space) when the coupling to the continuous states (P space) has been taken into account. Here, $H_{QQ}\equiv QHQ$, $H_{QP}\equiv QHP$, . . . , while

$$G_P^{(+)}=P\frac{1}{E^{(+)}-H_{PP}}P \tag{2}$$

is the Green's function in the P space. Q and P are the projection operators onto the corresponding subspaces. The P space contains all functions with one particle in the continuum where the particle may be either, e.g., a nucleon or an alpha particle. Decay channels different from the considered type are neglected. It is $P+Q=1$ in the framework of the model. The width of an isolated decaying state R given by the imaginary part of the eigenvalue of H_{QQ}^{eff} at energy E_R can be represented as

$$\Gamma_R=2\pi\sum_c \langle\phi_R\,|\,H\,|\,\xi_E^c\rangle\langle\xi_E^c\,|\,H\,|\,\phi_R\rangle\,|_{E=E_R}\ . \tag{3}$$

Here, E_R is the energy of the state. The function ϕ_R of the discrete state R follows from a usual nuclear structure calculation, e.g., from a shell model diagonalization with Woods-Saxon potential

$$(E-H_{QQ})\phi_R=0\ . \tag{4}$$

It is $P\phi_R=0$. The continuous functions ξ_E^c are solutions of the *system* of coupled channels equations

$$(E-H_{PP})\xi_E^c=0\ , \tag{5}$$

where c is the decay channel characterized by the quantum numbers of the residual nucleus and of the emitted particle and $Q\xi_E^c=0$. The ξ_E^c used for convenience here are normalized differently from the $\xi_c^{(c_0)}$ defined by Barz *et al.*[20] The $\xi_c^{(c_0)}$ are normalized asymptotically like $\sin kr$ in the entrance channel, while the ξ_E^c are normalized to $\delta(E-E')$.

The amplitude of the partial width with respect to the channel c follows from Eq. (3):

$$\Gamma_{R,c}^{1/2}=(2\pi)^{1/2}D_c^R\,|_{E=E_R}\ , \tag{6}$$

$$D_c^R=\langle\phi_R\,|\,H\,|\,\xi_E^c\rangle\ . \tag{7}$$

It should be noted here that the wave functions ξ_E^c do not describe the final states observed experimentally since they contain admixtures from other states due to the interaction operator V [Eq. (5)]. The final states are described by the basic wave functions in

the P space defined by the solutions of the equation

$$[E-(H_0)_{PP}]\chi_E^c=0 .$$

The decaying states are described by the wave functions[22,23]

$$\Omega_R=(1+G_PH_{PQ})\phi_R ,$$

i.e., by the wave function ϕ_R and some additional term which takes into account the modification of the wave function of a decaying state by the coupling to the continuum. It holds that the equality[32]

$$\langle\phi_R \mid V \mid \xi_E^c\rangle=\langle\Omega_R \mid V \mid \chi_E^c\rangle .$$

In the following, Eq. (7) will be used in accordance with the definition of a partial width given in Ref. 16. In the numerical calculations discussed in Sec. IV [example (*a*)] for the isospin forbidden decays of ^{13}N and ^{13}C with one open channel, the decay width (corresponding to the lifetime of the states) is obtained directly from the eigenvalue of the operator H_{QQ}^{eff} [Eq. (1); see Ref. 22].

The coefficients D_c^R can be determined in the following manner.[16] Using the definition

$$Q=\sum_R \mid\phi_R\rangle\langle\phi_R \mid \tag{8}$$

of the Q operator, Eq. (5) reads

$$(E-H)\xi_E^c=-QH\xi_E^c=-\sum_R D_c^R\phi_R . \tag{9}$$

The functions ξ_E^c can be represented as

$$\xi_E^c=\xi_E^{c,0}-\sum_R D_c^R\xi_E^R , \tag{10}$$

where the functions $\xi_E^{c,0}$ and ξ_E^R are defined by

$$(E-H)\xi_E^{c,0}=0 , \tag{11}$$

$$(E-H)\xi_E^R=\phi_R . \tag{12}$$

Using the orthogonality relation

$$\langle\phi_R \mid \xi_E^c\rangle=0 \tag{13}$$

between the scattering and bound states, it follows from Eq. (10) that

$$\langle\phi_R \mid \xi_E^{c,0}\rangle=\sum_{R'} D_c^{R'}\langle\phi_R \mid \xi_E^{R'}\rangle . \tag{14}$$

The coefficients D_c^R computed as solutions of Eq. (14) immediately determine the partial widths $\Gamma_{R,c}$ according to Eq. (6).

Numerical calculations are performed for isolated states R. In such a case Eq. (14) reads

$$\langle\phi_R \mid \xi_E^{c,0}\rangle=D_c^R\langle\phi_R \mid \xi_E^R\rangle . \tag{15}$$

Further, the wave function ϕ_R of the parent nucleus is represented in the same coordinates as the channel wave function, i.e., the spectroscopic representation

$$\phi_R=\sum_{c'} a_{R,c'}\phi_R^{c'} \tag{16}$$

of the function ϕ_R is used. The functions $\phi_R^{c'}$ are the projections of the function ϕ_R onto the channels c'. They are defined and normalized within the Q space (with $a_{R,c'}=1$ in the one-channel case). By Eq. (16), the totally antisymmetric states ϕ_R are transformed into the form of linear combinations $\phi_R^{c'}$ of antisymmetric states of $A-n$ particles vector coupled to antisymmetric states of the n particles and to the relative motion of the two nuclei $(A-n)$ and n. In $\phi_R^{c'}$, c' stands for the quantum numbers. The sum in Eq. (16) runs over all channels, independently of their energy. The spectroscopic representation (16) is used in the standard shell-model calculations for cluster spectroscopic factors. The amplitudes $a_{R,c'}$ contain the fractional parentage coefficients and the Moshinsky coefficients (if $n>1$), or they can be expressed by using the second quantization method. Equation (12) now reads

$$(E-H)\xi_E^R=\sum_{c'} a_{R,c'}\phi_R^{c'} , \tag{17}$$

where the sum on the right-hand side runs over all channels c' in the spectroscopic representation (16).

Since there is much discussion on the problem of antisymmetrization and renormalization in the literature (e.g., Refs. 13–15, 18, 17, and 21) some comments are helpful here. Equation (16) as it stands is an expansion of the wave function ϕ_R, antisymmetric in A nucleons, into a set of wave functions ϕ_R^c. The wave functions ϕ_R^c are not connected with any physical state observed experimentally. They are introduced only for the convenience of their channel dependence, i.e., for mathematical reasons. Indeed, it is relatively easy to calculate the overlap of ϕ_R^c with the functions ϕ_R and ξ_E^c. Therefore, there is no reason to renormalize or antisymmetrize the functions ϕ_R^c as long as the whole theory is formulated consistently. The functions which describe the final state observed experimentally are the channel wave functions ξ_E^c [see Eq. (7) and following]. These wave functions are antisymmetric in all A nucleons. Further, the Pauli principle between the wave functions ϕ_R and ξ_E^c is taken into account. Equation (17) clearly shows the difference of the formulation given here compared to other theoretical treatments of the alpha decay problem. There is some freedom to choose the basic set of wave functions ϕ_R^c on the right-hand side of Eq. (17). Changing this set into another one will alter, of course, the expansion coefficients $a_{R,c}$ also [Eq. (16)], but the wave functions ξ_E^R on the left-hand side of Eq. (17) and the wave functions ξ_E^c defined by Eq. (5) remain

unchanged. By this, the spectroscopic amplitudes $a_{R,c}$ defined here can differ from those used in other theoretical treatments. Fliessbach,[14,15] e.g., uses a set of antisymmetrized wave functions and therefore gets spectroscopic amplitudes different from those obtained here. The spectroscopic amplitudes $a_{R,c}$ defined by Eq. (16) characterize the nuclear structure and correspond to the spectroscopic amplitudes used successfully for many years in describing the relative probabilities of excitation and the relative decay widths on the basis of the R-matrix theory. The values $a_{R,c}$ do not imply any information about the reaction (in contrast to the definition of the spectroscopic factor by Fliessbach[15]) or even the continuous states, i.e., the final states observed experimentally. Such information is contained in the partial widths. The problem of obtaining the partial widths from the spectroscopic factors is not trivial. The factorization assumption $\Gamma_{R,c}^{1/2} \propto a_{R,c}$ will be considered in detail in Sec. III.

In the numerical calculations, Sandulescu *et al.*[16] solve Eq. (15) by restricting the equation to only one channel c. Furthermore, they also restrict the sum over c' to one channel. Such a procedure would be a good approximation in cases where $\langle \phi_R^{c'} \mid \xi_E^{c,0} \rangle$ as well as $\langle \phi_R^{c'} \mid \xi_E^R \rangle$ vanish for $c \neq c'$. The first condition is mostly fulfilled [see approximation (25)], while the second condition implies the restriction to one channel c in Eq. (17),

$$(E-H)\xi_E^{R,c} = a_{R,c}\phi_R^c \ . \tag{18}$$

Now, ξ^R depends on c. Using

$$\langle \phi_R^{c'} \mid \xi_E^{R,c} \rangle \propto \delta_{cc'}$$

it follows from Eq. (15) that

$$\langle \phi_R^c \mid \xi_E^{c,0} \rangle = D_c^R \langle \phi_R^c \mid \xi_E^{R,c} \rangle \ . \tag{19}$$

According to Eq. (18), $\xi_E^{R,c}$ in Eq. (19) is proportional to $a_{R,c}\phi_R^c$, while it should be proportional to $\sum_{c'} a_{R,c'}\phi_R^c$. Therefore, the values D_c^R calculated numerically from Eq. (19) are generally too large by a factor of the order of magnitude $a_{R,c}{}^{-1}$.

Other calculations for alpha widths on the basis of Eq. (6) gave values smaller than the experimental results. Thus, the problem of the large experimental widths is numerically unsolved. Calculations with channel coupling were not performed up to now.

III. THE PROBLEM OF FACTORIZATION OF THE DECAY WIDTHS

The partial widths $\Gamma_{R,c}^{1/2}$ are calculated mostly from the overlap integrals of the wave functions of the nuclei before and after the decay. The relation between the $\Gamma_{R,c}^{1/2}$ and the overlap integrals can be obtained in the following manner. Using Eq. (12) and the representation of the Green's function, one obtains from Eq. (14) (see Ref. 22)

$$\langle \phi_R \mid \xi_E^{c,0} \rangle = \sum_{R'} D_c^{R'} \langle \phi_R \mid (E - H_{QQ}^{\text{eff}})^{-1} \mid \phi_{R'} \rangle \ . \tag{20}$$

For isolated decaying states it is

$$\langle \phi_R \mid (E - H_{QQ}^{\text{eff}})^{-1} \mid \phi_{R'} \rangle \mid_{E=E_R} = -(2i/\Gamma_R)\delta_{RR'} \ . \tag{21}$$

Therefore Eq. (20) reads

$$\langle \phi_R \mid \xi_E^{c,0} \rangle \mid_{E=E_R} = -2iD_c^R/\Gamma_R \ . \tag{22}$$

By means of the spectroscopic representation (16) it follows from Eq. (22) that

$$D_c^R = \frac{i}{2}\Gamma_R \sum_{c'} a_{R,c'} \langle \phi_R^{c'} \mid \xi_E^{c,0} \rangle \mid_{E=E_R} \ , \tag{23}$$

or by using definitions (6) and (7),

$$\Gamma_{R,c}^{1/2} = i \left[\frac{\pi}{2} \right]^{1/2} \Gamma_R \sum_{c'} a_{R,c'} \langle \phi_R^{c'} \mid \xi_E^{c,0} \rangle \mid_{E=E_R} \ . \tag{24}$$

Equation (24) gives the relation between the amplitudes $\Gamma_{R,c}^{1/2}$ of the partial widths of (isolated) decaying states and the spectroscopic amplitudes $a_{R,c'}$ defined as an overlap of the function ϕ_R of the parent nucleus with the functions ϕ_R^c, which have the same channel coordinates as the wave functions of the two nuclei in the final state. If

$$\langle \phi_R^{c'} \mid \xi_E^{c,0} \rangle \mid_{E=E_R} \approx \langle \phi_R^c \mid \xi_E^{c,0} \rangle \mid_{E=E_R} \delta_{cc'} \tag{25}$$

holds, then the amplitudes of the partial widths can be factorized into two parts

$$\Gamma_{R,c}^{1/2} = a_{R,c}\beta_{R,c} \ , \tag{26}$$

where $a_{R,c}$ contains the nuclear spectroscopic information, obtained from nuclear structure calculations, and the coefficient

$$\beta_{R,c} = i \left[\frac{\pi}{2} \right]^{1/2} \Gamma_R \langle \phi_R^c \mid \xi_E^{c,0} \rangle \mid_{E=E_R} \tag{27}$$

is the amplitude of the partial width for a preformed particle in the nuclear state R. The function $\xi_E^{c,0}$ may be considered, according to Eq. (10), as the wave function of the continuum modified by the discrete states or, according to Eq. (11), as the total solution ψ_E^c of the problem (see Barz *et al.*[20]):

$$\xi_E^{c,0}=\xi_E^c+\sum_R (E-\tilde{E}_R+\frac{i}{2}\tilde{\Gamma}_R)^{-1} \times(1+G_P H_{PQ})\phi_R\langle\phi_R\,|\,H\,|\,\xi_E^c\rangle\ . \tag{28}$$

Here

$$\tilde{E}_R(E=E_R)=E_R$$

and

$$\tilde{\Gamma}_R(E=E_R)=\Gamma_R\ .$$

Equation (24) can also be obtained directly from Eq. (28):

$$\langle\phi_R\,|\,\xi_E^{c,0}\rangle=\sum_R\left[E-\tilde{E}_R+\frac{i}{2}\tilde{\Gamma}_R\right]^{-1}\langle\phi_R\,|\,H\,|\,\xi_E^c\rangle\ . \tag{29}$$

Using definitions (6), (7), and (16), Eq. (29) can be rewritten in Eq. (24) for isolated resonance states.

In the traditional calculations, $\beta_{R,c}$ is approximated by a penetration factor by means of Coulomb wave functions where the factor Γ_R does not appear explicitly. The $\Gamma_{R,c}^{1/2}$ of a state R relative to the various final states, according to the different channels, are proportional therefore to $a_{R,c}$.

Approximation (25) seems to be fulfilled for reactions with knockout of particles or particle groups by incident particles with intermediate energy. In such a case, the nucleon groups performed in configuration (channel) c seem to be knocked out with a large probability in comparison with that for the other preformed configurations c'. In the decay problem approximation (25) seems to be more poorly fulfilled. Nevertheless, it is used generally. That means $\sum_{c'} a_{R,c'}\phi_R^{c'}$ is replaced by $a_{Rc}\phi_R^c$ usually in the same manner as was done by Sandulescu *et al.*[16] As a consequence, the functions ϕ_R^c have to be renormalized since otherwise the $\Gamma_{R,c}^{1/2}$ would be too small according to Eq. (24). The renormalization is of no importance so long as $a_{R,c}\phi_R^c$ is large in comparison with the whole sum (16). But for small $a_{R,c}$, and above all for the low-lying states ϕ_R,[23] the renormalization plays an important role.

That means channel coupling according to Eq. (24) should be taken into account in a theory formulated consistently. This problem will be illustrated by means of examples in Sec. IV. It should merely be noted here that the partial widths $\Gamma_{R,c}$ obey the condition $\Gamma_R=\sum_c\Gamma_{R,c}$ for isolated decaying states in a nontrivial manner[22] according to Eqs. (3) and (6). The best method for numerical calculations including all the effects discussed is the calculation of Γ_R (inverse lifetime) by means of diagonalization of H_{QQ}^{eff} [Eq. (1)].

IV. DISCUSSION OF THE FACTORIZATION CONDITION

In the foregoing it has been shown that the partial widths for particle decay do not factorize into a spectroscopic factor and a penetration factor in a trivial manner. Nevertheless, the factorization assumption (26) can be used in most cases in order to determine the relative values of the widths if the spectroscopic factors are known and vice versa. In the following, some examples are considered, however, in which the factorization assumption (26) is not justified, even for light nuclei.

a. Isospin forbidden proton and neutron widths of ^{13}N *and* ^{13}C. Numerical calculations for the eigenvalues of the operator H_{QQ}^{eff} have shown that channel coupling effects may play an important role in a manner similar to configuration mixing.[23] In the considered case closed channels change the value of the calculated width by more than one order of magnitude since the absolute value itself is small.

b. Isospin allowed reduced alpha widths of the 1*p-shell nuclei.* In light nuclei, many cluster spectroscopic factors relative to both the ground state and the excited states are calculated theoretically and determined experimentally from transfer and knockout reactions. Mostly, the fractional parentage connection of the ground state of the parent nucleus with the excited states of the residual nucleus is comparable to that with the ground state. Even in the case of $^{12}C_{g.s.}$ the alpha spectroscopic factor relative to $^{8}Be_{g.s.}$ is theoretically only about 25% of the sum of the alpha spectroscopic factors while the remaining 75% connect $^{12}C_{g.s.}$ with ^{8}Be in its excited states, mainly with the first 2^+ and 4^+ states.[24,25] The experimental data have verified the relative values for the spectroscopic connection of a nucleus with the ground and excited states of the residual nucleus, i.e., the fragmentation of the strength. Thus, the shell model approach for the description of cluster properties in nuclei which takes directly into account the antisymmetrization of the different cluster wave functions has been supported by the experimental data. An exception is the alpha spectroscopic factor $^{10}B_{g.s.}\rightarrow{}^{6}Li_{g.s.}+\alpha$. According to the two calculations with wave functions of intermediate coupling,[24,26] this value is very small: about 1% of the sum of alpha spectroscopic factors relative to all states of ^{6}Li. The experimental value is much larger.[27] Obviously, coupling between the different channels $^{6}Li+\alpha$ leads to an enlargement of the small width for the channel $^{6}Li_{g.s.}+\alpha$.

c. Isospin forbidden reduced alpha widths of ^{12}C.

The isospin-forbidden alpha widths of some $T=1$ states in ^{12}C are calculated in the framework of the traditional shell model by taking into account the isospin mixing due to Coulomb forces but neglecting both the differences in the neutron and proton wave functions and the channel coupling.[28] The spectroscopic factors obtained are on the order of magnitude 10^{-6}. They are in the correct order of magnitude for the 1^+,1 state at 15.1 MeV but much too small for the 2^+,1 state at 16.1 MeV. The different agreement between theory and experiment in both cases is surely connected to the fact that 1^+ states with an alpha particle structure are forbidden by reason of symmetry. Therefore, coupling of the alpha channels does not play an important role for the 1^+,1 states in contrast to the 2^+,1 states. Consequently, the spectroscopic factors of the 2^+,1 states are enlarged due to channel coupling while there are no favored channels for the 1^+,1 states.

d. Enhancement of the absolute alpha widths of the 1p-shell nuclei in knockout and transfer reactions. Chant *et al.*[29] recently discussed the data in direct reaction theories of alpha transfer and knockout reactions for light nuclei ($A \leq 16$). They pointed out that wave functions with excessive rms radii are needed in order to reproduce predicted $1p$ shell absolute spectroscopic factors. In shell model language this result implies admixtures of four nucleon components from higher shells which would presumably enhance the wave function in the surface region. As an alternative to this interpretation Chant *et al.* considered significant inelastic coupling to excited states having large alpha widths. These couplings remain fairly constant from one reaction to another. From the point of view of the continuum shell model, as discussed in this paper, the two interpretations are not contradictory. Higher shell model states, being unbound, are taken into account together with the bound shell model states in solving the Schrödinger equation $H\psi = E\psi$:

$$\psi_E^{c(+)} = \sum_R b_E^c(R)\phi_R + \sum_{c'} \int_{\epsilon_{c'}}^{\infty} dE' a_E^c(E';c')\xi_{E'}^{c'} . \tag{30}$$

The wave function of an isolated decaying state is modified by the continuum [in analogy to the wave function of the continuum which is modified by the discrete states, Eq. (28)]. It consists of two parts (see Barz *et al.*[20]):

$$\Omega_R = \phi_R + \omega_R , \tag{31}$$

$$\omega_R = G_P H_{PQ}\phi_R = \sum_c \int_{\epsilon_c}^{\infty} dE'(E^+ - E')^{-1}\xi_{E'}^c \langle \xi_{E'}^c \mid H \mid \phi_R \rangle , \tag{32}$$

where ϕ_R is the usual shell model wave function (in a Woods-Saxon potential) and ω_R describes the coupling to the unbound shell model states (continuum), i.e., the asymptotic behavior. The part ω_R contains the wave functions ξ_E^c, and therefore the channel coupling. The form factor is determined by the wave function Ω_R since ϕ_R as well as ω_R are coupled to the external field by means of which the state R is investigated. While ϕ_R describes the direct excitation of the state R, the part ω_R describes the excitation of R via the continuum (channel-resonance scattering). Thus, channel coupling is not only projectile induced but determines the properties of the very state investigated. Therefore, it remains fairly constant from one reaction to another. Channel resonance scattering leads to a virtual enhanced alpha clustering in the nuclear periphery. Its relative role depends on the reaction investigated. It is presumably relatively more important in $(\alpha,2\alpha)$ reactions than in $(p,2p)$ reactions at intermediate energies, since in $(\alpha,2\alpha)$ reactions alpha channels mainly contribute in Eq. (31).

In the considered examples, channel coupling changes the widths even in light nuclei if they are small in absolute value. Channel coupling is contained in the wave functions ξ_E^c, which are solutions of the system (5) of coupled channels equations, in a manner similar as to how configuration mixing is taken into account in the functions ϕ_R defined by Eq. (4). Both functions are important for the diagonalization of the operator H_{QQ}^{eff} [Eq. (1)] from which the widths Γ_R are obtained. Since the alpha widths of heavy nuclei themselves are small, configuration mixing and channel coupling can lead to large effects. As to configuration mixing, this fact is very well known from numerical calculations.[8] For channel coupling a similar effect is expected according to the factorization condition (25) together with the many results obtained numerically on the basis of different approaches which all neglect channel coupling.

The calculated (favored) alpha widths of different states R in heavy nuclei show in most cases good agreement for relative values with the experimental data, although the absolute values are too small.[7] Experimental data are also obtained from an analysis of alpha transfer reactions[30] and elastic and total reaction cross sections[31] in the lead region. The absolute reduced alpha widths obtained from the reactions are in agreement with the experimental values obtained from the decay. This fact suggests that some averaging process is responsible for the deviations in the absolute values. Channel coupling may be such an averaging process, since it takes into account the contribution of every open as well as every closed channel in an averaged manner.

The existence of some surface alpha clustering is discussed not only for light nuclei[29] but also for heavy nuclei.[6] From the point of view of the continuum shell model, surface alpha clustering is directly connected with the coupling of different alpha channels as well as configuration mixing in the higher shells which are both concentrated in the surface region.

V. CONCLUSIONS

On the basis of a theory which describes nuclear structure and continuum coupling effects with comparable accuracy, the problem of alpha widths was discussed. The alpha widths of isolated states can be expressed by means of spectroscopic amplitudes (overlap integrals) characterizing the nuclear structure properties. The R-matrix approach has the advantage over the non-R-matrix approach without factorization of the width in that configuration mixing can easily be taken into account in the numerical calculations. However, channel coupling should additionally be taken into account to configuration mixing in calculating the absolute alpha widths which are fragmented over several states of the final nucleus (channels). A formula for calculating alpha widths by taking into account configurational mixing as well as channel coupling is given by Eq. (14) or (29) with functions obtained from Eq. (16) and the method described by Barz *et al.*[20] A factorization of the partial width is possible only in special cases.

Numerical calculations have been performed for nucleon widths by diagonalizing the Hamiltonian operator H_{QQ}^{eff} [Eq. (1)]. They led to the result that in spite of a small spectroscopic amplitude, the partial width of a certain resonance state may be relatively large due to channel coupling to which open as well as closed channels contribute. Channel coupling should be taken into account therefore at least in an averaged manner in calculating alpha widths of heavy nuclei. Both types of mixing (configurational mixing and channel coupling) are concentrated in the surface region and lead to a surface alpha clustering.

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