Fermi motion and Pauli exclusion principle effects in $d(\pi^{\pm},\pi^{\pm}p)n$ in the Δ-resonance region

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An *ab initio* method of taking Fermi motion effects in pion scattering into account combined with proper antisymmetrization of the two nucleon final state in $d(\pi^{\pm},\pi^{\pm}p)n$ shows that the reaction amplitude consists of not only a contribution from pion scattering on a proton, but also pion scattering on a neutron, and that the two amplitudes differ considerably in their π-N c.m. energies. The neutron scattering contribution to the differential cross section exceeds 10% in as many as 51 out of 195 π^- events recorded in a recent kinematically complete experiment by Hoftiezer *et al.* in the Δ-resonance region. In several kinematical situations the neutron contribution exceeds 100% and could even be as high as $\sim$400%.

[NUCLEAR REACTIONS $d(\pi^{\pm},\pi^{\pm}p)n$, *ab initio* method for Fermi motion effects, Pauli principle, effective mass, exchange amplitude.]

Recently, a kinematically complete experiment in $d(\pi^{\pm},\pi^{\pm}p)n$ was reported[1] in the Δ-resonance region which offers good scope for an incisive theoretical analysis for the Fermi motion and the Pauli exclusion principle (PEP) effects since the experiment determines precisely (i) the π-p c.m. energy W_p at which the scattering takes place on the proton, (ii) the initial Fermi momentum of the proton, and (iii) the extent to which the π-p scattering is off the energy shell. If we follow the *ab initio* method developed recently[2] for taking the Fermi motion into consideration, and take due account of the PEP acting on the two nucleons in the final state, the amplitude for $d(\pi^{\pm},\pi^{\pm}p)n$ involves terms which represent the pion scattering on the neutron in addition to the terms representing the pion scattering on the proton. The kinematically complete experiment determines also (iv) the π-n c.m. energy W_n, (v) the initial Fermi momentum of the neutron and, (vi) the extent to which the π-n scattering is off the energy shell. Since the $(\frac{3}{2},\frac{3}{2})$ scattering phase shift is dominant in the energy region under consideration, and since the π^- scattering on the neutron is completely in the isospin $\frac{3}{2}$ channel, one expects the Pauli term contribution to be sizable at least in the case of π^- scattering.

The purpose of this paper is primarily to draw immediate attention to the importance of the PEP correction. We find that it could be as high as $\sim$400% in certain kinematical situations. Attention is also drawn to the effects of Fermi motion by taking it exactly into consideration right from the beginning. The effects of the deuteron D state, pion distortion, final state interactions, and possible dibaryon contributions are completely neglected for the present since the purpose here is to focus attention on the PEP corrections.

If $q_i(\vec{q}_i,i\omega_i)$, $p_d(0,iM_d)$, $q_f(\vec{q}_f,i\omega_f)$, $p_p(\vec{p}_p,iE_p)$, and $p_n(\vec{p}_n,iE_n)$ denote, respectively, the four-momenta of the incident pion, the target deuteron, the scattered pion, the "knockout" proton, and the "spectator" neutron in the laboratory frame, the differential cross section as measured in the correlation experiment is given by

$$\frac{d^3\sigma}{dp_p d\Omega_\pi d\Omega_p}=(2\pi)^{-5}\frac{\omega_i}{q_i}\frac{{p_p}^2{q_f}^3E_n\omega_f}{E_n{q_f}^2-\omega_f(\vec{q}_f\cdot\vec{p}_n)} \times\frac{1}{3}\sum_{sm\mu}|\langle s\mu|M|1m\rangle|^2 , \quad (1)$$

where s denotes the final spin state of the two nucleons with spin projection μ and m denotes the deuteron spin projection. The on-energy-shell T matrix for the process is given in the impulse approximation by

$$\langle f|T|i\rangle=(2\pi)^3\delta(\vec{k}-\vec{p}_p-\vec{p}_n)\langle s\mu|M|1m\rangle =\left\langle f\left|\sum_{j=1}^{2}t_j\exp(i\vec{k}\cdot\vec{r}_j)\right|i\right\rangle ,$$

$$\vec{k}=\vec{q}_1-\vec{q}_2 , \quad (2a)$$

where the π-N scattering amplitude t has the form

$$t=t^p\left[\frac{1+\tau_z}{2}\right]+t^n\left[\frac{1-\tau_z}{2}\right] \tag{2b}$$

in terms of the π-p and π-n scattering amplitudes t^p and t^n and the isospin Pauli matrix τ_z of the nucleon. If the initial deuteron state $|i\rangle$ is written in terms of its momentum space wave function $G(p)$ as

$$|i\rangle=\frac{1}{(2\pi)^{3/2}}\times\int d^3p'G(p')\exp[i\vec{p}\,'\cdot(\vec{r}_1-\vec{r}_2)]\,|\,1m\rangle\times\left[\frac{p(1)n(2)-p(2)n(1)}{\sqrt{2}}\right] \tag{3}$$

and the fully antisymmetric final state $|f\rangle$ of the two nucleons as

$$|f\rangle=\frac{1}{\sqrt{2}}[\exp(i\vec{p}_p\cdot\vec{r}_1)\exp(i\vec{p}_n\cdot\vec{r}_2)p(1)n(2)+(-1)^s\exp(i\vec{p}_p\cdot\vec{r}_2)\exp(i\vec{p}_n\cdot\vec{r}_1)p(2)n(1)]\,|\,s\mu\rangle\ , \tag{4}$$

it is clear that

$$\begin{aligned}\langle f\,|\,T\,|\,i\rangle=\frac{1}{2}\frac{1}{(2\pi)^{3/2}}\int d^3p'G(p')(2\pi)^6 &\times[\delta(\vec{p}\,'+\vec{k}-\vec{p}_p)\delta(-\vec{p}\,'-\vec{p}_n)\langle s\mu\,|\,t_1^p\,|\,1m\rangle\\ &-(-1)^s\delta(\vec{p}\,'+\vec{k}-\vec{p}_n)\delta(-\vec{p}\,'-\vec{p}_p)\langle s\mu\,|\,t_2^p\,|\,1m\rangle\\ &-(-1)^s\delta(\vec{p}\,'-\vec{p}_n)\delta(-\vec{p}\,'+\vec{k}-\vec{p}_p)\langle s\mu\,|\,t_1^n\,|\,1m\rangle\\ &+\delta(\vec{p}\,'-\vec{p}_p)\delta(-\vec{p}\,'+\vec{k}-\vec{p}_n)\langle s\mu\,|\,t_2^n\,|\,1m\rangle]\ .\end{aligned} \tag{5}$$

Integration over $\vec{p}\,'$ gives the overall momentum conservation factor $\delta(\vec{k}-\vec{p}_p-\vec{p}_n)$ in each of the four terms of which the first two correspond to π-p scattering while the last two correspond to the π-n scattering. Noting that the initial Fermi momentum of the nucleon labeled 1 is $\vec{p}\,'$ while that of 2 is $-\vec{p}\,'$, we observe that the initial and final momenta $\vec{p}_i$ and $\vec{p}_f$ of the nucleon are $-\vec{p}_n$ and $\vec{p}_p$ in t^p while those in t^n are $-\vec{p}_p$ and $\vec{p}_n$. It follows that t^p is characterized by

$$W_p=[-(p_p+q_f)^2]^{1/2}$$

while t^n is characterized by

$$W_n=[-(p_n+q_f)^2]^{1/2}\ .$$

The momentum transfer in either case is, however, the same. Moreover, the deuteron structure function $G(p_n)$ multiplies the first two terms, and the last two terms are multiplied by $G(p_p)$. We thus have

$$\begin{aligned}\langle s\mu\,|\,M\,|\,1m\rangle=\frac{(2\pi)^{3/2}}{2}&[G(p_n)\langle s\mu\,|\,t_1^p(-\vec{p}_n)-(-1)^s t_2^p(-\vec{p}_n)\,|\,1m\rangle\\ &-(-1)^sG(p_p)\langle s\mu\,|\,t_1^n(-\vec{p}_p)-(-1)^s t_2^n(-\vec{p}_p)\,|\,1m\rangle]\ ,\end{aligned} \tag{6}$$

where $t(\vec{p}_F)$ are taken, *ab initio*, to be dependent on the Fermi motion of the nucleon which is shown inside the brackets. Following Ref. 2, the initial energy E_i of the nucleon may be determined assuming the conservation law

$$E_i=E_f+\omega_f-\omega_i \tag{7}$$

but attributing an effective mass $M^*<M$ to the bound nucleon (the Archimedes effect). In Eq. (7), E_f is, respectively, E_p and E_n in t^p and t^n. M^* is given by

$$E_i=[\,|\,\vec{p}_F\,|^2+M^{*2}]^{1/2}$$

and is Lorentz invariant. The extent ΔE is to which the amplitude t in Eq. (6) is the off energy shell in the laboratory can easily be computed now through

$$\Delta E=(p_F{}^2+M^2)^{1/2}-(p_F{}^2+M^{*2})^{1/2}\ . \tag{8}$$

We may now relate the *ab initio* Fermi momentum dependent amplitudes to the corresponding Feynman amplitudes $\tilde{t}$ through

$$t=\left[\frac{MM^*}{4E_iE_f\omega_i\omega_f}\right]^{1/2}\tilde{t}\ , \tag{9}$$

where the invariant amplitudes have to be considered as functions of M^* in addition to the usual kinematical variables of Mandelstam.

We observe that the differential cross section for π-N scattering in the c.m. frame is given now as

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\sum_{\text{spins}}\frac{M^*M}{(4\pi W)^2}\frac{q_2}{q_1}|\tilde{t}|^2\ , \tag{10}$$

where $\vec{q}_1$ and $\vec{q}_2$ denote the initial and final pion momentum in the c.m. frame. Since the explicit dependence of $\tilde{t}$ on M^* is not known at present, we equate (10) to the conventional expression

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\sum_{\text{spins}}|f|^2\ , \tag{11}$$

where f is expressible in terms of the phase shift and has the standard form

$$f=f_1+f_2(\vec{\sigma}\cdot\hat{q}_2)(\vec{\sigma}\cdot\hat{q}_1)=g+ih\vec{\sigma}\cdot\hat{n}\ . \tag{12}$$

We thus write

$$t=4\pi W\left[\frac{q_1}{q_2}\frac{1}{4E_iE_f\omega_i\omega_f}\right]^{1/2}(g+ih\vec{\sigma}\cdot\hat{n})$$
$$=i\vec{\sigma}\cdot\vec{K}+L\ , \tag{13}$$

which defines the spin dependent and the spin independent amplitudes $\vec{K}$ and L, respectively.

The spin summation over $|\langle s\mu|M|1m\rangle|^2$ can now be carried out in a straightforward manner to yield

$$\frac{1}{3}\sum_{s\mu m}|\langle s\mu|M|1m\rangle|^2=\frac{1}{3}(2\pi)^3[3G(p_n)^2(|L_p|^2+|\vec{K}_p|^2)+3G(p_p)^2(|L_n|^2+|\vec{K}_n|^2)$$
$$+2G(p_p)G(p_n)\{\mathrm{Re}(\vec{K}_p^*\cdot\vec{K}_n)+3\,\mathrm{Re}(L_p^*L_n)\}] \tag{14}$$

so that we can now write the differential cross section as

$$\frac{d^3\sigma}{dp_pd\Omega_pd\Omega_\pi}=\frac{1}{128\pi^5}\frac{q_f{}^3p_p{}^2}{q_iE_p}\frac{1}{E_nq_f{}^2-\omega_f(\vec{q}_f\cdot\vec{p}_n)}$$
$$\times\left[(4\pi W_p)^2\phi_d(p_n)^2\frac{E_n}{E_p^*}\left[\frac{q_1}{q_2}\frac{d\sigma}{d\Omega}\right]^{\text{c.m.}}_{\pi p\to\pi p}+(4\pi W_n)^2\phi_d(p_p)^2\frac{E_p}{E_n^*}\left[\frac{q_1}{q_2}\frac{d\sigma}{d\Omega}\right]^{\text{c.m.}}_{\pi n\to\pi n}\right.$$
$$\left.+\frac{2}{3}4\omega_i\omega_fE_pE_n\phi_d(p_p)\phi_d(p_n)\{\mathrm{Re}((\vec{K}_p^*\cdot\vec{K}_n)+3(L_p^*L_n))\}\right]\ , \tag{15}$$

where the initial energies E_i of the proton and the neutron are obtained using (7) and are denoted by E_p^* and E_n^*, respectively. ϕ_d denotes $(2\pi)^{3/2}G$. If we set $E_p^*=E_n$ and $E_n^*=E_p$ and neglect altogether the second and third terms in (15), we obtain expression (1) used by Hoftiezer *et al.* for their impulse approximation (IA) calculation.[3] As we have already remarked, the reaction amplitude for $d(\pi^\pm,\pi^\pm p)n$ consists of two amplitudes, one corresponding to the pion scattering on the proton and the other on neutron, and the first term in (15) results purely from pion scattering on the proton. The second term in (15) represents purely the scattering of pion on the neutron. In view of the fact that the structure function $\phi_d(p)$ is a decreasing function of p and since $p_p>p_n$ in the experiment under consideration, one might expect the second term to be comparatively unimportant since it is multiplied by $\phi_d(p_p)^2$. However, one has to bear in mind here that the π-N scattering amplitude in the resonance region could be quite considerable for π^- scattering on neutrons, owing to the isospin $\frac{3}{2}$ channel dominance. The third term in (15) represents the interference between the π-p and π-n scattering amplitudes and this naturally contains $\phi_d(p_p)$ in the first degree only. We shall denote these three terms by D, E, and I, respectively. The relative importance of the neutron contribution to the cross section can therefore be assessed by estimating the ratio

$$R=\frac{E+I}{D}\ . \tag{16}$$

Since the purpose of this paper is to bring out the importance of PEP, and in view of the finding of Jackson *et al.*,[4] in a different context, that off-shell effects are not important as compared to the PEP, we neglect for the present off-shell deviations in π-p and π-n scattering amplitudes and represent them by their respective on-energy-shell values for which we use the phase shift formulas given by Berends and

TABLE I. Distribution of events for π^+ and π^- scattering over the range of values for R.

Range of R (%)	No. of events π^+	π^-
−10—−5	0	5
−5—0	49	46
0—5	153	60
5—10	16	33
10—50	9	37
50—100	0	7
100—400	0	7

Donnachie.[5] It may be noted that we have not completely neglected the Fermi motion effects since these amplitudes are evaluated at different π-N c.m. energies W_p and W_n. For the deuteron structure function, we use the simple Hulthen wave function.

A more detailed analysis of the reaction, taking into account the D state contribution and other possibilities like dibaryon resonance contributions, is in progress. For the present, we note that the ratio R is quite appreciable in the case of π^- scattering, as anticipated. It is clear from Table I that in as many as 51 out of 195 π^- scattering events, R exceeds 10%, while in the case of π^+ scattering R exceeds 10% only for nine out of 227 events. Our calculations show also that R is not necessarily positive but can also assume negative values due to the appearance of the interference term I. In fact, R assumes negative values to the extent of −7% in the case of π^- scattering and −4% in the case of π^+ scattering. It is very interesting to note that in as many as 14 π^- events R exceeds 100% and goes almost up to 400%, as can be seen from Table II which lists some of these interesting features. It is worth noting that

$$\phi_d(p_p)/\phi_d(p_n)$$

does not exceed a value of 0.35 even when R is 385%. It is also interesting to note that while W_p lies between 1124.3 and 1277.6 MeV, W_n varies over a range from 1081.1 to 1238.8 MeV, and $W_n < W_p$ always. The effective proton mass M_p^* decreases up to 788 MeV while M_n^* goes as low as 538.2 MeV. We have, however, not studied the dependence of the π-N amplitude on M^* here. This will be taken up elsewhere.

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TABLE II. The π-p and π-n c.m. energies and effective proton and neutron masses along with the ratio of the structure functions and $R(\pi^{\pm})$ for a select number of interesting events.

p_p (MeV/c)	θ_p^0	θ_π^0	p_n (MeV/c)	W_p (MeV)	W_n (MeV)	M_p^* (MeV)	M_n^* (MeV)	$\phi_d(p_p)/\phi_d(p_n)$	$R(\pi^+)$ (%)	$R(\pi^-)$ (%)
325	30	105	156.2	1269.6	1238.2	909.8	820.6	0.14	10.43	84.4
325	50	95	154.6	1277.4	1218.1	910.3	820.6	0.14	8.56	71.8
325	50	105	197.0	1275.1	1218.1	894.1	820.6	0.25	22.18	250.9
325	60	100	223.7	1271.1	1205.1	881.8	820.6	0.34	21.27	384.6
335	30	105	144.9	1270.3	1234.5	913.5	813.0	0.11	8.20	60.9
335	50	105	190.4	1275.6	1213.8	896.9	813.0	0.20	17.50	179.9
335	60	100	220.5	1270.9	1200.3	883.4	813.0	0.30	16.33	279.2
345	50	105	184.3	1275.8	1209.2	899.4	805.1	0.17	13.55	125.4
345	60	100	218.1	1270.4	1195.3	884.5	805.1	0.27	12.35	199.0
355	50	105	178.9	1275.7	1204.6	901.6	796.9	0.15	10.29	84.3
355	60	100	216.6	1269.7	1190.2	885.3	796.9	0.24	9.20	139.4
365	50	105	174.2	1275.3	1199.7	903.4	788.4	0.13	7.66	54.1
365	60	100	215.9	1268.3	1184.8	885.6	788.4	0.22	6.77	96.0
375	60	100	216.3	1266.6	1179.3	885.4	779.6	0.20	4.92	65.2
475	60	100	289.7	1214.8	1113.5	844.0	673.2	0.20	−3.64	10.7
485	60	50	314.8	1166.0	1105.8	826.8	660.4	0.24	9.84	4.4
495	60	50	337.2	1153.6	1097.9	809.9	647.7	0.28	13.83	−7.6
495	60	100	325.6	1189.7	1097.9	818.8	647.1	0.25	−1.41	11.6
505	60	100	349.9	1171.7	1089.8	799.8	633.3	0.29	−3.35	8.5
535	50	105	311.9	1171.6	1088.4	828.8	588.7	0.17	−4.28	10.7
545	50	105	358.7	1136.0	1080.1	792.4	572.6	0.24	−19.20	−2.6

[1]J. H. Hoftiezer *et al.*, Phys. Rev. C 23, 407 (1981).

[2]G. Ramachandran, R. S. Keshavamurthy, and M.V.N. Murthy, Phys. Lett. B77, 65 (1978); J. Phys. G 5, 1525 (1979).

[3]In Eq. (1) of Hoftiezer *et al.*, the phase space factor contains q_f^2 instead of q_f^3, which is presumably a printing error. They also mention, after their Eq. (3), that they take into account neutron scattering.

[4]D. F. Jackson, A. A. Ionnides, and A. W. Thomas, Nucl. Phys. A322, 493 (1979).

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