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Coupled bound and continuum eigenstates in momentum space

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Described is a procedure for solving in momentum space the Coulomb plus nuclear problem for coupled bound and continuum eigenstates. The method is an extension of the Kwon-Tabakin formulation of the Lande subtraction technique. Complex eigenenergies are now determined after formulating and incorporating the correct momentum space "boundary conditions" into the Schrödinger equation. This exact numerical procedure can be applied to local or nonlocal, complex or real potentials even with relativistic kinematics. Application to the K^-p problem and other systems with strongly coupled absorptive channels is indicated.

[NUCLEAR STRUCTURE Hadronic atoms, momentum space formulations, coupled channels.]

I. INTRODUCTION

Since the strong interaction of two charged hadrons must be accompanied by their Coulomb interaction, a great deal of effort has gone into solving the combined Coulomb plus nuclear force problem. Although its solution in coordinate space is a classic,[1] a number of physical effects such as nonlocality and relativistic kinematics are handled more directly in momentum space. This paper continues the development of momentum space techniques.

The momentum space solution of the Coulomb plus nuclear problem for scattering boundary conditions was found rather recently by Vincent and Phatak,[2] and for bound states by Kwon and Tabakin.[3] By using a Coulomb potential with a large r cutoff R, Vincent and Phatak could deal with a nonsingular momentum space transform. They then could solve for the phase shift, which in turn yields the wave function $\Psi(r)$ for $r<R$, and by matching to Coulomb waves at $r=R$, produces the full solution.[4]

By employing an ingenious subtraction technique suggested by Lande,[5] Kwon and Tabakin[3] solved a matrix eigenvalue problem for the bound state energies of hadronic atoms (the KT-L technique). This singularity removal is similar to the principal value subtraction introduced by Haftel and Tabakin[6] to permit direct solution of the Lippmann-Schwinger equation. Both the Vincent-Phatak and KT-L techniques are "exact"—albeit numerical—with no restriction to local or separable interactions.

In actual hadronic atom calculations, the nuclear potential usually contains an imaginary part to approximate the absorption of flux into another channel. Although these complex potentials lead naturally to complex "eigenenergies," with E_I related to the lifetime of the state,[7] it would be more correct to solve for a "bound" state in one channel which has a certain lifetime to decay into a different, but open continuum channel[8] (in both cases we are really dealing with poles of the S matrix). While the coupled-channel nuclear problem in momentum space has been studied extensively, particularly by Dalitz *et al.*[9] and Gal *et al.*,[10] we are not aware of any discussion of the extra complications introduced by including the Coulomb force when there are both open and closed channels. Since the "true absorption" of hadrons, such as K^-'s, $\bar{p}$'s, and π's from atomic levels into coupled channels are important processes for us to understand, we wish to describe here some techniques we have developed for this problem. In separate works we describe the application of this technique to the K^--p system[11] and the K^--nuclear system.[12]

A phenomenon related to the Coulomb plus coupled channel nuclear problem is the Krell oscillations.[13] This is a nonmonotonic dependence of the shift of a K^- Coulomb level upon the depth of a *strongly* absorptive optical potential. Koch *et al.*[14] reproduced these oscillations with a schematic, coordinate-space coupled channels model in which the Coulomb force was simulated by a shallow, long-ranged square well. In a more recent, related work, Barrett[15] calculated the shift and width of the $1S$ state in hydrogen—where experiments[16] indicate a possible anomaly. He used a coupled channels

model with a *separable* potential to simulate the Coulomb potential. Although no conclusion was drawn as to why the experimental results were not reproduced, it is possible that the approximation for the Coulomb potential caused the disagreement.

In Sec. II we review the single channel KT-L formulation. In Sec. III A we introduce the coupled channels Schrödinger equations and in Sec. III B discuss the sometimes subtle connection between boundary conditions and eigenenergies. In Sec. III C we present the major new development of this paper, a Green's function formulation which permits coupling to continuum channels with the appropriate boundary conditions in momentum space. In Sec. IV we describe a few numerical tests of our method and in Sec. V we summarize our findings.

II. SINGLE CHANNEL FORMULATION: A REVIEW

A. Schrödinger equation

We wish to solve the momentum space Schrödinger equation,

$$\frac{p^2}{2\mu}\Psi(\vec{p})+\int d^3p'\langle \vec{p}\,|\,V\,|\,\vec{p}\,'\rangle\Psi(\vec{p}\,')=E\Psi(\vec{p}\,') , \tag{2.1}$$

where $\langle \vec{p}\,|\,V\,|\,\vec{p}\,'\rangle$ contains both the Coulomb and (nonlocal) strong potentials. In the Landau-Phatak-Tabakin[17] (LPT) conventions (which are consistent with LPOTT[18]), the potential

$$\langle \vec{p}\,|\,V\,|\,\vec{p}\,'\rangle=\int d^3r\,d^3r'\frac{e^{-i\vec{p}\cdot\vec{r}}}{(2\pi)^{3/2}} \times\langle \vec{r}\,|\,V\,|\,\vec{r}\,'\rangle\frac{e^{i\vec{p}\,'\cdot\vec{r}\,'}}{(2\pi)^{3/2}} , \tag{2.2}$$

has the Legendre polynomial expansion

$$\langle \vec{p}\,|\,V\,|\,\vec{p}\,'\rangle=\frac{1}{2\pi^2}\sum_{L=0}^{\infty}(2L+1)V_L(p\,|\,p') \times P_L(\cos\theta_{pp'}) . \tag{2.3}$$

The Schrödinger equation (2.1) is then

$$\frac{p^2}{2\mu}\Psi_L(p)+\frac{2}{\pi}\int_0^{\infty}V_L(p\,|\,p')\Psi_L(p')p'^2dp' =E\Psi_L(p') , \tag{2.4}$$

where $V_L(p\,|\,p')$ can contain nuclear and Coulomb parts.

If we consider an attractive Coulomb potential,

$$\langle \vec{r}\,|\,V^C\,|\,\vec{r}\,'\rangle=-\frac{Ze^2}{r}\delta(\vec{r}-\vec{r}\,') , \tag{2.5}$$

it has the momentum space forms

$$\langle \vec{p}\,|\,V^C\,|\,\vec{p}\,'\rangle=-\frac{Ze^2}{2\pi^2}\frac{1}{|\,\vec{p}-\vec{p}\,'\,|^2} , \tag{2.6}$$

$$V_L^C(p\,|\,p')=-\frac{Ze^2}{2}\int_{-1}^{1}\frac{P_L(x)dx}{p^2+p'^2-2pp'x} \tag{2.7}$$

$$=-\frac{Ze^2}{2pp'}Q_L(z_{pp'}) , \tag{2.8}$$

$$z'_{pp}=(p^2+p'^2)/2pp' , \tag{2.9}$$

where Q_L is the Legendre function of the second kind, e.g.,

$$Q_0(z_{pp'})=\tfrac{1}{2}\ln\left|\frac{z_{pp'}+1}{z_{pp'}-1}\right|=\ln\left|\frac{p+p'}{p-p'}\right| , \tag{2.10}$$

and our $V_L(p\,|\,p')$ is $\pi/2$ times the KT (Ref. 3) V_L.

B. The Kwon Tabakin-Lande technique

The logarithmic singularity of the Coulomb potential (2.8) at $p=p'$ makes a numerical solution of the Schrödinger equation (2.4) difficult. KT removed this singularity by subtracting a term from the integrand in (2.4) (the Lande subtraction) which makes it nonsingular, and then adding a simple correction term $S_L(p)$. For the pure Coulomb problem these steps take the following form:

$$\frac{p^2}{2\mu}\Psi_L(p)+\frac{2}{\pi}\int_0^{\infty}V_L^C(p\,|\,p')\left[\Psi_L(p')p'^2-\frac{\Psi_L(p)p^2}{P_L(z_{pp'})}\right]dp'+\frac{2}{\pi}\Psi_L(p)p^2S_L(p)=E\Psi_L(p) , \tag{2.11}$$

$$S_L(p)=\int_0^{\infty}\frac{V_L^C(p\,|\,p')}{P_L(z_{pp'})}dp' . \tag{2.12}$$

Amazingly, the integral $S_L(p)$ can be evaluated analytically.[3]

Since the bracketed term in (2.11) vanishes for $p=p'$, this integral equation can be converted to a matrix equation by replacing the integral over p' with a sum over N grid points and evaluating the equation for $\Psi(p)$ on the grid, $p=p_m$:

$$\int_0^\infty dp' f(p') \to \sum_{n=1}^{N} W_n f(p_n) , \tag{2.13}$$

$$\frac{p_m^2}{2\mu}\Psi_L(p_m)+\frac{2}{\pi}\sum_{n\neq m}^{N} W_n V_L^C(p_m \mid p_n) p_n^2 \Psi_L(p_n)+\frac{2}{\pi}\left\{\sum_{n\neq m}^{N}\frac{-V_L^C(p_m \mid p_n)}{P_L(z_{mn})}+S_L(p_m)\right\} p_m^2 \Psi_L(p_m^2) = E\Psi_L(p_m) . \tag{2.14}$$

Note that the diagonal ($n=m$) part of the Coulomb potential makes no contribution to the sum in (2.14), and so by defining a diagonal term to include the subtraction term in braces,

$$V_L^C(p_m \mid p_m) \equiv \frac{1}{W_m}\left\{S_L(p_m)-\sum_{n\neq m}^{N}\frac{V_L^C(p_m \mid p_n)}{P_L(z_{mn})}\right\} , \tag{2.15}$$

(2.14) reduces to a conventional eigenvalue problem:

$$[K_1+V_{11}]_{N\times N}[\Psi_1]_{N\times 1}=E[\Psi_1] , \tag{2.16}$$

$$K_1(p_m \mid p_n)=\frac{p_m^2}{2\mu_1}\delta_{mn} , \tag{2.17}$$

$$V_{11}(p_m \mid p_n)=\frac{2}{\pi}V_L(p_m \mid p_n) W_n p_n^2 , \tag{2.18}$$

where it is obvious that a relativistic definition of the kinetic energy operator can be substituted easily into (2.17).

III. COUPLED CHANNELS EXTENSION

A. Eigenvalue equations

We wish to extend the KT-L procedure to solve for a bound state in one channel (e.g., $K^- p$), coupled to one or more open channels (e.g., $\Sigma\pi$) and possibly closed channels (e.g., $\bar{K}^0 n$). For simplicity we consider only two channels:

$$\frac{p^2}{2\mu_1}\Psi_1(p)+\frac{2}{\pi}\sum_{i=1}^{2}\int_0^\infty V_1(p \mid p')\Psi_i(p')p'^2 dp' = E\Psi_1(p) ,$$

$$\left[\frac{p^2}{2\mu_2}-\Delta M_{12}\right]\Psi_2(p)+\frac{2}{\pi}\sum_{i=1}^{2}\int_0^\infty V_{2i}(p \mid p')\Psi_i(p')p'^2 dp' = E\Psi_2(p) , \tag{3.1}$$

$$\Delta M_{1i}=M_1-M_i . \tag{3.2}$$

M_i is the total mass and μ_i the reduced mass in channel i. In supermatrix form analogous to (2.16), this is the eigenvalue equation

$$\begin{bmatrix} K_1+V_{11} & V_{12} \\ V_{21} & K_2+V_{22}-\Delta M_{12} \end{bmatrix}\begin{bmatrix}\Psi_1 \\ \Psi_2\end{bmatrix}=E\begin{bmatrix}\Psi_1 \\ \Psi_2\end{bmatrix} , \tag{3.3}$$

where the KT-L subtraction can be included in the V's, and where $K_2=P^2/2\mu_2$.

B. Boundary conditions and complex eigenenergies

As is true in coordinate space—but rarely discussed for momentum space—correct boundary conditions are essential in obtaining correct solutions of the Schrödinger equation.[19] When applying the KT-L momentum space method to the single channel equation, (2.4) or (2.16), the mere finding of a stable, normalizable eigenvector with finite eigenvalue is equivalent to the usual bound state conditions. Specifically, any solution containing e^{+Kr} for large r, or $r^{-(L+1)}$ for small r, would *not* be normalizable (the small r behavior, r^{-1}, for $L=0$ would be normalizable, but its eigenvalue would be infinite[19]).

The single channel Schrödinger equation yields real eigenvalues $k=iK$ for real potentials, and complex eigenvalues,

$$E=E_R+iE_I ,$$

for complex potentials. As we shall see, the coupled-channels Schrödinger equation can produce complex energies even for real potentials. If we assume the conventional time dependence,

$$\tilde{\Psi}_1(\vec{x},t)=e^{-iEt}\tilde{\Psi}_1(\vec{x})=e^{E_I t}(e^{-iE_R t}\tilde{\Psi}_1(\vec{x})) , \tag{3.4}$$

E_I must be <0 to produce the lifetime τ of the complex energy eigenstate:

$$\tau^{-1}=\Gamma=-2E_I \ . \tag{3.5}$$

Yet since

$$E=k^2/2\mu$$
$$=[k_R{}^2-k_I{}^2+2ik_Rk_I]/2\mu \ , \tag{3.6}$$

the condition

$$E_I(=2k_Rk_I/2\mu)<0$$

also determines the asymptotic wave functions to be one of two forms:

$$\tilde{\Psi}_A(r)\sim e^{-|k_I|r}e^{-i|k_R|r} , \tag{3.7}$$

$$\tilde{\Psi}_B(r)\sim e^{i|k_R|r}e^{|k_I|r} . \tag{3.8}$$

Since $\tilde{\Psi}_A$ decays exponentially in space, it represents a normalizable bound state appropriate to the closed channel 1 (its *incoming* wave part is required to "feed" this state so that it can maintain the exponential decay with time).

Since $\tilde{\Psi}_B$ contains an outgoing wave part, it represents the wave which "leaks" into the open channel 2 and escapes. It is a non-normalizable wave function and can represent a coupled continuum channel or a resonance (its exponentially growing space part indicates a state decaying with time which was stronger at an earlier time—as seen at larger r).

Although the above asymptotic forms may seem unusual, it is in fact the pure outgoing wave condition in channel 2 which leads to the imaginary parts of the eigenenergies. The relevance of the above discussion to our problem is that we cannot use the usual complex energy eigenvalue procedures to include open channels. These procedures yield normalizable solutions, such as Ψ_A, Eq. (3.7), whereas we need include the non-normalizable states Ψ_B, Eq. (3.8).

C. Green's function approach

Rather than try to build boundary conditions directly into a p space wave function,[20,11] it is best when solving integral equations to place them into the Green's functions. The Lippman-Schwinger equation appropriate for our problem is the following[9,21]:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} + \begin{bmatrix} V_{11}G_1 & V_{12}G_2 \\ V_{21}G_1 & V_{22}G_2 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} . \tag{3.9}$$

Yet since there are no incident waves in the continuum and bound channels, the potential (homogeneous) term on the rhs of Eq. (3.9) must be set to zero:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} V_{11}G_1 & V_{12}G_2 \\ V_{21}G_1 & V_{22}G_2 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} . \tag{3.10}$$

To make connection with the previous formalism, we write the wave function form of the Lippmann-Schwinger equation with no incident waves:

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} G_1V_{11} & G_1V_{12} \\ G_2V_{21} & G_2V_{22} \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} . \tag{3.11}$$

The appropriate Green's functions for Eqs. (3.9)–(3.11) are

$$G_1=(E-\bar{K}_1)^{-1} , \tag{3.12a}$$

$$G_2=(E+\Delta M_{12}+i\epsilon-\bar{K}_2)^{-1} . \tag{3.12b}$$

The V's, $\bar{K}$'s, and G's can be considered operators, or the matrices (2.16). The condition for the existence of nontrivial solutions of a transformed version of (3.11) yields the bound state energies via:

$$\det \begin{bmatrix} 1-V_{11}G_1 & -V_{21}G_1 \\ -V_{12}G_2 & 1-V_{22}G_2 \end{bmatrix} = 0 , \tag{3.13}$$

To be more explicit, we rewrite the Lippmann-Schwinger equation (3.11) as the following:

$$\begin{aligned} \Psi_1(p)-\frac{2}{\pi}\int_0^\infty \frac{dp'V_L{}^{11}(p\,|\,p')p'^2\Psi_1(p')}{E-p'^2/2\mu_1}-\frac{2}{\pi}\int_0^\infty \frac{dp'V_L{}^{12}(p\,|\,p')p'^2\Psi_2(p')}{E-p'^2/2\mu_1}=0 , \\ \Psi_2(p)-\frac{2}{\pi}\int_0^\infty \frac{dp'V_L{}^{21}(p\,|\,p')p'^2\Psi_1(p')}{E+\Delta M_{12}+i\epsilon-p'^2/2\mu_2}-\frac{2}{\pi}\int_0^\infty \frac{dp'V_L{}^{22}(p\,|\,p')p'^2\Psi_2(p')}{E+\Delta M_{12}+i\epsilon-p'^2/2\mu_2}=0 . \end{aligned} \tag{3.14}$$

If we replace the integrals by sums over grid points p_n and define[6,18]

$$D_i(p_n)=\frac{\frac{2}{\pi}W_n{p_n}^2}{({p_n}^2-{k_i}^2)/2\mu_i}\ , \tag{3.15}$$

$${k_i}^2/2\mu_i\equiv E+\Delta M_1\ , \tag{3.16}$$

the bound state condition (3.13) becomes the matrix equation

$$\det\begin{bmatrix}\delta_{mn}+{V_L}^{11}(p_m\,|\,p_n)D_1(p_n) & {V_L}^{12}(p_m\,|\,p_n)D_1(p_n)\\ V^{21}(p_m\,|\,p_n)D_2(p_n) & \delta_{mn}+{V_L}^{22}(p_m\,|\,p_n)D_2(p_n)\end{bmatrix}_{2N\times 2N}=0\ . \tag{3.17}$$

A new advance introduced into the above formulation is the possibility that the Coulomb potential be included in the $V(p_M\,|\,p_n)$ via the Kwon, Tabakin-Lande subtraction procedure as summarized in (2.8) and (2.15).

To apply (3.17) to problems in which channel 2 is open, $\mathrm{Re}({k_2}^2)>0$, we must evaluate explicitly the $i\epsilon$ prescription of (3.14). [For ${k_1}^2<0$ no special treatment is required since it is not possible to reach the singular point $p'=k_1$ in (3.14).] This means that we are treating k_2 and E as *real* numbers until after the $i\epsilon$ prescription is completed—analogous to the Kapur-Peierls theory[7] of resonances.[20] Thus if channel 2 is open the $-VG_2\Psi$ terms take the following form:

$$-\langle p\,|\,VG_2\Psi\rangle=\frac{-2}{\pi}\int_0^\infty\frac{dp'{p'}^2V(p\,|\,p')\Psi(p')}{({k_2}^2-{p'}^2)/2\mu_2+i\epsilon}=\frac{-2}{\pi}\int_0^\infty\frac{dp'{p'}^2V(p\,|\,p')\Psi(p')}{(k_2-p'+i\epsilon)(k_2+p')/2\mu_2} \tag{3.18}$$

$$=\frac{-2}{\pi}P\int_0^\infty\frac{dp'{p'}^2V(p\,|\,p')\Psi(p')}{({k_2}^2-{p'}^2)/2\mu_2}+2i\mu_2k_2V(p\,|\,k_2)\Psi(k_2) \tag{3.19}$$

$$=\frac{2}{\pi}\int_0^\infty\frac{dp'[{p'}^2V(p\,|\,p')\Psi(p')-{k_2}^2V(p\,|\,k_2)\Psi(k_2)]}{({p'}^2-{k_2}^2)/2\mu_2}+2i\mu_2k_2V(p\,|\,k_2)\Psi(k_2)\ , \tag{3.20}$$

where we have made the Haftel-Tabakin subtraction[6] in computing the principal value prescription P. We can thus extend the technique introduced in Ref. 6, and define an extra grid point p_{N+1}, and $D(p_{N+1})$ as:

$$p_{N+1}\equiv k_2\ , \tag{3.21}$$

$$D_2(p_{N+1})=-\frac{2}{\pi}{k_2}^2\left[\sum_{n=1}^{N}\frac{W_n}{({p_n}^2-{k_2}^2)/2\mu_2}\right]+2i\mu_2k_2\ , \tag{3.22}$$

$$D_1(p_{N+1})=0\ . \tag{3.23}$$

If we now simply enlarge our grid to include the k_2 point, the same equation (3.17) can be used to find Coulomb plus nuclear bound states coupled to open channels.

Since the $i\epsilon$ prescription makes $D_2(p_{N+1})$ a complex number, (3.17) will produce complex "eigenenergies" even for pure real potentials—if there is an open channel. Of course, if E is a complex number, Eq. (3.16) requires k_2 to be complex but with the boundary condition (3.8) demanding a purely outgoing wave in channel 2:

$$k_2=|\,k_R\,|-i\,|\,k_I\,|\ . \tag{3.24}$$

Yet if k_2 is complex we must evaluate the potentials in (3.17) at complex momenta. In practice, however, this just means that we search in the complex E (and k_2) plane for a self-consistent solution of (3.17).

IV. NUMERICAL TEST

As a first test of the above method we studied the single channel pure Coulomb problem. We found that searching for zeros of the determinant, (3.17), produced bound state energies numerically identical to those found as eigenvalues of the Schrödinger equation (2.16)—provided the same grids were used (see the Appendix). Typical results for an increasing number of grid points are given in Table I.

To test the method's ability to describe a coupled channels problem, we studied[11] K^-p bound states for the coupled system:

$$K^-p\leftrightarrow\begin{cases}K^-p+0\ \text{MeV}\\ \bar{K}^0n-5\ \text{MeV}\\ \Sigma\pi+100\ \text{MeV}\end{cases}\ . \tag{4.1}$$

In Fig. 1 we present a theoretical experiment which demonstrates the sensitivity of the K^-p $1S$ level shift ϵ and width Γ to the strength Λ of the cou-

TABLE I. Pure Coulomb binding energies in eV for K^--proton, nonrelativistic kinematics, point proton.

Atomic state	Exact	$N=8$	$N=16$	$N=20$	$N=32$	$N=40$
$1S$	−8613	−9557	−9083	−8652	−8623	−8613
$2S$	−2153	−2550	−2112	−2341	−2192	−2154

pling to the $\bar{K}^0 n$ and $\Sigma\pi$ channels ($\Delta E = -\epsilon - i\Gamma/2$). In Fig. 1 $\Lambda=0$ is a pure single channel with real nuclear potentials, whereas $\Lambda=1$ is the full coupling strength of the separable potential model[22] C of Henley, Alberg, and Wilets.[21] For weak coupling we see a large positive (more bound) shift and small width, whereas as $\Lambda\to 0.65$ there is a sign change in ϵ and a maximum in Γ. This behavior is an example of a level crossing effect and is just the coupled channels version of the Krell oscillation.[13] The sign change in ϵ occurs at the point where the channel coupling becomes strong enough to bind a nuclear (inner) state, Y_0^*, in addition to the kaonic hydrogen Coulomb (outer) state.[14]

IV. CONCLUSIONS

We have reviewed the Kwon, Tabakin-Lande exact momentum-space solution of the Coulomb plus nuclear bound state problem, and indicated how it could be extended to include coupling to open, unbound channels. Since many attributes of nuclear potentials can be incorporated best in momentum space, the extension of precision momentum space techniques of the types introduced in Pittsburgh[3,6,2] is valuable. In particular, since the numerical solutions are exact we believe they can be developed to the point where the exotic hydrogen atoms such as K^-p, $p\bar{p}$, and Σp, which have strong absorption into coupled channels, can yield more reliable information on the two body interactions.

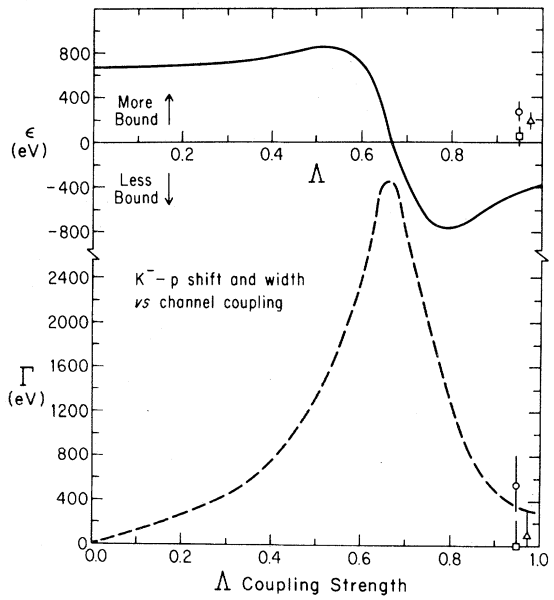

FIG. 1. The shift ϵ and width Γ of the $1S$ kaonic hydrogen level as a function of the coupling strength to the $\Sigma\pi$, $\bar{K}^0 n$, and $\Lambda\pi$ channels. $\Lambda=1$ is the potential C of Ref. 21. The data are from Davies *et al.*[16] (boxes), Izychi *et al.*[16] (circles), and Bird *et al.*[16] (triangles).

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APPENDIX: CHOICE OF GRID

As Kwon and Tabakin indicate[3], the choice of grid points to use in (2.13) can be crucial—particularly for a problem like K^--p where there is a strong imbalance in the ranges and strengths of the Coulomb and nuclear potentials. We generally tried several grids, with the requirement that we reproduce separately the pure strong and pure Coulomb bound states so that—presumably—we have a good grid for the sum. The grids we used are the following: the standard choice of LPOTT,[18] the one outlined in Appendix A of Ref. 3, and a combination of the two in which the standard Gaussian points are scaled in a more linear fashion:

Interval	Number of points	Scaling $(-1<x_m<1)$
$0\rightarrow C_A$	$N_A/2$	$\dfrac{C_A C_S(1+x_m)}{C_S-(C_S-2C_A)x_m}$
$C_A\rightarrow C_S$	$N_A/2$	
$C_S\rightarrow C_N$	$N_N/2$	$\dfrac{C_S+C_N}{2}+\dfrac{C_N-C_S}{2}x_m$
$C_N\rightarrow C_M$	$N_N/2$	$\dfrac{C_N+C_M}{2}+\dfrac{C_M-C_N}{2}x_m$

The A, S, N, and M denote atomic, size, nuclear, and maximum, respectively, and for a typical K^-p case we used $N_A=40$, $N_N=24$, $C_A=2.4$ MeV, $C_s=157$ MeV, $C_N=2400$ MeV, and $C_M=10^8$ MeV.

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