Exit doorway states in nuclear reactions

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Compound nuclear processes should in fact exhibit all the resonances of the underlying compound nucleus. In intermediate resolution experiments where the energy resolution ΔE is considerably large compared to the width and spacing of compound nuclear states but small compared to those of optical model shape resonances, one observes the well-known intermediate structure. In the case of compound elastic processes a dynamically reasonable account of intermediate structure resonances was developed by various authors, where one assumes the existence of a doorway state through which the incident (or the final) state couples to the compound nuclear states. An energy average of the resultant amplitude exhibits intermediate structure. In the case of compound inelastic reactions the incident and the final states are different and the doorway states that relate the final state to the compound nuclear states will in general be different from the doorway states which relate the incident state to the compound nuclear states. The former doorway states will be referred to as the exit doorway states and the latter as the entrance doorway states. A dynamical theory is developed acknowledging the notion of these two kinds of doorway states. The energy averaged transition amplitude shows two sets of intermediate structure resonances corresponding to these two kinds of doorway states. It is expected that one of these sets of resonances will usually dominate a reaction in an energy domain. The present formulation can possibly explain the different sets of intermediate structure resonances observed in different exit channels of a nuclear reaction, for example, those observed in various exit channels of the $^{12}C+^{16}O$ system.

[NUCLEAR REACTIONS Intermediate structure resonances, entrance and exit doorway states, inelastic compound nuclear processes.]

I. INTRODUCTION

The resonancelike bumps in the average cross section for a nuclear reaction are called intermediate structure resonances and have been a topic of discussion in reaction theory for the last two decades. The appearance of intermediate structure can be explained by assuming the existence of doorway states which couple the incident state to the compound nuclear states of the system.[1-7] The hypothesis of doorway states has been a powerful tool in explaining intermediate structure in compound elastic processes. Many authors have discussed the theory of intermediate structure and doorway states in nuclear reactions.[1-7]

But in compound inelastic processes, experiment has revealed that in certain systems the intermediate structure in various exit channels are not consistent with each other.[8] This is true, for example, in the $^{12}C+^{16}O$ system. This work may illuminate the understanding of the appearance of intermediate structure in such cases where various exit channels exhibit different intermediate structure resonances.

The formation of the compound nucleus explains sharp peaks in the energy dependence of the cross section of a nuclear reaction.[1] Such compound nuclear resonances usually have very narrow spacing and a small width ($\sim\Gamma$). On the energy scale Γ can be as low as a few keV. The optical potential on the other hand, deals with average features of the cross section over an energy interval $\Gamma_{op}\sim 1-2$ MeV. In order to observe the compound nuclear resonances one needs to perform a very high resolution experiment where energy resolution $\Delta E \ll \Gamma$. In poor resolution experiments where ΔE is large one observes only the optical potential peaks. Usual experiments will have an energy resolution ΔE which is intermediate, where $\Gamma_{op} \gg \Delta E \gg \Gamma$. Such an experiment will exhibit intermediate structure resonances with characteristic widths and spacings much larger than those of compound nuclear resonances and much smaller than those of optical model shape resonances.[1]

Intermediate structure in compound elastic processes is explained by the doorway state hypothesis which assumes that the system has simple modes of excitation called doorway states which are the only states having strong coupling with the entrance (or the exit) channel.[1,2] Hence the incident state passes through the doorway state to the complicated compound nuclear states. The compound nuclear resonances arising from compound nuclear states having a strong coupling with the doorway states will be more strongly excited than those having a weak coupling with the doorway states. If the compound nuclear states which couple strongly with a doorway state are located in a relatively narrow region one observes a smooth peak whose width Γ_d will be intermediate, i.e., $\Gamma_{\text{op}} \gg \Gamma_d \gg \Gamma$. Such a resonance will be called an intermediate structure resonance. If the energy resolution ΔE of the experiment is large compared to the spacing of doorway states one will have the poor resolution of the optical model.

Now let us consider the compound inelastic process where the incident and the final states are different. Then one can generalize the concept of usual doorway states and introduce two types of doorway states—the entrance and the exit doorway states. The entrance doorway states are the only states which are strongly coupled to the entrance channel. Analogously the exit doorway states are the only states which are strongly coupled to the exit channel. In the case of the compound elastic process these doorway states are the same. Both doorway states also couple to the complicated compound nuclear states. The entrance and the exit states, on the other hand, do not couple with the compound nuclear states. So in the time development of the nuclear process the system will pass through the entrance doorway states before forming the complicated compound nuclear states. Subsequently such compound nuclear states have to pass through the exit doorway state before decaying to the exit channel state. In this paper, using the concept of entrance and exit doorway states, we develop a dynamical formalism in order to explain the intermediate structure resonances observed in various exit channels of a nuclear reaction.

With this introduction to the entrance and exit doorway states we now discuss the associated modification that one would expect in the observed intermediate structure resonances in a compound inelastic process. The compound nuclear states which strongly couple with an entrance doorway state will be easily excited. If such states are located in a narrow region one again observes the intermediate structure resonance corresponding to the entrance doorway state. Similarly an exit doorway state will also couple strongly with some of the compound nuclear states. If such states are located in a narrow interval of energy, one will again observe an intermediate structure that now corresponds to the exit doorway state. When there are many doorway states of both types one may observe intermediate structure resonances corresponding to both types of doorway states. Of course, both of these types of intermediate structure resonances may not be simultaneously observed in a reaction. Depending on the nature of couplings of these doorway states with the compound nuclear states on one hand and with the entrance and exit channel states on the other hand, one type of intermediate structure resonance may dominate over the other type.

The importance of such exit doorway states can be made more explicit if we look at the reaction in a time reversed way. As the transition amplitude is time reversal invariant we can also study the time reversed process. In a time reversed picture the exit doorway state becomes the entrance doorway state. The importance of the entrance doorway state or the "first collision" in nuclear reaction has been emphasized by various workers.[7] The exit doorway state corresponds to the "last collision" of the system and should be equally important as is obvious from the time reversed picture. Of course one should remember that the existence of doorway states does not necessarily imply the appearance of intermediate structure resonances. For the intermediate structure resonances to appear, the coupling of the entrance (exit) doorway states with the entrance (exit) channel should be strong and the coupling of the doorway states to the compound nuclear resonances should not be spread out.

In this work we particularly emphasize the importance of exit doorway states and develop a dynamical formalism including the effect of such states. Since the transition matrix is time reversal invariant, one can easily write a formulation for the entrance doorway state. The transition matrix for the compound inelastic process is written as a sum of three terms. The first one is a nonresonant term which varies with energy on the same scale as optical model shape resonances. The second term varies with energy on the same scale as the doorway states and is called the doorway state term. The third term is a rapidly varying term and varies with energy on the time scale as the compound nuclear resonances. The last term after energy averaging varies with energy on the same scale as the doorway states, and when combined with the second term, will lead to the intermediate structure resonances of the process. In cases of compound inelastic processes we develop the formalism for the exit doorway state and using the time reversal invariance one can easily write the result where the effect of the entrance doorway state is important. The formalism of the exit doorway state is particularly important because it can possibly explain the inconsistency in the observed intermediate structures in the various exit channels of a nuclear reaction.[8]

A striking example of this phenomena is the nuclear reaction involving ^{12}C and ^{16}O in the initial state, where the observed intermediate structures in various exit channels are found to be inconsistent with each other.[8] We would like to suggest an explanation of this inconsistency in terms of the exit doorway states. As for various exit channels the exit doorway states may be different, the observed intermediate structure can also be different in various exit channels. The importance of exit doorway states in explaining this inconsistency has been suggested in a recent letter.[8] If on the other hand the entrance doorway state is the dominant reaction mechanism, the intermediate structures in various exit channels are supposed to be the same and this is true in most of the reactions studied so far.

In this work we shall show how the entrance and exit doorway states provide two competing mechanisms for the formation of intermediate structure resonances. We shall be mainly concerned with compound inelastic processes. Our main mathematical tool will be the rearrangement scattering theory and we present a brief summary of the same in Sec. II, which is appropriate to the present formulation.[9] In Sec. III we explain the appearance of intermediate structure resonances using the idea of entrance and exit doorway states. Finally in Sec. IV we present a brief discussion.

II. SCATTERING FORMALISM

Let us consider the transition of a system from an initial state b to a final state a. Both of these states are assumed to be two-fragment states. The full Hamiltonian is usually broken into two parts

$$H=H_a+V^a=H_b+V^b=\cdots \; , \tag{2.1}$$

where V^a and V^b are the interactions external to the channels a and b, respectively. The Hamiltonians H_a and H_b contain the full kinetic energy and interactions internal to channels a and b, respectively. The transition operator T for a transition from channel b to channel a is defined by[10]

$$T=V^b+V^aGV^b \; , \tag{2.2}$$

and can be easily shown to satisfy

$$T=V^b+V^aG_aT \; , \tag{2.3}$$

where G is the full resolvent operator defined by

$$G=(E+i\epsilon-H)^{-1}$$

and G_a is the resolvent operator for channel a defined by

$$G_a=(E+i\epsilon-H_a)^{-1}$$

with E the center of mass energy of the system.

Now we introduce the projection operators P_a and Q_a such that P_a projects onto the open channel part of the channel a and Q_a projects on the remainder of the space. Thus P_a has the following structure:

$$P_a=\int d\vec{p}\,|\,\vec{p}\Phi_a\rangle\langle\Phi_a\vec{p}\,| \; , \tag{2.4}$$

where Φ_a is the product of the bound state of the two fragments comprising the channel a and $\vec{p}$ is a plane wave of relative motion between them. The operator Q_a is defined by

$$Q_a=1-P_a \; .$$

The operators P_b and Q_b are defined similarly for channel b.

In Eq. (2.3) the intermediate state propagation includes both P and Q spaces. We can separate the intermediate propagations in P and Q spaces by breaking up Eq. (2.3) into the following two sets of equations

$$T=M+NPG_aT \; , \tag{2.5}$$

$$M=V^b+V^aQG_aM \; , \tag{2.6}$$

and

$$N=V^a+V^aQG_aN \; . \tag{2.7}$$

In Eqs. (2.5)–(2.7) and in the following, we suppress the suffix a on P_a and Q_a and represent them simply by P and Q. The suffix b on the projection operators P_b and Q_b will, however, be explicitly shown. Equations (2.6) and (2.7) have the following formal solutions:

$$M=V^b+V^aQ\frac{1}{E-H_{QQ}}QV^b \tag{2.8}$$

and

$$N=V^a+V^aQ\frac{1}{E-H_{QQ}}QV^a\ , \tag{2.9}$$

where

$$H_{QQ}\equiv Q_aHQ_a=QHQ\ . \tag{2.10}$$

In Eqs. (2.8) and (2.9) and in the following the energy parameter E of all the resolvent operators is assumed to have an infinitesimally small positive imaginary part.

Next we break up the transition amplitude T into two parts. One of the parts represents the scattering by an average field and is expected to vary slowly with energy and the second part varies rapidly with energy and resonates if the energy is appropriate. From Eqs. (2.8) and (2.9) it is obvious that such resonances appear as E passes through one of the eigenvalues E_S of H_{QQ}

$$(E_S-H_{QQ})\phi_S=0\ . \tag{2.11}$$

Before writing a formal solution of Eq. (2.5) we separate the group of states E_S in the neighborhood of E, which contribute to rapid fluctuations in E from the distant states which contribute to smooth variations in E. Specifically we rewrite Eqs. (2.8) and (2.9) as:

$$M=V^{b'}+V^aQ\frac{1}{E-H_{QQ}}QV^b \tag{2.12}$$

and

$$N=V^{a'}+V^aQ\frac{1}{E-H_{QQ}}QV^a\ . \tag{2.13}$$

In Eq. (2.12) $V^{b'}$ includes V^b and the nonresonant part of the last term of Eq. (2.8). The second term on the right hand side of Eq. (2.12) projects onto the resonant part of M. In Eqs. (2.12) and (2.13) and in the following Q has been redefined so that it projects onto the group of resonant states around E. The same comment applies to Eq. (2.13) where $V^{a'}$ contains the nonresonant part of N and the last term contains the group of resonant states around E.

Substituting Eqs. (2.12) and (2.13) in Eq. (2.5) and inverting the nonresonant part of the kernel corresponding to the term $V^{a'}$ of Eq. (2.13) we get

$$\begin{aligned}T=&(1-V^{a'}PG_a)^{-1}PM\\&+(1-V^{a'}PG_a)^{-1}PV^aQ(E-H_{QQ})^{-1}\\&\times QV^aPG_aT\ .\end{aligned} \tag{2.14}$$

Equation (2.14) has the following formal solution (see Appendix A for details):

$$\begin{aligned}T=&(1-V^{a'}PG_a)^{-1}PV^{b'}\\&+(1-V^{a'}PG_a)^{-1}PV^aQ(E-H_{QQ}-W_{QQ})^{-1}\\&\times QV^aPe_P{}^{-1}PV^{b'}\\&+(1-V^{a'}PG_a)^{-1}PV^aQ\\&\times(E-H_{QQ}-W_{QQ})^{-1}QV^b\ ,\end{aligned} \tag{2.15}$$

where

$$e_P=E-H'_{PP}\equiv E-P(H_a+V^{a'})P \tag{2.16}$$

and

$$W_{QQ}\equiv QV^aPe_P{}^{-1}PV^aQ=H_{QP}e_P{}^{-1}H_{PQ}\ . \tag{2.17}$$

The channel states ϕ_a and ϕ_b satisfy

$$\begin{aligned}&(E-H_a)\phi_a=0\\&\text{and}\\&(E-H_b)\phi_b=0\ .\end{aligned} \tag{2.18}$$

As P and Q are projection operators for two orthogonal spaces and as they commute with the channel Hamiltonian H_a, we have

$$PQ=QP=0$$

and

$$PH_aQ=QH_aP=0\ .$$

Hence,

$$H_{PQ}\equiv PHQ=PV^aQ$$

and

$$H_{QP}\equiv QHP=QV^aP$$

and the transition matrix element for transition from channel b to channel a is given by

$$\langle\phi_a|T|\phi_b\rangle=\langle\psi_a^{(-)}|V^{b'}|\phi_b\rangle+\langle\psi_a^{(-)}|H_{PQ}(E-H_{QQ}-W_{QQ})^{-1}(QV^b+H_{QP}e_P{}^{-1}V^{b'})|\phi_b\rangle\ , \tag{2.19}$$

where $\psi_a^{(-)}$ is the distorted state defined by

$$\langle\psi_a^{(-)}|=\langle\phi_a|(1-V^{a'}PG_a)^{-1}$$

and satisfies

$$(E-H'_{PP})\psi_a^{(-)}=0 , \tag{2.20}$$

where H'_{PP} is defined by Eq. (2.16). Equation (2.19) provides the desired separation of the transition matrix element into its nonresonant and resonant parts. The first term on the right hand side of Eq. (2.19) varies slowly with energy whereas the last term varies rapidly with energy. In a compound elastic process the exit channel a is identical to the entrance channel b and in this case Eq. (2.19) reduces to[1]

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \psi_a^{(-)} \mid V^{a'} \mid \phi_a \rangle + \langle \psi_a^{(-)} \mid H_{PQ}(E-H_{QQ}-W_{QQ})^{-1}H_{QP} \mid \psi_a^{(+)} \rangle , \tag{2.21}$$

where

$$\mid \psi_a^{(+)} \rangle = (1+e_P^{-1}V^{a'}) \mid \phi_a \rangle ,$$

and satisfies

$$(E-H'_{PP})\psi_a^{(+)}=0 . \tag{2.22}$$

Equation (2.21) is the well known separation of the transition amplitude for the compound elastic process into its resonant and nonresonant parts and was first derived by Feshbach.[9] Equation (2.19) is the generalization of Eq. (2.21) for the case of compound inelastic scattering.

We have derived Eq. (2.19) from Eqs. (2.2) and (2.3) for the transition operator. But we could have started with the equivalent definition[10]

$$T=V^a+V^aGV^b , \tag{2.23}$$

for the transition operator which satisfies

$$T=V^a+TG_bV^b , \tag{2.24}$$

where $G_b=(E+i\epsilon-H_b)^{-1}$. We can break up Eq. (2.24) into the following sets of equations

$$T=\overline{M}+TG_bP_b\overline{N} , \tag{2.25}$$

where

$$\overline{M}=V^a+\overline{M}G_bQ_bV^b , \tag{2.26}$$

and

$$\overline{N}=V^b+\overline{N}G_bQ_bV^b . \tag{2.27}$$

Again Eqs. (2.26) and (2.27) have the following formal solutions

$$\overline{M}=V^a+V^aQ_b(E-H_{QQ}^{(b)})^{-1}Q_bV^b , \tag{2.28}$$

$$\overline{N}=V^b+V^bQ_b(E-H_{QQ}^{(b)})^{-1}Q_bV^b , \tag{2.29}$$

where

$$H_{QQ}^{(b)}=Q_bHQ_b .$$

As in Eqs. (2.8) and (2.9) $\overline{M}$ and $\overline{N}$ of Eqs. (2.28) and (2.29) will resonate as E passes through one of the eigenvalues $E_s^{(b)}$ of $H_{QQ}^{(b)}$ given by

$$(E_S^{(b)}-H_{QQ}^{(b)})\phi_s^{(b)}=0 . \tag{2.30}$$

Again in Eqs. (2.28) and (2.29) we separate the group of states of energy $E^{(b)}$ in the neighborhood of E, which contribute to rapid fluctuations in energy, from the distant states which contribute to a smooth variation in energy. Then we rewrite Eqs. (2.28) and (2.29) as

$$\overline{M}=\overline{V}^{a'}+V^aQ_b(E-H_{QQ}^{(b)})^{-1}Q_bV^b \tag{2.31}$$

and

$$\overline{N}=\overline{V}^{b'}+V^bQ_b(E-H_{QQ}^{(b)})^{-1}Q_bV^b . \tag{2.32}$$

In Eqs. (2.31) and (2.32) $\overline{V}^{a'}$ (or $\overline{V}^{b'}$) includes V^a (or V^b) and the nonresonant part of the second term on the right hand side of Eq. (2.28) [or Eq. (2.29)]. In Eqs. (2.31) and (2.32) and in the following the operator Q_b has been redefined so that it projects onto the group of resonant states around E. Now performing exactly the same algebra needed to derive Eq. (2.19), we can deduce an expression for the transition matrix element, where the resonant and the nonresonant terms are separated. Without repeating essentially the same steps, we can read off the final expression in this case by using time reversal symmetry of the transition matrix element. Then we have

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \phi_a \mid \overline{V}^{a'} \mid \psi_b^{(+)} \rangle + \langle \phi_a \mid (V^aQ_b+\overline{V}^{a'}e_{P_b}{}^{-1}H_{PQ}^{(b)})(E-H_{QQ}^{(b)}-W_{QQ}^{(b)})^{-1}H_{QP}^{(b)} \mid \psi_b^{(+)} \rangle , \tag{2.33}$$

where $H_{PQ}^{(b)}=P_bHQ_b$, $H_{QP}^{(b)}=Q_bHP_b$,

$$e_{P_b} \equiv E-H_{PP}^{(b)'}=E-P_b(H_b+\overline{V}^{b'})P_b , \tag{2.34}$$

and

$$W_{QQ}^{(b)} \equiv Q_bV^bP_be_{P_b}{}^{-1}P_bV^bQ_b=H_{QP}^{(b)}e_{P_b}{}^{-1}H_{PQ}^{(b)} . \tag{2.35}$$

In Eq. (2.33) $\psi_b^{(+)}$ is the outgoing scattering state of $H_{PP}^{(b)'}$ satisfying

$$(E-H_{PP}^{(b)'})\psi_b^{(+)}=0 \ . \tag{2.36}$$

In Eq. (2.33) the first term on the right hand side is the nonresonant term and varies smoothly with energy, whereas the last term is the resonant term and varies rapidly with energy. In the case of a compound elastic process the entrance channel b is identical to the exit channel a and Eq. (2.33) reduces again to Eq. (2.21).

Following Feshbach[9] we now study how resonances appear in Eqs. (2.19) and (2.33). Let us study Eq. (2.19) first and assume that H_{QQ} has a single eigenvalue E_S near E such that Eq. (2.11) is satisfied. In order to keep the algebra and the discussion simple we shall not consider the possibility that H_{QQ} has many eigenvalues near E. This later possibility only increases mathematical complication and can be treated as in the formulation by Feshbach. Then the transition matrix element of Eq. (2.19) can be written as

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \psi_a^{(-)} \mid V^{b'} \mid \phi_b \rangle + \langle \psi_a^{(-)} \mid H_{PQ} \mid \phi_S \rangle \times \frac{1}{E-E_S-\Delta_S+\frac{1}{2}i\Gamma_S} \langle \phi_S \mid (QV^b + H_{QP}e_P{}^{-1}V^{b'}) \mid \phi_b \rangle \ , \tag{2.37}$$

where Δ_S and $-\Gamma_S/2$ are the real and the imaginary parts of the diagonal matrix elements of the operator W_{QQ}

$$\langle \phi_S \mid W_{QQ} \mid \phi_S \rangle = \Delta_S - \tfrac{1}{2} i \Gamma_S \ , \tag{2.38}$$

where

$$\Delta_S = \langle \phi_S \mid H_{QP} \frac{\mathscr{P}}{e_P} H_{PQ} \mid \phi_S \rangle \ , \tag{2.39}$$

and

$$\Gamma_S = 2\pi \langle \phi_S \mid H_{QP}\delta(e_P) H_{PQ} \mid \phi_S \rangle \ , \tag{2.40}$$

where $\mathscr{P}$ and δ represent the principal value and the δ function parts of $e_P{}^{-1}$. We should recall that the eigenvalue E_S is real and the term W_{QQ} provides a non-negative width Γ_S for the resonance and hence allows the system to decay from the compound state ϕ_S.

Analogously we could have studied the transition matrix element of Eq. (2.33). Let us assume that $H_{QQ}^{(b)}$ has a single eigenvalue $E_S^{(b)}$ near E such that $H_{QQ}^{(b)} \mid \phi_S^{(b)} \rangle = E_S^{(b)} \mid \phi_S^{(b)} \rangle$. Then the transition matrix element of Eq. (2.33) can be written as

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \phi_a \mid \bar{V}^{a'} \mid \psi_b^{(+)} \rangle + \langle \phi_a \mid (V^a Q_b + \bar{V}^{a'} e_{P_b}{}^{-1} H_{PQ}^{(b)}) \times \mid \phi_S^{(b)} \rangle \frac{1}{E-E_S^{(b)}-\Delta_S^{(b)}+\frac{1}{2}i\Gamma_S^{(b)}} \langle \phi_S^{(b)} \mid H_{QP}^{(b)} \mid \psi_b^{(+)} \rangle \ , \tag{2.41}$$

where $\Delta_S^{(b)}$ and $-\Gamma_S^{(b)}/2$ are matrix elements of the real and imaginary parts of the operator $W_{QQ}^{(b)}$ defined as in Eqs. (2.38)–(2.40). Again the term $W_{QQ}^{(b)}$ provides a non-negative width $\Gamma_S^{(b)}$ for the resonance and hence allows the system to decay.

Both Eqs. (2.37) and (2.41) show a typical Breit-Wigner form for the resonance amplitude. In these cases the resonance energies are shifted from E_S and $E_S^{(b)}$ by Δ_S and $\Delta_S^{(b)}$, respectively. The energy spectrum of H_{QQ} and $H_{QQ}^{(b)}$ are usually very complicated at moderate excitation energies. The cross sections for such processes show a fine structure in a good resolution experiment. As pointed out in the Introduction the observed intermediate structure corresponds to certain energy averaging of the scattering amplitude to be carried out in the next section. We recall that Eqs. (2.19) and (2.33) are identities but we shall see in the next section that after introducing the concept of entrance and exit doorway states and after performing the energy averaging, Eqs. (2.19) and (2.33) or Eqs. (2.37) and (2.41) will correspond to different intermediate structure resonances—one corresponding to entrance doorway states and the other corresponding to exit doorway states. But depending on the residue and width of such resonances only one of the two sets of intermdiate structure resonances is expected to dominate a reaction amplitude for a certain domain of energy.

III. DOORWAY STATES AND INTERMEDIATE RESONANCES

The projection operator P projects onto the exit channel part of the nuclear wave function. Now we introduce the projection operator corresponding to the exit doorway state by d. The operator d is considered to be orthogonal to P and hence it belongs to Q. The rest of the Q space is denoted by q such that

$$P+d+q=1 \tag{3.1}$$

and

$$Q=d+q\ . \tag{3.2}$$

The projection operators P, d, and q are mutually orthogonal to each other. Now the essence of the doorway state hypothesis is that the more complicated states corresponding to the projection operator q do not couple directly to the exit channel by the Hamiltonian, i.e.,

$$H_{Pq}=0\ ,\quad H_{Pd}\neq 0\ ,\quad H_{dq}\neq 0\ , \tag{3.3}$$

so that

$$H_{PQ}=H_{Pd},\quad H_{QP}=H_{dP}\ . \tag{3.4}$$

Introducing the concept of exit doorway states to Eq. (2.19) we have

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \psi_a^{(-)} \mid V^{b'} \mid \phi_b \rangle + \langle \psi_a^{(-)} \mid H_{Pd} \frac{1}{E-H_{QQ}-W_{dd}} \{(d+q)V^b + H_{QP}e_P{}^{-1}V^{b'}\} \mid \phi_b \rangle\ . \tag{3.5}$$

Now using the following identities (see Appendix B for details)

$$d\frac{1}{E-H_{QQ}-W_{dd}}d = d\frac{1}{E-H_{dd}-W_{dd}-H_{dq}e_q{}^{-1}H_{qd}}d \tag{3.6}$$

and

$$d\frac{1}{E-H_{QQ}-W_{dd}}q = d\frac{1}{E-H_{dd}-W_{dd}-H_{dq}e_q{}^{-1}H_{qd}} \times H_{dq}e_q{}^{-1}q\ , \tag{3.7}$$

where $e_q = E - qHq = E - H_{qq}$ and

$$W_{dd} = H_{dP}e_P{}^{-1}H_{Pd}\ . \tag{3.8}$$

Equation (3.5) reduces to

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \psi_a^{(-)} \mid V^{b'} \mid \phi_b \rangle + \langle \psi_a^{(-)} \mid H_{Pd} \frac{1}{E-H_{dd}-W_{dd}-H_{dq}e_q{}^{-1}H_{qd}} (dV^b + H_{dP}e_P{}^{-1}V^{b'} + H_{dq}e_q{}^{-1}qV^b) \mid \phi_b \rangle\ . \tag{3.9}$$

Now using a second identity[1]

$$\frac{1}{E-H_{dd}-W_{dd}-H_{dq}e_q{}^{-1}H_{qd}} = \frac{1}{E-H_{dd}-W_{dd}} + \frac{1}{E-H_{dd}-W_{dd}} H_{dq} \frac{1}{E-H_{qq}-W_{qq}} H_{qd} \frac{1}{E-H_{dd}-W_{dd}}\ , \tag{3.10}$$

where

$$W_{qq} = H_{qd}\frac{1}{E-H_{dd}-W_{dd}}H_{dq}\ , \tag{3.11}$$

Eq. (3.9) reduces to

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \psi_a^{(-)} \mid V^{b'} \mid \phi_b \rangle + T_d + T_q\ , \tag{3.12}$$

where

$$T_d = \langle \psi_a^{(-)} \mid H_{Pd} \frac{1}{E-H_{dd}-W_{dd}} (dV^b + H_{dP}e_P{}^{-1}V^{b'} + H_{dq}e_q{}^{-1}qV^b) \mid \phi_b \rangle \tag{3.13}$$

and

$$T_q = \langle \psi_a^{(-)} | H_{Pd} \frac{1}{E - H_{dd} - W_{dd}} H_{dq} \frac{1}{E - H_{qq} - W_{qq}} \times H_{qd} \frac{1}{E - H_{dd} - W_{dd}} (dV^b + H_{dP} e_P{}^{-1} V^{b'} + H_{dq} e_q{}^{-1} q V^b) | \phi_b \rangle \; . \tag{3.14}$$

It should be noted that Eq. (3.12) is an exact identity, where T_d gives the resonant contribution from the doorway states and T_q is the resonant contribution from the compound nuclear processes. Equation (3.12) reduces to similar amplitudes discussed by Feshbach, Kerman, and Lemmer[1] in the case of compound elastic processes. The doorway state term T_d varies on an energy scale many times that of the widths of compound nuclear resonances which appear in T_q. The first two terms on the right hand side of Eq. (3.12) are nonresonant amplitudes on the energy scale of compound nuclear resonances.

As has been explained in the Introduction, the intermediate structure will correspond to an energy averaged transition amplitude. This averaging procedure has essentially been carried through in many places[1] and we quote the essential results here. We write the q space propagator in T_q as

$$\frac{1}{E - H_{qq} - W_{qq}} = \sum_\mu | \phi_\mu \rangle \frac{1}{E - \epsilon_\mu} \langle \phi_\mu^A | \; , \tag{3.15}$$

where ϕ_μ and ϕ_μ^A are the biorthogonal set of eigenstates of the operator $(H_{qq} + W_{qq})$ corresponding to the complex eigenvalue ϵ_μ. Following Ref. 1 we define the energy averaged propagator corresponding to Eq. (3.15) as

$$\Lambda_{qq} = \sum_\mu \int | \phi_\mu \rangle \frac{\rho(E,E')}{E' - \epsilon_\mu} dE' \langle \phi_\mu^A | \; , \tag{3.16}$$

where the weight function ρ is only appreciably different from zero in a region ΔE surrounding E and is normalized to unity over ΔE. We consider in this paper the Lorentz weight function defined by

$$\rho(E,E') = \frac{I}{2\pi} \frac{1}{(E - E')^2 + \frac{1}{4} I^2} \; , \tag{3.17}$$

with

$$I \sim 2\Delta E / \pi \; . \tag{3.18}$$

This function is employed mainly because of its analytic convenience.[1] If it is used to average a function $F(E)$ which is regular in the upper half complex E plane and decreases rapidly enough for large $|E|$, then the energy averaging of $F(E)$ yields

$$F(E + iI/2) \; .$$

Hence the energy average of the operator

$$(E - H_{qq} - W_{qq})^{-1}$$

is Λ_{qq} and that of $(E - H_{qq})^{-1}$ is $(\Lambda_{qq}{}^{-1} + W_{qq})^{-1}$ which has been established in detail by Feshbach, Kerman, and Lemmer[1] and by de Toledo Piza and Kerman.[5] Then after energy averaging, the terms T_d and T_q of Eq. (3.12) can be combined and we arrive at

$$\langle \phi_a | T | \phi_b \rangle = \langle \psi_a^{(-)} | V^{b'} | \phi_b \rangle + \langle \psi_a^{(-)} | H_{Pd} \frac{1}{E - H_{dd} - W_{dd} - \hat{W}_{dd}} \{ dV^b + H_{dP} e_P{}^{-1} V_b' + H_{dq} (\Lambda_{qq}{}^{-1} + W_{qq})^{-1} q V^b \} | \phi_b \rangle \; , \tag{3.19}$$

where

$$\hat{W}_{dd} = H_{dq} (\Lambda_{qq}{}^{-1} + W_{qq})^{-1} H_{qd} \; . \tag{3.20}$$

The main resonance feature of the amplitude given by Eq. (3.19) is contained in the energy denominator $(E - H_{dd} - W_{dd} - \hat{W}_{dd})$, which is similar to the energy denominator $(E - H_{dd} - W_{dd})$ of T_d of Eq. (3.13). In Eq. (3.19) $\hat{W}_{dd}$ is an additional complex interaction for doorway states. It is obvious that $\hat{W}_{dd}$ is nonlocal, complex, and energy dependent. The imaginary part of W_{dd} provides the doorway state with a decay channel back to the exit channel in addition to the imaginary part of $\hat{W}_{dd}$ which provides the doorway state with a decay channel down into the compound nuclear states. A particularly simple form for $\hat{W}_{dd}$ results if we recall that

$$\frac{1}{E-H_{qq}-W_{qq}}=\sum_q \frac{|\phi_q\rangle\langle\phi_q|}{E-E_q-\langle\phi_q|W_{qq}|\phi_q\rangle}, \tag{3.21}$$

where ϕ_q are eigenfunctions and E_q are real eigenvalues of the Hamiltonian H_{qq}: $(E_q-H_{qq})\phi_q=0$. We know that the effect of energy averaging with Lorentz weight factor (3.17) is to introduce an imaginary term $iI/2$ to the energy denominator. Hence

$$(E-H_{qq}-W_{qq})^{-1}\xrightarrow[\text{average}]{\text{Lorentz}}\Lambda_{qq}\equiv(E+\tfrac{1}{2}iI-H_{qq}-W_{qq})^{-1}, \tag{3.22}$$

$$(E-H_{qq})^{-1}\xrightarrow[\text{average}]{\text{Lorentz}}(\Lambda_{qq}{}^{-1}+W_{qq})^{-1}\equiv(E+\tfrac{1}{2}iI-H_{qq})^{-1}. \tag{3.23}$$

Equation (3.19) for the average transition matrix demonstrates that the exit doorway state resonances are given by the poles of the complex propagator $(E-H_{dd}-W_{dd}-\hat{W}_{dd})^{-1}$. The analysis of the resonance behavior of this averaged transition operator is similar to the analysis of Sec. II after Eq. (2.37) for the actual t matrix. For the sake of simplicity we consider again the case of an isolated exit doorway state ψ_d satisfying

$$(E_d-H_{dd})\psi_d=0, \tag{3.24}$$

where E_d is a real energy eigenvalue. In this case Eq. (3.19) becomes

$$\begin{aligned}\langle\phi_a|T|\phi_b\rangle=&\langle\psi_a^{(-)}|V^{b'}|\phi_b\rangle+\langle\psi_a^{(-)}|H_{Pd}|\psi_d\rangle\\ &\times\frac{1}{E-E_d-\langle\psi_d|W_{dd}|\psi_d\rangle-\sum_q\frac{|\langle\psi_d|H_{dq}|\phi_q\rangle|^2}{E-E_q+\frac{1}{2}iI}}\\ &\times\langle\psi_d|\left[V^b+H_{dP}e_p{}^{-1}V^{b'}+\sum_q H_{dq}\frac{|\phi_q\rangle\langle\phi_q|}{E-E_q+\frac{1}{2}iI}qV^b\right]|\phi_b\rangle.\end{aligned} \tag{3.25}$$

As in the usual doorway state formulation[1] let

$$\langle\psi_d|W_{dd}|\psi_d\rangle=\Delta_d^{\uparrow}-\tfrac{1}{2}i\Gamma_d^{\uparrow} \tag{3.26}$$

and

$$\sum_q\frac{|\langle\psi_d|H_{dq}|\phi_q\rangle|^2}{E-E_q+\frac{1}{2}iI}=\Delta_d^{\downarrow}-\tfrac{1}{2}i\Gamma_d^{\downarrow}. \tag{3.27}$$

$\Gamma_d^{\downarrow}$ and $\Gamma_d^{\uparrow}$ contribute to the width of the exit doorway state. $\Gamma_d^{\downarrow}$ is the width acquired by the doorway state because of its coupling to the compound nuclear states and $\Gamma_d^{\uparrow}$ is the width acquired because of its coupling to the exit channel. The width Γ_d and the energy ϵ_d of the resonance are

$$\Gamma_d=\Gamma_d^{\uparrow}+\Gamma_d^{\downarrow} \tag{3.28}$$

and

$$\epsilon_d=E_d+\Delta_d^{\uparrow}+\Delta_d^{\downarrow}. \tag{3.29}$$

Hence if the exit doorway states are important the transition amplitude is expected to exhibit a resonance at an energy ϵ_d of width Γ_d.

We have deduced Eq. (3.25) starting from Eq. (2.19) and stressing the importance of the exit doorway state. But we could also have started from Eq. (2.33) and stressed the importance of entrance doorway states and deduced another set of intermediate structure resonances similar to those of Eq. (3.25). We do this in the following. Unless confusion arise we shall suppress the exit channel index a in the following and represent the entrance channel explicitly by the index b.

In place of Eqs. (3.1) and (3.2) we now have

$$P_b + d_b + q_b = 1 \,, \tag{3.30}$$

$$Q_b = d_b + q_b \,, \tag{3.31}$$

for the entrance channel. Again the projection operators P_b, d_b, and q_b are mutually orthogonal to each other. The doorway state hypothesis amounts to

$$H_{Pd}^{(b)} \neq 0, \;\; H_{dq}^{(b)} \neq 0, \;\; \text{but } H_{Pq}^{(b)} = 0 \,. \tag{3.32}$$

Performing the same steps needed to deduce Eqs. (3.19) and (3.20), we have

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \phi_a \mid \bar{V}^{a\prime} \mid \psi_b^{(+)} \rangle + \langle \phi_a \mid \{ V^a q_b (\Lambda_{qq}^{(b)-1} + W_{qq}^{(b)})^{-1} H_{qd}^{(b)} + \bar{V}^{a\prime} e_P^{(b)-1} H_{Pd}^{(b)} + V^a d_b \} \times \frac{1}{E - H_{dd}^{(b)} - W_{dd}^{(b)} - \hat{W}_{dd}^{(b)}} H_{dP}^{(b)} \mid \psi_b^{(+)} \rangle \,. \tag{3.33}$$

Equation (3.33) is self-explanatory once one recalls the content of Eq. (3.19). All the variables of Eq. (3.33) can be defined in a straightforward way once one remembers the definition of the variables associated with Eq. (3.19). The resonance feature of Eq. (3.33) is contained in the energy denominator of this equation. As in Eq. (3.19) the imaginary part of $W_{dd}^{(b)}$ will provide the entrance doorway state with a width because of its coupling to the entrance channel, and the imaginary part of $\hat{W}_{dd}^{(b)}$ will provide it with a width because of its coupling to the compound nuclear states.

Once again we consider one isolated entrance doorway state $\psi_d^{(b)}$ satisfying

$$(E_d^{(b)} - H_{dd}^{(b)}) \psi_d^{(b)} = 0 \,, \tag{3.34}$$

where $E_d^{(b)}$ is a real energy eigenvalue. Now introducing the entrance doorway state $\psi_d^{(b)}$ in the formulation and performing the energy average as in the case of exit doorway state, we have for the transition matrix element

$$\langle \phi_a \mid T \mid \phi_b \rangle = \langle \phi_a \mid \bar{V}^{a\prime} \mid \psi_b^{(+)} \rangle + \langle \phi_a \mid \{ V^a q_b \frac{1}{E - E_q^{(b)} + \frac{1}{2} i I^{(b)}} H_{qd}^{(b)} + \bar{V}^{a\prime} e_P^{(b)-1} H_{Pd}^{(b)} + V^a d_b \} \mid \psi_d^{(b)} \rangle \times \frac{1}{E - E_d^{(b)} - \langle \psi_d^{(b)} \mid W_{dd}^{(b)} \mid \psi_d^{(b)} \rangle - \sum_q \frac{|\langle \psi_d^{(b)} \mid H_{dq}^{(b)} \mid \phi_q^{(b)} \rangle|^2}{E - E_q^{(b)} + \frac{1}{2} i I^{(b)}}} \times \langle \psi_d^{(b)} \mid H_{dP}^{(b)} \mid \psi_b^{(+)} \rangle \,, \tag{3.35}$$

where the index b again explicitly refers to the entrance channel. As before let

$$\langle \psi_d^{(b)} \mid W_{dd}^{(b)} \mid \psi_d^{(b)} \rangle = \Delta_d^{(b)\uparrow} - \tfrac{1}{2} i \Gamma_d^{(b)\uparrow} \tag{3.36}$$

and

$$\sum_{q_b} \frac{|\langle \psi_d^{(b)} \mid H_{dq}^{(b)} \mid \phi_q^{(b)} \rangle|^2}{E - E_q^{(b)} + \frac{1}{2} i I^{(b)}} = \Delta_d^{(b)\downarrow} - \tfrac{1}{2} i \Gamma_d^{(b)\downarrow} \,, \tag{3.37}$$

where $\Gamma_d^{(b)\downarrow}$ is the width acquired by the entrance doorway state because of its coupling to the compound nuclear states and $\Gamma_d^{(b)\uparrow}$ is the width acquired by it because of its coupling to the entrance channel. The energy and the width of the resonance are given by

$$\epsilon_d^{(b)} = E_d^{(b)} + \Delta_d^{(b)\uparrow} + \Delta_d^{(b)\downarrow} \tag{3.38}$$

and

$$\Gamma_d^{(b)} = \Gamma_d^{(b)\uparrow} + \Gamma_d^{(b)\downarrow} \,. \tag{3.39}$$

If the entrance doorway states are important the transition amplitude is expected to show a resonance at energy $\epsilon_d^{(b)}$ and of width $\Gamma_d^{(b)}$.

Equations (3.25) and (3.35) are the main equations of this work. If conditions are favorable these equations will represent intermediate structure resonances corresponding to exit and entrance doorway states. In the case of a compound elastic process both Eqs. (3.25) and (3.35) reduce to the usual doorway state formulation of Ref. 1.

Writing Eqs. (3.25) and (3.35) formally and exploiting the exit and entrance doorway states does not guarantee the intermediate structure resonances. As is well known we must have certain conditions for the intermediate resonances to be observed. The first condition is that the residues at the poles of the transition amplitudes given by Eqs. (3.25) and (3.35) be large. In Eq. (3.25) the necessary condition for this to happen is that the exit doorway state should have strong coupling with the exit channel, i.e., the term H_{Pd} is reasonable. The second condition is that the coupling of the doorway state with the compound nuclear states not be too spread out. In Eqs. (3.25) and (3.26) it means that $\Gamma_d^{\downarrow}$ should be small. If $\Gamma_d^{\downarrow}$ is large and approaches the spacing and width of exit doorway states, intermediate structure in the average cross section will not be observable. In such cases the doorway state will be shared among compound states which are too far away in energy for the doorway state strength to be gathered up without averaging over many doorway states. Similarly in Eqs. (3.35) in order to observe intermediate structure resonances we must have a large residue at the pole of the energy denominator and a small compound nuclear width $\Gamma_d^{(b)\downarrow}$.

If conditions are favorable both sets of intermediate structure resonances given by Eqs. (3.25) and (3.35) will be observed in a reaction. In the case of a compound elastic process in channel a the intermediate structure resonances are given by[1]

$$\langle \phi_a \mid T \mid \phi_a \rangle = \langle \psi_a^{(-)} \mid V^{a\prime} \mid \phi_a \rangle + \frac{\langle \psi_a^{(-)} \mid H_{Pd} \mid \psi_d \rangle \langle \psi_d \mid H_{dP} \mid \psi_a^{(+)} \rangle}{E - E_d - \Delta_d^{\uparrow} - \Delta_d^{\downarrow} + \frac{1}{2} i (\Gamma_d^{\uparrow} + \Gamma_d^{\downarrow})}, \tag{3.40}$$

where $\Delta_d^{\uparrow}$, $\Delta_d^{\downarrow}$, $\Gamma_d^{\uparrow}$, and $\Gamma_d^{\downarrow}$ are defined by Eqs. (3.26) and (3.27). Hence the resonance energies and the widths given by Eq. (3.40) are identical to those given by Eqs. (3.25). But the resonance energies and widths given by Eq. (3.35) are different and are identical to those of the usual doorway state resonances for a compound elastic process in channel b.

Now let us consider the physical transition from channel b to channel a, and as a reference, let us keep in mind the compound elastic process in channel b. Now it is obvious that if the entrance doorway state is the dominant reaction mechanism, both the compound elastic and inelastic processes will show an intermediate structure resonance at energy $\epsilon_d^{(b)}$ and of width $\Gamma_d^{(b)}$ given by Eqs. (3.38) and (3.39). In the case of the compound elastic process the exit and the entrance doorway states are the same. But in the inelastic process from channel b to channel a the exit doorway states are, in general, different from the entrance doorway states. If the exit doorway state is the dominant reaction mechanism the compound inelastic process will show an intermediate structure resonance at energy ϵ_d and of width Γ_d given by Eqs. (3.26) and (3.27). This resonance will not be observed in the compound elastic process in channel b.

In a compound inelastic process in a certain domain of energy one of the two sets of intermediate structure resonances may dominate a reaction. The present formulation is aimed at explaining reactions where the intermediate structure resonances corresponding to the exit doorway states dominate a reaction. This may be the explanation for the different sets of intermediate structures observed in various exit channels of a nuclear reaction, for example, in the system $^{16}O + {}^{12}C$.

IV. DISCUSSION

In this paper we developed a formalism for the compound inelastic process including the effect of entrance and exit doorway states. The present formalism does not depend on the use of a special model for a nuclear reaction, for example, a shell model or an alpha particle model, etc.

The doorway states are not eigenstates of the full Hamiltonian, but they are eigenstates of parts of the Hamiltonian. The doorway states are, however, states of definite angular momentum, parity, etc. Hence the observed intermediate structure corresponding to a doorway state formalism will have definite angular momentum, parity, etc. The same will be true, in particular, for the case of exit doorway states.

In their discussion of doorway states Feshbach, Kerman, and Lemmer[1] also considered the case of true inelastic reaction. But in such cases they assumed that the exit doorway states are identical to the entrance doorway states and the observed intermediate structure contains the intermediate resonances of the entrance channel. In the present formalism we include the possibility that the exit doorway state can be different from the entrance doorway state. It is not, however, expected that the exit doorway states will be different for all exit channels. It is very reasonable to have the same exit doorway state for more than one exit channel. In that event the intermediate structure resonances arising from the exit doorway formalism for these exit channels are expected to be approximately the same. The small difference will be due to different coupling of the exit doorway state with the exit channel and the compound nuclear states.

We can illustrate this considering the reaction channels in the system $^{12}C + {}^{16}O$. Experiments show that the intermediate structure resonances for this system are not consistent with each other for various exit channels.[8,11] A rotational bandlike structure is observed in the elastic scattering of this system. But the intermediate structure observed in the reaction $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ is completely distinct from the intermediate structure observed in the elastic channel. For another exit channel, for example, in the reaction $^{12}C(^{16}O,^{4}He)^{24}Mg$, the intermediate structure resonances can be different from both the intermediate structure resonances mentioned above. It is to explain these discrepancies that we formulate the present hypothesis of exit doorway states. We hope that the present formalism will be useful in explaining intermediate structure resonances in various other reactions.

Another example of a recent light ion experimental work which may illustrate the exit doorway effects is given in Ref. 12. In this work the authors measured the decay width of compound nuclear states to various exit channels and the experimental results may imply the existence of exit doorway states. Further experimental works are needed in order to verify the hypothesis of exit doorway states.

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APPENDIX A

In this appendix we present a formal proof of Eq. (2.15) starting from Eq. (2.14). For this purpose we consider the following formal Neumann series solution of Eq. (2.14):

$$\begin{aligned}T=&(1-V^{a\prime}PG_a)^{-1}PM+(1-V^{a\prime}PG_a)^{-1}PV^aQ\frac{1}{E-H_{QQ}}\\&\times QV^aPe_P{}^{-1}PM+(1-V^{a\prime}PG_a)^{-1}PV^aQ\left[\frac{1}{E-H_{QQ}}QV^aPe_P{}^{-1}PV^aQ\frac{1}{E-H_{QQ}}\right]QV^aPe_P{}^{-1}PM\\&+(1-V^{a\prime}PG_a)^{-1}PV^aQ\left[\frac{1}{E-H_{QQ}}QV^aPe_P{}^{-1}PV^aQ\frac{1}{E-H_{QQ}}QV^aPe_P{}^{-1}PV^aQ\frac{1}{E-H_{QQ}}\right]QV^aP\\&\times e_P{}^{-1}PM+\cdots\ ,\end{aligned}\tag{A1}$$

where

$$e_P=E-H'_{PP}\equiv E-P(H_a+V^{a\prime})P\ .\tag{A2}$$

Now using the identity

$$\begin{aligned}Q\frac{1}{E-H_{QQ}-W_{QQ}}Q=Q\Bigg[&\frac{1}{E-H_{QQ}}+\frac{1}{E-H_{QQ}}W_{QQ}\frac{1}{E-H_{QQ}}\\&+\frac{1}{E-H_{QQ}}W_{QQ}\frac{1}{E-H_{QQ}}W_{QQ}\frac{1}{E-H_{QQ}}+\cdots\Bigg]Q\ ,\end{aligned}\tag{A3}$$

where

$$W_{QQ}=QV^aPe_P{}^{-1}PV^aQ\ ,\tag{A4}$$

we can sum up the series (A1) and get

$$T=(1-V^{a\prime}PG_a)^{-1}PM+(1-V^{a\prime}PG_a)^{-1}PV^aQ\frac{1}{E-H_{QQ}-W_{QQ}}QV^aPe_P{}^{-1}PM\ .\tag{A5}$$

Now substituting for M in Eq. (A5) from Eq. (2.12) yields

$$T=(1-V^{a'}PG_a)^{-1}PV^{b'}+(1-V^{a'}PG_a)^{-1}PV^aQ\frac{1}{E-H_{QQ}-W_{QQ}}QV^aPe_P{}^{-1}PV^{b'}+(1-V^{a'}PG_a)^{-1}$$
$$\times PV^aQ\left[\frac{1}{E-H_{QQ}}+\frac{1}{E-H_{QQ}-W_{QQ}}W_{QQ}\frac{1}{E-H_{QQ}}\right]QV^b\,, \tag{A6}$$

or,

$$T=(1-V^{a'}PG_a)^{-1}PV^{b'}+(1-V^{a'}PG_a)^{-1}PV^aQ\frac{1}{E-H_{QQ}-W_{QQ}}$$
$$\times QV^aPe_P{}^{-1}PV^{b'}+(1-V^{a'}PG_a)^{-1}PV^aQ\frac{1}{E-H_{QQ}-W_{QQ}}QV^b\,. \tag{A7}$$

Equation (A7) is the equation we wished to prove.

APPENDIX B

In this appendix we would like to prove Eqs. (3.6) and (3.7). A proof of Eq. (3.6) appears in the Appendix of Ref. 1. Here we provide a similar proof of Eq. (3.7). Let us define

$$e_Q=QE-H_{QQ}-W_{QQ}\,, \tag{B1}$$

$$e_d=dE-H_{dd}-W_{dd}\,, \tag{B2}$$

$$e_q=qE-H_{qq}\,. \tag{B3}$$

Then using the definition of d and q spaces and their associated properties, we have

$$e_Q=e_d+e_q-H_{dq}-H_{qd}\,. \tag{B4}$$

Now we have

$$Q=(e_Qd)\left[d\frac{1}{e_Q}\right]+(e_Qq)\left[q\frac{1}{e_Q}\right]. \tag{B5}$$

Now premultiplying and postmultiplying Eq. (B5) by operators d and q, we arrive at

$$e_d\left[d\frac{1}{e_Q}q\right]-H_{dq}\left[q\frac{1}{e_Q}q\right]=0 \tag{B6}$$

and

$$-H_{qd}\left[d\frac{1}{e_Q}q\right]+e_q\left[q\frac{1}{e_Q}q\right]=q\,. \tag{B7}$$

Eliminating $q(1/e_Q)q$ between Eqs. (B6) and (B7) we arrive at

$$-H_{dq}\frac{1}{e_q}H_{qd}\left[d\frac{1}{e_Q}q\right]+e_d\left[d\frac{1}{e_Q}q\right]=H_{dq}\frac{1}{e_q}q \tag{B8}$$

or,

$$d\frac{1}{e_Q}q=d\frac{1}{E-H_{dd}-W_{dd}-H_{dq}\frac{1}{e_q}H_{qd}}dH_{dq}\frac{1}{e_q}q\,, \tag{B9}$$

which proves Eq. (3.7).

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