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Binding energy estimates for charmed few-body systems

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By solving exact three- and four-body equations, we obtain estimates of the binding energies of light nuclei containing one charmed C_0^+ baryon.

[NUCLEAR STRUCTURE Few-body hypernuclei, charmed analogs of hypernuclei, binding energy estimates.]

I. INTRODUCTION

The existence of baryons possessing net charm ($C=+1$) has been established,[1] in particular that of the C_0 and C_1 (the Λ_C and Σ_C). The discovery of such charmed hadrons fostered conjectures[2–5] about the possible existence of charm analogs of hypernuclei ($S=-1$). The observation of a candidate event,[6] which might possibly be interpreted in terms of the decay of a charmed nucleus, led to further speculation concerning the properties of such bound systems.[7,8] In view of the future prospects for the experimental study of charmed nuclei, we have reexamined the spectroscopy of S-shell systems, using proper three- and four-body equations to obtain estimates of ground-state binding energies.

We restrict our considerations to bound systems involving the C_0^+ charmed baryon (the Λ_C), the analog of the isospin-zero Λ hyperon. It is this $C=+1$ baryon which is stable with respect to strong decay. The C_0 decays weakly in free space into $\Lambda\pi^+\pi^+\pi^-$, for instance, with $\tau \cong 7\times 10^{-13}$ sec; in nuclear matter additional decay modes are available such as $C_0+N\to N+N$. The C_1 is unstable with respect to strong decay into $C_0\pi$. However, the energy release in $C_1\to C_0\pi$ is only about 28 MeV, so that the width of a C_1-nuclear state could be of the order of a few MeV. Indeed, relatively narrow Σ-hypernuclear states have been seen,[9] even though the strong decay $\Sigma\to\Lambda\pi$ is allowed. The spectroscopy of C_1 nuclei is treated in Ref. 3. Because of its heavy mass, $M_C=2.27$ GeV, we investigate the question of the binding of the C_0 to a nuclear core in terms of a nonrelativistic framework, where the C_0N interaction is represented by a potential based upon one-boson-exchange models. We approximate the one-boson-exchange potentials by rank-one separable potentials in order to facilitate the solution of the exact three-body and four-body equations which describe the $A=3$ and 4 bound state systems. Such an approximation has proven reliable and useful in nuclear and hypernuclear studies.[10] For our purpose of estimating the possible ranges of binding energies to be expected for the $A=3$ and 4 charmed nuclei, this separable potential model approach should be quite adequate.

Before discussing in detail the results of our study, we emphasize two characteristics of these binding energy estimates which differ from those found in the more familiar studies of Λ hypernuclei. The mass of the C_0 is approximately twice that of the Λ. Thus, the kinetic energy associated with the C_0 is reduced compared with that of the Λ. This implies that the C_0 will be more strongly bound than the Λ in the case that both have identical interactions with the nucleon.[3] The C_0 also has a positive charge, whereas the Λ is neutral. Therefore, Coulomb effects will play a non-negligible role in charmed nuclei. The hypertriton (${}^3_\Lambda$H) would be unbound if the Λ were to have a positive charge. The Coulomb energies in the $A=4$ isodoublet ${}^4_C\mathrm{Li}$-${}^4_C\mathrm{He}$ (the ground states of the C_0^+npp and C_0^+pnn systems, respectively) will differ significantly:

$$|E_{\mathrm{Coul}}({}^4_C\mathrm{Li})| \cong 3\,|E_{\mathrm{Coul}}({}^4_C\mathrm{He})| \ .$$

The corresponding C_0-separation energies

$$B_C({}^4_C\mathrm{Li}) = B({}^4_C\mathrm{Li}) - B({}^3\mathrm{He})$$

and

$$B_C({}^4_C\mathrm{He}) = B({}^4_C\mathrm{He}) - B({}^3\mathrm{H})$$

will differ by approximately

$$2\,|E_{\mathrm{Coul}}({}^3\mathrm{He})| \cong 1.5 \text{ MeV} \ ,$$

assuming that wave function distortions are not significant so that

$$\tfrac{1}{3}E_{\mathrm{Coul}}({}^4_C\mathrm{Li}) \cong E_{\mathrm{Coul}}({}^4_C\mathrm{He}) \cong E_{\mathrm{Coul}}({}^3\mathrm{He}) \ .$$

(The Coulomb energy is essentially determined by size, or total binding energy; hence, E_{Coul} of ^{3}He is more appropriate than E_{Coul} of the more strongly bound ^{4}He.) This situation differs dramatically from that found in the case of the $A=4$ Λ-hypernuclear isodoublet, where Coulomb effects in the Λ separation energies are essentially negligible.

In Sec. II, we discuss the C_0N potential model assumptions made in this study of the binding energies of charmed nuclei. The three-body and four-body equations are summarized in Sec. III, and our numerical results are presented in Sec. IV. We conclude in Sec. V with a brief summary of $A=3$ and 4 and an estimate of what might be expected for $A=5$.

II. THE C_0^+N POTENTIAL MODELS

As remarked above, our investigation is based upon one-boson-exchange (OBE) models of the C_0N potential. Such models have been successfully applied in describing low-energy nucleon-nucleon (NN) scattering.[11] Extensions of the OBE model to incorporate SU(3) symmetry in coupling constants have been utilized to provide a description of low energy hyperon-nucleon (YN, where $Y=\Lambda,\Sigma$) scattering data.[12–14] We extend these models of the NN and YN baryon-baryon interaction in a simple way to an SU(4) picture, in order to include the C_0N force.

As a first approximation, we assume that the ΛN and C_0N interaction potentials are identical. As noted in the Introduction, the heavier mass of the C_0 compared to that of the Λ implies that the C_0 will be more strongly bound in a nucleus than the Λ. This is reflected in the scattering lengths listed in Table I for models 1–3, each one of which yields ΛN scattering lengths of the order of -2 fm. In other words, assuming that $V_{C_0N} \equiv V_{\Lambda N}$, as in models 1–3, implies that the scattering length is more negative (more attractive, more nearly bound) for the two-body system with the larger reduced mass. We include the two most recent models of Nagels *et al.*[12,13] along with that of Brown *et al.*[14] in order to provide an indication of the uncertainty based upon the parametrization of the limited hyperon-nucleon data.

We also consider a model that results from including SU(4) symmetry breaking in the masses of the exchanged mesons as well as the baryon masses,[3,7] while retaining SU(4) invariance for the coupling constants. Because the model of Ref. 14 is most easily modified, we shall restrict our study of this assumption to that particular one-boson-exchange potential.[7] A comparison of scattering lengths for model 4 with those of model 3 shows that the resulting C_0N interaction is somewhat weaker. Therefore, we anticipate that the binding energies of the $A=3$ and 4 charmed nuclei calculated with model 4 will be similar to those obtained for the analogous Λ hypernuclei.

For calculational purposes we have represented the baryon-baryon interactions by rank-one separable potentials of the form

$$V_{\alpha N} = -\frac{\lambda_{\alpha i}}{2\mu_\alpha} g_{\alpha i}(k) g_{\alpha i}(k'), \quad i=s,t; \quad \alpha = N, C_0$$

$$g_{\alpha i}(k) = (k^2 + \beta_{\alpha i}{}^2)^{-1} \ .$$

The s and t subscripts refer to the spin-singlet and spin-triplet states of the NN and C_0N interactions. The C_0N potential parameters are uniquely specified by the scattering lengths and effective ranges listed in Table I. The NN central potential parameters correspond to a singlet scattering length and effective range of -17.0 and 2.84 fm and to triplet values of 5.42 and 1.76 fm. A Yamaguchi-Yamaguchi tensor potential[15] is also used; the form factors can be found in Ref. 16 along with the parameters for the Phillips model[17] in which the deuteron has a 7% D-state probability.

Our model of the C_0N potential neglects explicit

TABLE I. Scattering lengths and effective ranges for the one-boson-exchange potential models of the C_0N force derived from the ΛN models referenced; units are fm.

Model	Ref.	a_s	r_s	a_t	r_t
1 (Dutch *D*)	12	−3.83	3.05	−4.24	2.60
2 (Dutch *F*)	13	−5.63	2.60	−3.99	2.75
3 (BDI-II)	14	−3.74	2.60	−3.75	2.49
4 (BDI-II-*m*)	7,14	−1.075	1.665	−0.828	2.015

C_0N-C_1N coupling. The coupling is, of course, included implicitly. However, in hypernuclear ground states this approximation leads to overbinding in the $A=4$ system.[18] Thus, our binding energy estimates based upon models 1–3 are most likely to be upper bounds on the ground state energies of the $A=3$ and 4 charmed nuclei.

III. THE BOUND STATE EQUATIONS

The exact equations describing the ground states of 3 and 4 spin-$\frac{1}{2}$ fermions in which one is "tagged" are well known. We summarize them here in schematic form as a reminder of the structure of the coupled, linear integral equations which must be solved numerically. For a detailed discussion of the equations as well as our numerical approach to their solution, the reader is referred to Refs. 16 and 18.

For the ${}^3_C\mathrm{He}$ ground state, the Schrödinger equation can be decomposed into a set of one-variable coupled equations of the form

$$G_t=\tau_t^G \int \sum_j I_{tj}^{NC} H_j \ ,$$

$$H_i=\tau_i^H \int [I_{it}^{CN} G_t + \sum_j I_{ij}^{CC} H_j] \ ,$$

where $i,j=s,t$. Because only the spin triplet np (deuteron) pair enters the calculation, there is only a single spectator function G_t describing the C_0 motion relative to that of the np pair. Both H_s and H_t spectator functions (N relative to the NC_0 interacting pair) appear. The kernels of the integral equations (I_{ij}^{CN}, I_{ij}^{NC}, I_{ij}^{CC}) are integral functions of the separable potential form factors. The τ functions contain the potential strengths and simple integrals over the form factors. When one includes tensor forces in the NN system, there are a total of six coupled equations to be solved.[16]

For the $A=4$ isodoublet ground states, the Schrödinger equation can be decomposed, after projecting to $l=0$ spectator functions, into a set of two-variable coupled equations of the form

$$A_i=\tau_i^A \int\int \sum_j [X_{ij} B_j + X_{ij} D_j] \ ,$$

$$B_i=\tau_i^B \int\int \sum_j [X_{ij}^{NN} A_j + X_{ij}^{NN} D_j + X_{ij}^{NC} C_j + X_{ij}^{NC} F_j] \ ,$$

$$C_i=\tau_i^C \int\int \sum_j [X_{ij}^{CN} A_j + X_{ij}^{CN} D_j + X_{ij}^{CC} C_j + X_{ij}^{CC} F_j] \ ,$$

$$D_i=\tau_i^D \int\int \sum_j [Y_{ij}^{NN} A_j + Y_{ij}^{NN} B_j + 2Y_{ij}^{NC} C_j] \ ,$$

$$F_i=\tau_i^F \int\int \sum_j [Y_{ij}^{CN} A_j + Y_{ij}^{CN} B_j + 2Y_{ij}^{CC} C_j] \ ,$$

where $i,j=s,t$. The A, B, and C are the spectator functions having [3,1] symmetry (e.g., $C_0 + {}^3\mathrm{H}$ or $n + {}^3_C\mathrm{He}$), while the D and F are the spectator functions having [2,2] symmetry (e.g., $NN + NC_0$). The kernels ($X, X^{\alpha\beta}, Y^{\alpha\beta}$) of the integral equations are themselves solutions of coupled inhomogeneous integral equations. Allowing for spin-singlet and spin-triplet NN and C_0N forces, one much solve a set of ten coupled equations.[18]

Because it is well known that the use of a central potential approximation to the NN force in a separable potential model overbinds ${}^3\mathrm{H}$ and ${}^4\mathrm{He}$, we also use the truncated t-matrix approximation in the NN channel for the $A=4$ calculation. This use of only the t_{00} component of the tensor force t matrix was found to be quite satisfactory by Tjon.[19] It leads to a small ($<8\%$) error in results for the ${}^3\mathrm{H}$ bound state.[20] Our binding energy estimates using this approximation should be more realistic than those obtained when the NN interaction is treated as a pure central force.

IV. MODEL RESULTS

The scattering lengths and effective ranges determined by our various SU(4) extensions of the OBE model are listed in Table I. Using the C_0N separable potentials generated from those low-energy scattering parameters in the three-body and four-body equations outlined in the previous section, we have calculated the binding energies of the ground states of the corresponding nuclei containing one charmed baryon, namely a C_0, while *neglecting* Coulomb effects. Our results are summarized in Table II, where the C_0-separation energies are listed. (That is, we have subtracted the model deuteron and triton binding energies.) We have included results in the $A=3$ case only for the np tensor force (7% deuteron D state) calculation.

Recall that models 1 and 2 correspond to the potentials D and F of Nagels *et al.*[12,13] The binding energy differences are an indication of the uncertainties in our estimates due to ambiguities in our knowledge of the YN interaction. The results of model 3 fall between those of models 1 and 2. Comparison of the $A=4$ results for central and tensor forces provides an indication of the overbinding which arises in the case that one utilizes a central force approximation to the NN interaction. This well-known effect is due to the long range nature of the tensor force coming from one pion exchange; the $A=4$ system, which has an $A=3$ nuclear "core," is much more compact than the deuteron, and the

TABLE II. Non-Coulomb C_0-separation energies in MeV calculated for the potential model parameters in Table I.

Model	B_C ($A=3$) (np tensor force)	B_C ($A=4$) (np tensor force)	B_C ($A=4$) (np central force)
1	1.1	6.6	8.7
2	1.8	7.8	10.2
3	1.3	7.3	9.9
4	<0	2.0	3.8

long-range tensor force is less effective in binding that system. For each of the models 1–3, the $A=3$ charmed nucleus has a small separation energy, approximately 1–2 MeV. The $A=4$ system has a separation energy of some 7–8 MeV. However, as noted above, these results most likely represent upper bounds on the binding energies.

The binding energies resulting from model 4 are smaller. The $A=3$ system is just unbound; a 4% D state tensor force model, instead of 7%, would just bind. The $A=4$ system shows a C_0-separation energy of about 2 MeV. Here, we have assumed that the SU(4) symmetry is broken by meson masses as well as baryon masses. The size of this effect due to the meson masses can be seen from a comparison of results for models 3 and 4.

As was emphasized above, Coulomb effects have been ignored up to this point, but they must be included in any realistic estimate. They are significant in splitting the $A=4$ isodoublet, and they most likely ensure that the $A=3$ charmed nucleus is unbound. For our purposes, we shall assume that radial scaling gives

$$E_{\mathrm{Coul}}({}^3_C\mathrm{He})\cong[\langle r^2({}^3\mathrm{He}-{}^3\mathrm{H})\rangle/\langle r^2({}^2\mathrm{H})\rangle]^{1/2} \times E_{\mathrm{Coul}}({}^3\mathrm{He})\cong -0.6\ \mathrm{MeV}\ ,$$

while

$$E_{\mathrm{Coul}}({}^4_C\mathrm{He})\cong E_{\mathrm{Coul}}({}^3\mathrm{He})=-0.76\ \mathrm{MeV}$$

and

$$E_{\mathrm{Coul}}({}^4_C\mathrm{Li})\cong 3E_{\mathrm{Coul}}({}^3\mathrm{He})=-2.3\ \mathrm{MeV}\ .$$

Thus, one must subtract some 0.6 MeV from results quoted in Table II for the $A=3$ system to obtain the C_0-separation energy for ${}^3_C$He. Likewise, one must subtract 0.76 and 2.3 MeV, respectively, from the $A=4$ results of Table II in order to obtain the C_0-separation energies for ${}^4_C$He and ${}^4_C$Li.

V. CONCLUSIONS

Based upon the numerical results quoted in Table II and our simple estimates of the Coulomb energies for each of the systems, we conclude the following. ${}^3_C$He is quite likely to be unbound. Certainly, if it is bound, the C_0-separation energy will be small and this species will be very difficult to observe. The ground state of ${}^4_C$He should have a C_0-separation energy of the order of 1–3 MeV. The large Coulomb energy of ${}^4_C$Li will most likely place this species in the same category as ${}^3_C$He—unbound or very weakly bound at best.

If the assumption of SU(4) symmetry for coupling constants is a good guide, then it is possible to make an estimate of the ${}^5_C$Li ground state energy (C_0^+ plus an alpha particle). The results of Table II indicate that the non-Coulomb binding energies of the S-shell charmed nuclei will be similar to those of the corresponding Λ hypernuclei. Since the Λ-separation energy of ${}^5_\Lambda$He is about an MeV larger than that of ${}^4_\Lambda$He, whereas the Coulomb energy of ${}^4_C$Li is about an MeV larger than that of ${}^4_C$He, one would expect the C_0 separation of ${}^5_C$Li to be similar to that of ${}^4_C$He; i.e.,

$$B_C({}^5_C\mathrm{Li})\cong B_C({}^4_C\mathrm{He})\ .$$

However, if the C_0-C_1 conversion is not as large an effect as the Λ-Σ conversion, then the $A=5$ C_0-separation energy could be as much as 2–3 MeV larger.

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