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Classification of exchange currents

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After expansion of the vector and axial vector currents in powers of (v/c), a heretofore unremarked regularity results. Meson exchange currents can be classified into types I and II, according to the way they satisfy the constraints of special relativity. The archetypes of these two categories are the impulse approximation to the vector and axial vector currents. After a brief discussion of these constraints, the $(\rho\pi\gamma)$ and $(\omega\sigma\gamma)$ exchange currents are constructed and classified, and used to illustrate a number of important points which are often overlooked.

[NUCLEAR REACTIONS Meson exchange currents, classification scheme.]

I. INTRODUCTION

Classification schemes in physics serve two useful purposes. By dividing a complicated system into separate components, the body of knowledge of that system is more easily assimilated. In addition, the separate classified components of the system may have distinct important physical characteristics which obtain simply by belonging to those classifications. The classification scheme for meson exchange currents that we introduce below illustrates both points.

Meson exchange currents in nuclear physics were introduced many years ago[1,2] in order to satisfy an important physical principle: current conservation. As time passed the name "meson exchange currents" was adopted by other fields in nuclear physics to signify processes where mesons *explicitly* influence the physics, even when no conservation principle obtains. These processes have become an important topic of study.

The importance of such currents is quantitative, as well as qualitative. Because a nucleus is a weakly bound many-body system, we expect potential- and kinetic- (energy) dependent quantities to be roughly the same order of magnitude. Thus, meson-exchange (potential-dependent) and impulse approximation (kinetic or potential-independent) contributions to an observable of the same order (to be discussed below) should be roughly equal in magnitude in most cases.[3,4] This rough argument appears to be be borne out in practice. An example is the contribution of meson exchange currents to isovector magnetic moments. Although the relative contributions are approximately 10—20%, the impulse approximation result contains a dimensionless parameter (the nucleon isovector magnetic moment): 4.7 μ_N. Were it not for this factor, the exchange currents would be roughly comparable to the impulse approximation.

One important conceptual element of these "currents" which has not been emphasized is their role in maintaining Lorentz invariance.[4–8] Since it is conventional to treat nuclei as weakly bound systems of nucleons, which are slowly moving *on the average*, we will assume henceforth that we have expanded all the operators pertinent to the nucleus as a power series in (v/c), where v is a typical nuclear velocity. Since the scale of nucleon velocities is set by (p/M), where M is the nucleon mass and p is a typical nucleon momentum, an attractive alternative is to count relative powers of $(1/M)$ instead of $(1/c)$. In doing so, two caveats must be borne in mind:

(1) The nuclear potential should be treated as order $(1/M)$ for a weakly bound system such as a nucleus, since $\langle T\rangle \sim -\langle V\rangle$ and $T\sim 1/M$.

(2) For *specific processes* it is possible[4] for leading-order terms to be order $(1/c^2)$ without any explicit powers of $(1/M)$.

Caveat (1) is physically important because it shows that ordinary static meson exchange currents [no *explicit* powers of $(1/M)$] should be comparable to the usual convection and spin magnetization currents as argued previously; both are the same order in $(1/c)$.

This expansion is best illustrated by the nuclear electromagnetic current $J^\mu=(\rho,\vec{J})$. In what follows the subscript "0" indicates no potential dependence (i.e., it does not depend on meson-nucleon coupling constants), while "ex" signifies such a dependence. The charge operator of nonrelativistic order $(v/c)^0$ is ρ_0, while relativistic corrections are of order $(v/c)^2$ and higher: $\Delta\rho_0$ and $\Delta\rho_{ex}$. The leading order (v/c) nonrelativistic current operators are $\vec{J}_0$ and $\vec{J}_{ex}$, while corrections are of order $(v/c)^3$: $\Delta\vec{J}_0$ and $\Delta\vec{J}_{ex}$. Thus we have

$$J^\mu \cong (\rho_0+\Delta\rho_0+\Delta\rho_{ex}+\cdots, \vec{J}_0+\vec{J}_{ex}+\Delta\vec{J}_0+\Delta\vec{J}_{ex}+\cdots) .$$

The same type of decomposition applies to the axial vector current $(J^5)^\mu=(\rho^5,\vec{J}^5)$, except that $\vec{J}_0{}^5$ is of order (1), there is no $\vec{J}_{ex}{}^5$, while $\rho_0{}^5$ and $\rho_{ex}{}^5$ are of order (v/c), etc. Thus

$$(J^5)^\mu \cong (\rho_0{}^5+\rho_{ex}{}^5+\Delta\rho_0{}^5+\Delta\rho_{ex}{}^5+\cdots, \vec{J}_0{}^5+\Delta\vec{J}_0{}^5+\Delta\vec{J}_{ex}{}^5+\cdots) .$$

Both of the decompositions are illustrated in the tableaux of Fig. 1 with the electromagnetic (vector) and axial vector currents depicted on the left- and right-hand sides, respectively. Indeed, these tableaux are the essence of our extremely simple classification scheme. The electromagnetic current is the archetype of *type I* nuclear four-vectors: The nonrelativistic time component is dominant, while the nonrelativistic current operator is a "factor" of (v/c) smaller. The axial current is the archetype of *type II* four-vectors: The nonrelativistic space component is dominant, while the nonrelativistic time component is a factor of (v/c) smaller. We will show below that the structure of Fig. 1 follows from the constraints of special relativity.

What utility does such a classification have? One is usually interested in a specific physical process such as electron scattering via its interaction with the nuclear charge or current operator. Classification of meson exchange currents according to our scheme immediately tells us whether the leading-order meson exchange contribution is the charge or current density and which operator has nonstatic elements (i.e., momentum dependence).

Let us examine the electromagnetic current in some detail. The ordinary potential-independent parts of the current are type I, by *fiat.* The one-boson-exchange currents, such as π exchange, ρ exchange, etc., are of type II if the mesons are isovector, and generally of type I if the mesons are isoscalar. In the former case charged meson exchange dictates a static meson exchange current, which establishes the primacy of the space component of the four-vector; the exchange current is therefore of type II. In the latter case, the absence of a net exchange of charge requires the currents to be nonstatic [i.e., they have the form $(\Delta\rho_{ex},\Delta\vec{J}_{ex})$]. In such cases we expand our scheme slightly by denoting such currents type I′ or type II′, the prime denoting no static limit for *any* component of the four-vector. Thus the usual types of vector exchange currents are either type II or type I′.

It is clear, therefore, that if one is calculating magnetic moments, one should concentrate on type II exchange currents. Are there any exchange currents of type I? We will explicitly calculate such a current in Sec. III.

The primed (nonstatic) categories are special because of a serious technical problem associated with their calculation and implementation. Nonstatic current or charge operators of order $(v/c)^2$ beyond the nonrelativistic limit generate contributions to matrix elements of the same order. Wave functions will also have corrections of order $(v/c)^2$ and affect matrix elements in that order. A consistent treatment of matrix elements *requires* that both operators and wave functions be consistently calculated to the same order. It has been found that different calculational techniques in general will lead to different charge, current, and potential operators which are nevertheless part of a unitarily equivalent family.[8-10] In order to preserve the invariance of the unitarily transformed matrix elements, it is absolutely essential that the Hamiltonian be a part of the unitary family. If this is not so, and currently popular "realistic" potential models are not members of *any* such family, the numerical values of such matrix elements will be ambiguous. Regrettably, many such calculations have been performed. These remarks apply to the calculation of all operators in Fig. 1 preceded by a Δ. It is also possible to exploit this ambiguity. The isoscalar one-pion-exchange current is generally type I′. By a judicious choice of unitary representation this can be converted to type II′. By means of this trick the relativistic corrections to the isoscalar part of this $\Delta\rho_{ex}$ can be converted from order $(v/c)^2$ to $(v/c)^4$.

In what follows we briefly illustrate in Sec. II the role relativity plays in our classification scheme, and work out in Sec. III two simple examples: the $(\rho\pi\gamma)$ and $(\omega\sigma\gamma)$ exchange currents. The results of this calculation are not new, but exemplify many of our points.

II. RELATIVITY

Our goal is to illustrate the degree to which special relativity determines the basic structure of charge and current operators, and not to display the panoply of transformations of the Poincaré group.

Consequently, we will adopt a primitive viewpoint whose utility is simplicity.[11]

Consider a classical charge configuration at rest in a particular inertial reference frame: $\rho(\vec{x})$. In another such frame, slowly moving with velocity $-\vec{V}$ with respect to the first, the charge density develops a current component: $\rho(x)\vec{V}/c$, the convection current. Similar considerations apply to a static current configurations $\vec{J}(x)$ in the first frame with no net charge (e.g., a wire loop with a current inside). From the viewpoint of the second frame of reference, there is a nontrivial charge density: $\vec{V}\cdot\vec{J}(\vec{x})/c$. Our considerations have applied to slowly moving systems only; Lorentz contraction, the Thomas precession, and other phenomena will also affect[8,12] the basic densities ρ and $\vec{J}$ in order $(V/c)^2$.

Precisely the same structure obtains for the matrix elements of the charge and current operators in a quantum mechanical many-body system. The wave functions contribute terms of order $(V/c)^2$, which we may ignore. Thus the operators themselves must contain the terms discussed above. Examples are provided by the impulse approximation to the vector and axial vector currents:

$$\rho_0(\vec{x})=\sum_{i=1}^{A} e_i\delta^3(\vec{x}-\vec{x}_i)\ , \tag{1a}$$

$$\vec{J}_0(\vec{x})=\sum_{i=1}^{A} e_i\left\{\frac{\vec{p}_i}{2M_i},\delta^3(\vec{x}-\vec{x}_i)\right\} + \text{“spin terms”} \to \left\{\frac{\vec{P}}{2M_t},\rho_0(\vec{x})\right\}+\cdots\ , \tag{1b}$$

$$\vec{J}_0{}^5(\vec{x})=\sum_{i=1}^{A}\vec{\sigma}_i\delta^3(\vec{x}-\vec{x}_i)\tau_i^\alpha\ , \tag{2a}$$

$$\rho_0{}^5(\vec{x})=\sum_{i=1}^{A}\left\{\frac{\vec{\sigma}_i\cdot\vec{p}_i}{2M_i},\delta^3(\vec{x}-\vec{x}_i)\right\}\tau_i^\alpha \to \left\{\frac{\vec{P}}{2M_t}\cdot\vec{J}_0{}^5(\vec{x})\right\}+\cdots\ . \tag{2b}$$

Matrix elements of the total nuclear momentum $\vec{P}$ generate the expected forms with

$$\vec{V}\equiv(\vec{P}_f+\vec{P}_i)/2M_t\ ,$$

the average velocity of a nucleus undergoing a transition. In the above expressions, $\vec{x}_i$, $\vec{p}_i$, $\vec{\sigma}_i$, $\vec{\tau}_i$, M_i, and e_i are the coordinate, momentum, (Pauli) spin, isospin, mass, and charge of the ith particle while

$$M_t=\sum_{i=1}^{A}M_i$$

is the total mass of the A particles.

These two examples illustrate the two ways a slowly moving system which is “essentially” nonrelativistic can accommodate relativity: type I and type II. The order of the first two “rungs” of the ladder structure in Fig. 1 is determined by which member of the pair $(\rho,\vec{J})$ for a slowly moving system is given in terms of the other by a proportion involving V/c. This ladder structure continues to all orders in $(1/M)$ or (v/c). The more formal way of describing this is to write the conditions which guarantee that $(\rho,\vec{J})$ transform as a four-vector[5–8,12]:

$$[\vec{K},\rho(0)]=i\vec{J}(0)\ , \tag{3a}$$

$$[K^\alpha,J^\beta(0)]=i\delta^{\alpha\beta}\rho(0)\ . \tag{3b}$$

The “boost” operator, $\vec{K}$, can be expanded[5–7] in a series in $1/M$ also:

$$\vec{K}=M_t\vec{R}+\left[\sum_i\{\vec{p}_i{}^2,\vec{r}_i\}/2M_i - \sum_i\vec{\sigma}_i\times\vec{p}_i/4M_i+\vec{R}V_0+\vec{w}\right]+\cdots$$
$$=\vec{K}_0+\Delta\vec{K}+\cdots\ , \tag{3c}$$

where the first term is given by $\vec{K}_0=M_t\vec{R}$ in terms of the usual center-of-mass coordinate $\vec{R}$ of the system, V_0 is the nonrelativistic potential appropriate to the system, and $\vec{w}$ depends on the potential parameters. Note that $\vec{K}_0$ is of order M, while $\Delta\vec{K}$ is of order $1/M$, both explicitly and implicitly through V_0 and $\vec{w}$, while the unwritten higher-order terms indicated by the ellipsis are of order $(1/M^3)$ and higher. Examples of the potential-dependent boost $\vec{w}$ can be inferred from the results of Refs. 8–10 for several physical models. The import of

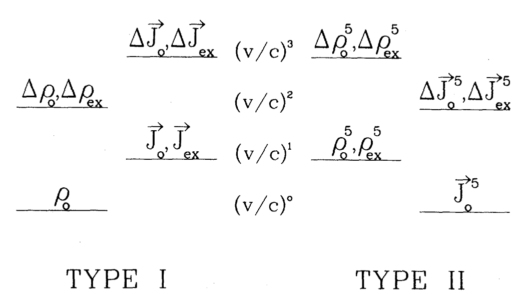

FIG. 1. Tableaux illustrating expansion of type I and type II currents in powers of (v/c). Quantities preceded by a Δ are relativistic corrections of relative order $(v/c)^2$.

this discussion is that $K \sim M + 1/M + 1/M^3 + \cdots$ [i.e., an expansion in powers of $(1/c)^2$] and this both determines and preserves the ladder structure. An example of this is provided by Eq. (3b). The commutation of $\vec{K}_0$ with the $\vec{P}/M_t$ part of $\vec{J}_0$ generates ρ_0, while commutators of $\Delta\vec{K}$ with $\vec{J}_0$ and $\vec{K}_0$ with $\Delta\vec{J}$ generate $\Delta\rho$, the latter terms clearly being of order $1/M^2$, as they must be. Since Eqs. (3) apply to any four-vector, it is only because we have expanded in powers of $(1/c)$ that any differentiation between types I and II arises. This differentiation may be appropriate to a nucleus, but hardly relevant to a strongly bound relativistic system where such expansions would be *a priori* nonsensical. It is because of this restriction that our classification scheme is not fundamental or especially profound. As we will show below, it has some practical utility for comparing and discussing exchange currents.

For systems having a strong interaction Hamiltonian H_0, which are more appropriately described by an electric dipole moment density $\vec{D}(\vec{x})$ than by $\rho(\vec{x})$, and by a magnetic dipole moment density $\vec{\mu}(\vec{x})$ than by $\vec{J}(\vec{x})$, the appropriate transformations are given by

$$\rho(\vec{x}) = -\vec{\nabla}\cdot\vec{D}(\vec{x}) \ , \tag{4a}$$

$$\vec{J}(\vec{x}) = \vec{\nabla}\times\vec{\mu}(\vec{x}) + i[H_0, \vec{D}(\vec{x})] \ , \tag{4b}$$

$$\vec{D} = \vec{\nabla}\times\vec{\mu}/c + \cdots \quad \text{or} \quad \rho = \vec{\nabla}\cdot\vec{J}/c + \cdots \ , \tag{4c}$$

$$\vec{\mu} = -\vec{\nabla}\times\vec{D}/c + \cdots \quad \text{or} \quad \vec{J} = \vec{\nabla}\rho/c + \cdots \ . \tag{4d}$$

The two forms given in each of Eqs. (4c) and (4d) are equivalent. Type I currents will satisfy Eq. (4d) in lowest order, while type II currents will satisfy Eq. (4c). The current defined in Eqs. (4) is obviously conserved for any $\vec{\mu}$ and $\vec{D}$.

III. MODEL CALCULATION

The model calculation we perform is for a class of meson exchange processes which involve virtual meson electromagnetic decays. Such a process is indicated in Fig. 2 where the nucleus (double lines) interacts with π and ρ mesons, which in turn interact with an external electromagnetic field (vertical wiggly line). This can be viewed as a ρ meson, which is being exchanged between nucleons, electromagnetically decaying into a photon and a pion which is later absorbed (or vice versa) by a different nucleon. We will restrict ourselves to $(VP\gamma)$ and $(VS\gamma)$ processes involving a vector meson (V) and a scalar (S) or pseudoscalar (P) meson. These processes have had a long and controversial history,[13–23] which is intertwined with that of the deuteron magnetic moment problem. It was realized rather early that ordinary one-boson-exchange currents of the true exchange type could not contribute to the deuteron. By true exchange we mean processes exemplified by Fig. 2, where the photon lands on a meson and *not* on a nucleon. Since a photon has charge conjugation -1, and the deuteron has isospin 0, it is rather easy to prove[24] that the incoming photon can only "dissociate" into mesons having a total G parity of -1. Since $G = (-1)^n$ for a system of n pions, only an odd number of pions can contribute; this is exemplified by the ρ-π system. The ω-π exchange currents with $G = +1$ contribute to the ${}^3S_1$-${}^1S_0$ deuteron transition, for example, but not to the deuteron ground state. Much of the controversy concerning calculations has dealt with the sizes of coupling constants and form factors.[25]

The two types of electromagnetic vertices we require are described by the following interaction Lagrangians:

$$L_{\rho\pi\gamma} = e_p \frac{g_{\rho\pi\gamma}}{2m_\rho} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} \rho^\gamma \frac{\partial \pi}{\partial x_\delta} \tag{5a}$$

and

$$L_{\omega\sigma\gamma} = e_p \frac{g_{\omega\sigma\gamma}}{m_\omega} F^{\alpha\beta} \omega_\alpha \frac{\partial \sigma}{\partial x^\beta} \ . \tag{5b}$$

In Eq. (5a) the isospin labels on the ρ and π field have been suppressed, and should have the form $\vec{\rho}\cdot\vec{\pi}$. The similar $(\omega\pi\gamma)$ process is structurally identical, with meson fields appearing in the form $\omega\pi_3$. In these equations $\epsilon_{\alpha\beta\gamma\delta}$ is the totally antisymmetric tensor and $F^{\alpha\beta}$ is the electromagnetic field tensor. The masses of the ρ and σ are m_ρ and m_σ, while the coupling constants are $g_{\rho\pi\gamma}$ and $g_{\omega\sigma\gamma}$. We have chosen to extract a factor of the proton charge e_p from $g_{\rho\pi\gamma}$, which is dimensionless. Because we are only interested in the lowest-order charge and current operators, the effects of retardation (finite meson propagation speed) can be ignored and the meson equations of motion become static and easy to deal with.

The elements of the electromagnetic field tensor are determined by the external electric and magnetic fields $\vec{E}$ and $\vec{B}$. We therefore write the energy in the form

$$H = -L = -e_p \int \vec{D}(\vec{x})\cdot\vec{E}(\vec{x})\,d^3x$$

$$-e_p \int \vec{\mu}(\vec{x})\cdot\vec{B}(\vec{x})\,d^3x \ .$$

We note that the space components of the vector meson fields are order $(1/M)$ with respect to the scalar component, and time derivatives of the meson fields are retardation corrections which we can ignore. We find

$$\vec{D}(\vec{x}) = -\frac{g_{\rho\pi\gamma}}{m_\rho}(\vec{\rho}(\vec{x})\times\vec{\nabla}\pi(\vec{x})) \to -\frac{g_{\rho\pi\gamma}g_\rho f}{m_\rho\mu_\pi}\sum_{i\neq j}\vec{\tau}_i\cdot\vec{\tau}_j\vec{\nabla}_i[\vec{\sigma}_i\cdot\vec{\nabla}_i h_0(\vec{x}-\vec{x}_i)] \times\left[\left\{\frac{\vec{p}_j}{2M}, h_0'(\vec{x}-\vec{x}_j)\right\} + \frac{\bar{\mu}}{2M}\vec{\sigma}_j\times\vec{\nabla}_j h_0'(\vec{x}-\vec{x}_j)\right] \tag{6a}$$

and

$$\vec{\bar{\mu}}(\vec{x}) = -\frac{g_{\rho\pi\gamma}}{m_\rho}\rho_0\vec{\nabla}\pi \to \frac{g_{\rho\pi\gamma}g_\rho f}{m_\rho\mu_\pi}\sum_{i\neq j}\vec{\tau}_i\cdot\vec{\tau}_j\vec{\nabla}_i[\vec{\sigma}_i\cdot\vec{\nabla}_i h_0(\vec{x}-\vec{x}_i)]h_0'(\vec{x}-\vec{x}_j) \tag{6b}$$

for the $(\rho\pi\gamma)$ case, while obtaining

$$\vec{D}(\vec{x}) = \frac{g_{\omega\sigma\gamma}}{m_\omega}\omega_0\vec{\nabla}\sigma \to \frac{g_{\omega\sigma\gamma}g_\sigma g_\omega}{m_\omega}\sum_{i\neq j}h_0(\vec{x}-\vec{x}_i)\vec{\nabla}_j h_0'(\vec{x}-\vec{x}_j) \tag{7a}$$

and

$$\vec{\bar{\mu}}(\vec{x}) = -\frac{g_{\omega\pi\gamma}}{m_\omega}\vec{\omega}\times\vec{\nabla}\sigma \to \frac{g_{\omega\sigma\gamma}g_\omega g_\sigma}{m_\omega}\sum_{i\neq j}\left[\left\{\frac{\vec{p}_i}{2M}\times, \vec{\nabla}_j h_0'(\vec{x}-\vec{x}_j)h_0(\vec{x}-\vec{x}_i)\right\} + \frac{\bar{\mu}}{2M}[\vec{\sigma}_i\times\vec{\nabla}_i h_0(\vec{x}-\vec{x}_i)]\times\vec{\nabla}_j h_0'(\vec{x}-\vec{x}_j)\right] \tag{7b}$$

for the $(\omega\sigma\gamma)$ case. We have written the static meson propagators as

$$h_0(\vec{z}) = e^{-mz}/4\pi z$$

for a meson with mass m. In Eq. (6) the primed h_0 refers to the ρ, and in Eq. (7) it refers to the σ. The coupling constants g_σ, g_ω, and g_π couple the σ, ω, and π mesons to the nucleons of mass M, and $f = \mu_\pi g_\pi/2M$ is the effective π-nucleon coupling constant for the pion of mass μ_π. The quantity $\bar{\mu}$ is the magnetic moment coupling parameter of ρ mesons to nucleons; in the vector dominance model its numerical value is $\mu_p - \mu_n$, the isovector nucleon magnetic moment.

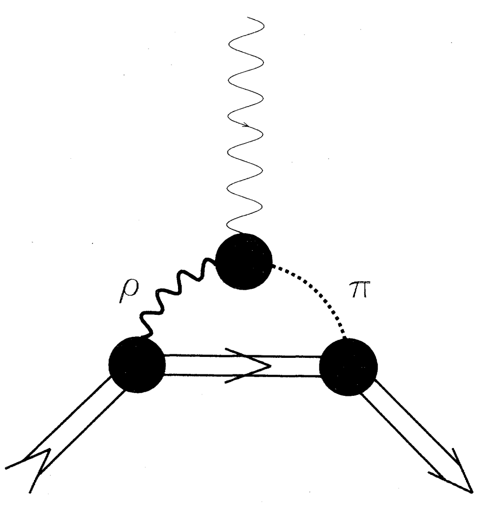

FIG. 2. Feynman diagram for $(\rho\pi\gamma)$ exchange currents. The double line depicts a nucleus, the dashed line a pion, and the heavy (lower) and light (upper) wiggly lines show the ρ meson and photon, respectively.

In order to verify Eq. (4), we need to substitute

$$\vec{p}_i = \vec{\pi}_i + M\vec{P}/M_t$$

for each $\vec{p}_i$. This is extremely easy to do and we find that the $(\rho\pi\gamma)$ case in Eqs. (6) and (7) satisfies (4c), while the $(\omega\sigma\gamma)$ case satisfies Eq. (4d). Thus $(\omega\sigma\gamma)$ exchange currents are type I, while $(\rho\pi\gamma)$ exchange currents are type II.

IV. DISCUSSION AND CONCLUSIONS

The preceding exercise illustrates a number of points: (1) The requirements of special relativity for the charge and current operators guarantee the existence of certain nonlocal terms which classify the charge-current operators as type I or type II; that is, their type predicts the location of the onset of nonlocality. (2) Nonlocal terms are usually considered to be a nuisance which can be ignored; we emphasize their fundamentality. (3) The original

Lagrangian was manifestly gauge and Lorentz invariant. The mapping of a four-dimensional form to an effective three-dimensional form maintains the Lorentz and gauge invariances, but it is no longer manifest. The requirements of special relativity become particularly complicated in form when one goes beyond[4–8,12] the (v/c) treatment we have made here. (4) The current is model dependent; no *ad hoc* argument or "minimal" substitution can guarantee the form of the magnetic interaction. (5) Our treatment of the charge and current operator in the model problems emphasized $\vec{\mu}$ and $\vec{D}$, because this simplified the formulae. If we had calculated $\vec{j}$ and ρ directly we would have found that the time derivative in $\vec{E}(-\vec{\nabla}\phi-\dot{\vec{A}})$ generated the $i[H_0,\vec{D}]$ term. This essential time dependence in the problem is the simplest example of retardation, and illustrates the mapping of a complicated time-dependent many-body (i.e., mesons) problem onto the purely nuclear Hilbert space. Moreover, the $\vec{p}^{\,2}/2M_t$ part of the kinetic energy in H_0 is essential in proving Eq. (4). (6) The potential part of H_0 in that commutator generates three-nucleon exchange currents, since V is a sum of two-body operators; they are essential for current conservation and illustrate how retardation naturally brings multinucleon exchange currents into any discussion of two-nucleon exchange currents. (7) Because meson propagators depend on $\hbar/mc$, the meson Compton wavelength, it is not entirely trivial to ascertain the order in $1/c^2$ of a particular multimeson exchange current. Nevertheless, following the rules advocated in Ref. 4, the charge density of the $(\omega\sigma\gamma)$ model is order $(1/c^2)$ in spite of the fact that this model is static in the nuclear parameters (no explicit factors of $1/M$) and of type I. It should be correspondingly small. (8) Most importantly, the classification into types I and II immediately shows which of ρ or $\vec{j}$ is the larger in powers of v/c.

As our last example we discuss the axial vector current. If one assumes a naive (and incomplete) $\gamma_\mu\gamma_5$ model of coupling the pion field to the nucleon and thereby to the axial current via the pair terms in that model, the latter are sufficiently weak that the pion exchange currents are of type I′. Thus they would be $(v/c)^2$ corrections to both ρ^5 and $\Delta\vec{J}^{\,5}$ and consequently negligible. Chiral symmetry dictates that in addition there should exist a seagull-type of coupling between axial current, nucleon, and pion.[26,27] This term generates $\rho_{\text{ex}}{}^5$ and the axial exchange current becomes type I ($\rho_{\text{ex}}{}^5$ and $\Delta\vec{J}_{\text{ex}}{}^5$). This has the consequence that the axial charge operator has exchange current contributions of the same order of magnitude as the impulse approximation, which may have important consequences in some applications. The same result is obtained in a different way in the σ model, via the strong pair terms of that model.

In atomic physics the same procedure can be applied to photon exchange as the genesis of the electron-electron force.[28] In that case it can be shown that the traditional method of calculating in Coulomb gauge leads to a nonstatic "photon exchange" charge operator of order $(v/c)^4$ and a corresponding current operator of order $(v/c)^3$. The photon-exchange currents are therefore of type II′. Finally, Table I classifies the impulse approximation and common exchange currents for the vector and axial vector currents.

TABLE I. Resolution of impulse approximation and common exchange current contributions to the vector and axial vector currents into types. The subscripts "s" and "v" refer to isoscalar and isovector contributions to the vector current, while "A" refers to a contribution to the axial vector current. The notation (M_1,M_2) refers to contributions of the true exchange currents of the type $(M_1M_2\gamma)$ to the vector current, while "I" refers to the impulse approximation. The prime on the last π_A entry refers to the unphysical (pure) $\gamma_\mu\gamma_5$ model of the axial exchange current. The $(\pi\pi)$ entry refers to the two-π exchange current calculated in Ref. 29, while "γ" refers to the photon-exchange current of atomic physics.

Type I:	I_s, I_v, (ω,σ), π_A, $(\pi\pi)_v$
Type II:	I_A, (ρ,π), (ω,π), (ρ,η), π_v,ρ_v
Type I′:	π_s, ω_s, σ_s, π_A'
Type II′:	γ

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$$\Gamma_{\text{th}} = \alpha g_{\rho\pi\gamma}{}^2 m_\rho (1 - \mu_\pi{}^2/m_\rho{}^2)^3/24$$
$$\cong (211.6 \text{ keV}) g_{\rho\pi\gamma}{}^2$$

and $\Gamma_{\text{exp}} = 67 \pm 7$ keV we obtain $|g_{\rho\pi\gamma}| = 0.56 \pm 0.03$, somewhat in excess of the old experimental value.

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