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Baryon rapidity distribution and stopping power of high-energy colliding nuclei

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A very simple estimate is made of the baryon rapidity distribution to be expected from central collisions of high energy nuclei. This estimate is based upon straight-line trajectories and some average rapidity loss per nucleon-nucleon collision. It is found that U + U will become transparent at a beam energy of about 500 GeV/nucleon, or 15 + 15 GeV/nucleon for colliding beams.

[NUCLEAR REACTIONS Relativistic nuclear collisions, straight-line trajectories, stopping power, baryon rapidity distribution.]

INTRODUCTION

The primary motivation for colliding nuclei at high energy is to create hadronic matter with a large energy density. At high enough beam energy we expect the nuclei (or, more accurately, the baryon numbers of the respective nuclei) to be transparent to each other. Goldhaber[1] has estimated that the maximum attainable baryon compression may occur for central collisions of uranium nuclei at center of mass (c.m.) kinetic energies of 1 to 4 GeV/nucleon. Even though the nuclei should be transparent at higher energy, the baryon compression in each nucleus may still continue to increase, albeit very slowly, with increasing energy. This is the result of arguments by Anishetty, Koehler, and McLerran[2] using the parton model. According to Mueller,[3] Bjorken,[4] and Kajantie and McLerran,[5] the energy density associated with the midrapidity region will continue to grow with beam energy, although the net baryon number there will be zero.

For central collisions of equal mass nuclei the baryon rapidity distribution should exhibit the following behavior. For low beam energies the nuclei will stop each other and there will be a peak in the rapidity distribution at midrapidity. For large beam energies the nuclei will be transparent to each other and there will be a hole in the rapidity distribution at midrapidity. The purpose of this paper is to discuss a simple, plausible model for the baryon rapidity distribution as a function of beam energy. The model has some nice scaling properties with respect to nuclear size and beam energy, and gives relativistically invariant results. Unfortunately it can make no definite predictions for the maximum baryon or energy densities attainable during the collision since it is formulated in rapidity space.

FORMULATION OF THE MODEL

Although we will quickly specialize to central collisions between equal mass nuclei, it may be worthwhile to first consider the general case. Let there be a collision between a beam nucleus B and a target nucleus T at impact parameter $\vec{b}$. Following Glauber's theory,[6] the rows on rows model of Hüfner and Knoll,[7] and the firestreak model of Myers,[8] we estimate the number $N_T(\vec{b}+\vec{s})$ of nucleon-nucleon collisions undergone by a nucleon from B by assuming that it follows a straight-line trajectory (see Fig. 1). This number is

$$N_T(\vec{b}+\vec{s})=\sigma_{NN}\int dz_T\rho_T(\vec{b}+\vec{s}+\vec{z}_T)\ . \tag{1}$$

(When numerical values are necessary we take $\sigma_{NN}=40$ mb.) Furthermore, we assume that the beam nucleon suffers a constant rapidity loss y_0 per collision. (It would be possible to relax this last as-

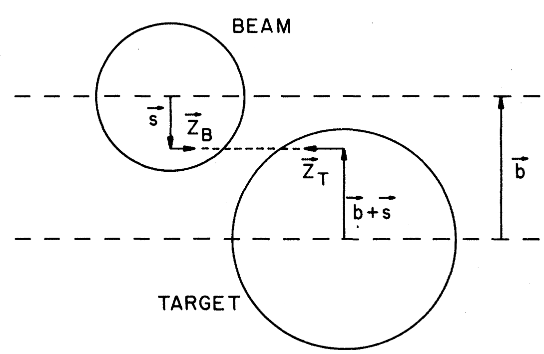

FIG. 1. Geometry of nucleus-nucleus collisions. The vector $\vec{s}$ is perpendicular to the beam axis; the vectors $\vec{z}_B$ and $\vec{z}_T$ are parallel to the beam axis.

sumption without seriously complicating the model.) Hence the rapidity loss suffered by the row of nucleons at fixed $\vec{s}$ in B is

$$y_{\rm loss}(\vec{b}+\vec{s})=y_0 N_T(\vec{b}+\vec{s}) . \quad (2)$$

It is not necessary to think of the dynamics as a sequence of binary, nucleon-nucleon collisions. It is possible to think of a beam nucleon interacting almost instantaneously or coherently with the whole row of target nucleons, as long as the rapidity loss is given by Eq. (2). Since the number of beam nucleons which suffer this rapidity loss is

$$d^2s \int dz_B \rho_B(\vec{s}+\vec{z}_B) , \quad (3)$$

we find the baryon rapidity distribution of beam nucleons to be

$$\frac{dN_B}{dy}(\vec{b})= \int d^2s\, dz_B \rho_B(\vec{s}+\vec{z}_B)\delta \left[y-y_B+y_0\sigma_{NN} \int dz_T \rho_T(\vec{b}+\vec{s}+\vec{z}_T)\right] , \quad (4)$$

where y_B is the initial beam rapidity. Similarly the baryon rapidity distribution of target nucleons is

$$\frac{dN_T}{dy}(\vec{b})= \int d^2s\, dz_T \rho_T(\vec{b}+\vec{s}+\vec{z}_T)\delta \left[y-y_0\sigma_{NN} \int dz_B \rho_B(\vec{s}+\vec{z}_B)\right] . \quad (5)$$

Here we have taken the target rapidity y_T to be zero.

The net baryon rapidity distribution is not necessarily given by the sum of Eqs. (4) and (5). The reason is simple. Consider a collision between a tube of beam nucleons at $\vec{s}$ and a tube of target nucleons at $\vec{b}+\vec{s}$ as shown in Fig. 1. It may happen that the beam energy is low enough so that the beam and target nucleons are able to stop each other in their c.m. frame. The condition for this to happen is

$$y_B \le y_0\sigma_{NN} \left[\int dz_T \rho_T(\vec{b}+\vec{s}+\vec{z}_T) + \int dz_B \rho_B(\vec{s}+\vec{z}_B) \right] . \quad (6)$$

In that case the assumption of straight-line trajectories, which led to Eq. (2) and its counterpart for the target nucleons, is no longer reasonable. These nucleons will probably suffer strong compression and interaction effects in position space. Therefore, we supplement the model by assuming that those nucleons will be emitted with a Gaussian rapidity distribution, centered at the c.m. rapidity of the colliding tube-tube system, and with a variance of $0.5y_0$.

ESTIMATING y_0

What is a reasonable numerical value of y_0? First suppose that a pair of nucleons scatters elastically. It is known experimentally that, for not too large momentum transfer, the cross section may be parametrized as[9]

$$\frac{d\sigma}{dt}=C\exp(bt) . \quad (7)$$

Here t is the four-momentum transfer squared. For beam energies in the range 1—10 GeV, $b \simeq 6$ GeV^{-2}. For higher energies b increases and seems to reach an asymptotic value of about 10 GeV^{-2}. In the rest frame of the projectile nucleon, and in the nonrelativistic limit, $t=-\vec{q}^{\,2}$ and

$$y_{\rm loss}=\tfrac{1}{2}\ln\left[\frac{E+q_{\parallel}}{E-q_{\parallel}}\right]\simeq\frac{q_{\parallel}}{m} . \quad (8)$$

Then, averaging over the cross section $\exp(-b\vec{q}^{\,2})$,

$$\langle y_{\rm loss}\rangle = \left[\frac{\pi}{b}\right]^{1/2}\frac{1}{2m} . \quad (9)$$

With $b=6$ or 8 GeV^{-2} this yields a rapidity loss of 0.38 or 0.33, respectively.

On the other hand, if the scattering is inelastic leading to multiple pion production, the rapidity loss is in general greater. For beam energies in the range 10—30 GeV the momentum of each nucleon after the collision in the c.m. frame is roughly one half of its initial momentum. This is a result of an approximately flat longitudinal momentum distribution.[10,11] From this we would estimate an average rapidity loss of between $\frac{1}{2}$ and ln2.

For numerical purposes we shall use the reasonable value of $y_0=0.4$. If anything, this value is probably even conservative.

CENTRAL COLLISIONS

As an interesting application of this simple model we will from now on focus on central collisions of equal mass nuclei. We will assume a uniform density distribution

$$\rho(\vec{r})=\rho_0\theta(R-r) , \quad (10)$$

with $\rho_0=0.15$ nucleons fm^{-3} and $R=1.168$ fm$A^{1/3}$.

At what beam energy do nuclei of a particular size first become completely transparent to each other? Of course the maximum stopping power is provided by the tube of nucleons which runs down the center of the nucleus. This corresponds to $\vec{s}=0$ in Eq. (6). Therefore

$$y_B^{\max}=4y_0\sigma_{NN}\rho_0 R=1.121A^{1/3}\ . \tag{11}$$

The laboratory beam momentum per nucleon may then be calculated from

$$p_B^{\max}=m\sinh(y_B^{\max})\ , \tag{12}$$

where $m=939$ MeV is the nucleon mass, and so on. A plot of $A^{1/3}$ versus laboratory beam energy or c.m. beam energy in GeV per nucleon is presented in Fig. 2. According to this simple model α particles should become completely transparent above a beam energy of 2.05 GeV/nucleon, carbon nuclei above 5.55 GeV/nucleon, and uranium nuclei above 520 GeV/nucleon.

The baryon rapidity distribution takes a particularly simple form. From Eqs. (4), (5), and (10),

$$\frac{dN_T}{dy}(\vec{b}=0)=\frac{\pi}{2\rho_0^2\sigma_{NN}^3y_0^3}y^2=17y^2 \tag{13}$$

and

$$\frac{dN_B}{dy}(\vec{b}=0)=17(y_B-y)^2\ . \tag{14}$$

Notice that the shape and magnitude of these distributions are independent of nuclear size and beam energy. However, the rapidities of emitted baryons do not take on arbitrary values, but fall within certain limits. For target baryons

$$0\le y\le \min\{\tfrac{1}{2}y_B,\tfrac{1}{2}y_B^{\max}\}\ , \tag{15}$$

and for beam baryons

$$\max\{\tfrac{1}{2}y_B,\tfrac{1}{2}y_B^{\max}\}\le y\le y_B\ . \tag{16}$$

Recalling an earlier discussion we see that if $y_B\ge y_B^{\max}$ then the nuclei are completely transparent and the observed rapidity distribution dB/dy is just the sum of Eqs. (13) and (14). If $y_B<y_B^{\max}$ then some of the nucleons will be stopped in the c.m. frame and these will be given a Gaussian rapidity distribution.

First consider a beam rapidity of $y_B=1.6$ corresponding to an energy of 1.5 GeV/nucleon in the laboratory frame. Then

$$\frac{dB}{dy}=\frac{dN_T}{dy}+\frac{dN_B}{dy}+\frac{dN_{\text{stopped}}}{dy}\ , \tag{17}$$

where

$$\frac{dN_{\text{stopped}}}{dy}=(A-2.91)\left[\frac{2}{\pi}\right]^{1/2}\frac{2}{y_0}\times\exp[-2(y-\tfrac{1}{2}yB)^2/y_0^2]\ . \tag{18}$$

The 2.91 comes from counting the number of target baryons in the range $0\le y\le 0.8$,

$$N_T(y\le 0.8)=\int_0^{0.8}\frac{dN_T}{dy}dy=2.91\ , \tag{19}$$

and similarly for the beam baryons. For $y_B=3.2$, corresponding to a beam energy of 10 GeV/nucleon, $N_T(y\le 1.6)=23.27$.

Numerical results at these two energies are plotted in Figs. 3 and 4 for ^{12}C, ^{40}Ar, and ^{238}U. At 1.5 GeV even C nuclei are large enough to essentially stop all nucleons. Therefore, the baryon distributions are dominantly Gaussian. At 10 GeV, though, C nuclei have already become transparent. There is a gap in the rapidity distribution. The size of this gap will increase with increasing energy but the shape of the distribution at low and high rapidity is fixed forevermore in this model. On the other hand, U nuclei stop each other almost completely at this energy, as can be seen from the huge peak at midrapidity. Furthermore, the scaling with respect to nuclear size at low and high rapidity is quite evident at 10 GeV.

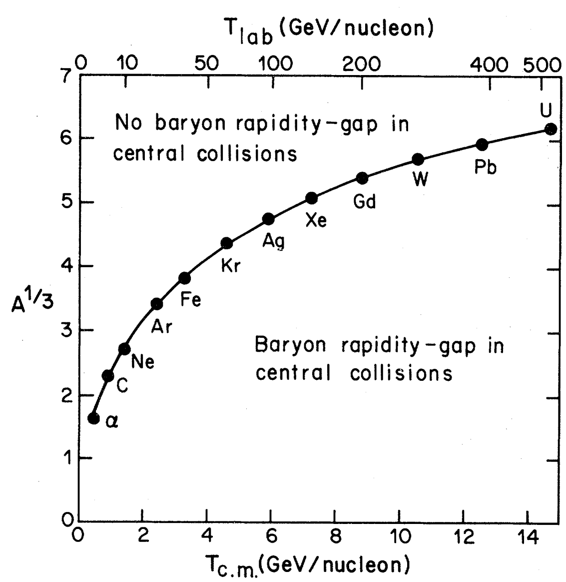

FIG. 2. A plot of the critical nuclear size as a function of beam energy below which a rapidity gap appears in the baryon distribution for central collisions.

CONCLUSION

Experiments on colliding nuclei at ultrarelativistic energies could conceivably be done at existing pro-

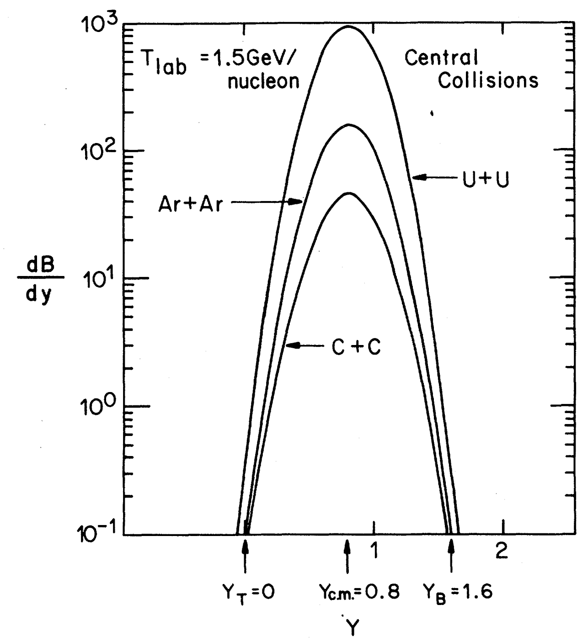

FIG. 3. A plot of the baryon rapidity distribution for central collisions of mass 12, 40, and 238 nuclei at 1.5 GeV/nucleon.

ton machines, such as the CERN PS ($\lesssim {}^{20}$Ne, $T_{\rm lab} \lesssim 12$ GeV/nucleon) or the CERN SPS ($\lesssim {}^{20}$Ne, $T_{\rm lab} \lesssim 200$ GeV/nucleon); or at dedicated heavy ion machines proposed for the future, such as an improved Bavalac (up to ^{238}U, $T_{\rm lab} \lesssim 10$ GeV/nucleon) or at VENUS (up to ^{238}U, $T_{\rm c.m.} \lesssim 20$ GeV/nucleon) at Lawrence Berkeley Laboratory (LBL). Clearly it is important to perform simple model estimates, such as in this paper, in order to clarify the physics questions for this new domain. Along this line the following remarks may be useful.

The model presented here is rather conventional, perhaps even conservative. Certainly one could make better estimates, such as using experimental *pp* rapidity distributions rather than taking a fixed value of y_0, and taking account of fluctuations in the number of nucleon-nucleon collisions. Since the rapidity distributions calculated here have referred to average or typical events, taking into account such fluctuations might tell us the probability of seeing very rare events, such as Ne nuclei stopping each other at $T_{\rm lab}=200$ GeV/nucleon. The assumption of straight-line trajectories can also be criticized, especially for energies below which the nuclei become completely transparent. If the central tubes of nucleons are able to stop each other then we may have the situation illustrated schematically in Fig. 5. During the collision there would be a buildup of pressure along the beam axis, making it favorable for the nucleons to move outward at 90° in the c.m. frame. Then this additional material may provide additional stopping power for the nucleons on the periphery of the nuclei. Indeed this effect has been seen in relativistic hydrodynamic calculations at lower energies.[12]

Although this model strongly suggests that high energy densities will be reached, possibly leading to the formation of quark-gluon matter at high enough energies, it can say nothing specific (at the moment)

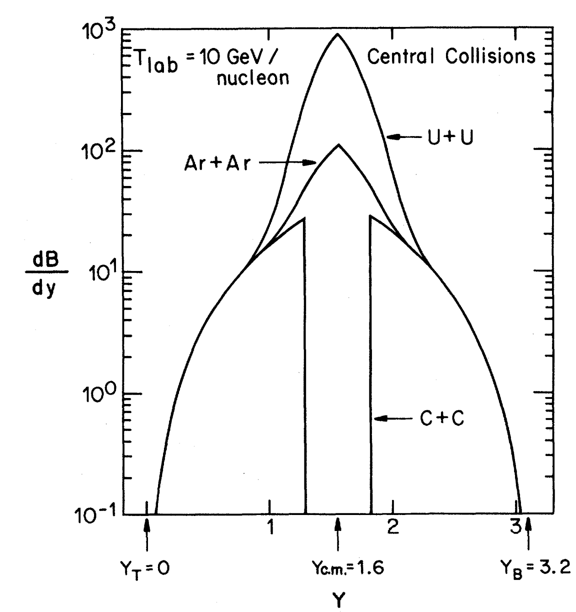

FIG. 4. A plot of the baryon rapidity distribution for central collisions of mass 12, 40, and 238 nuclei at 10 GeV/nucleon.

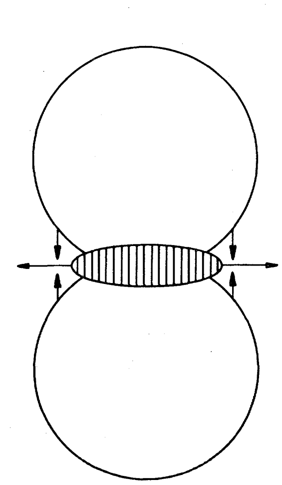

FIG. 5. Schematic illustration of how a pressure buildup along the beam axis could lead to additional stopping power for the nucleons on the periphery.

about this exciting new physics since it is formulated in rapidity space and not position space.

Finally it must be emphasized that even if a baryon rapidity gap opens up at high energy it may still be that the energy density associated with midrapidity will continue to grow. It would be filled with mesons and baryon-antibaryon pairs produced in the elementary nucleon-nucleon collisions. As is well known from hadron-nucleus experiments, these particles generally appear at later times outside the nuclear volume.

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