New class of supersymmetry in nuclei

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We suggest that a new class of supersymmetry may occur in nuclei. We construct energy formulas and classification schemes for two specific cases of this class. With this addition, supersymmetry considerations may be extended to many nuclei, thus providing a useful tool to analyze the corresponding spectra.

[NUCLEAR STRUCTURE Supersymmetries. Derived classification scheme and energy formulas for some odd-even nuclei. Application to ^{195}Pt.]

Supersymmetries have been introduced in the context of dual models,[1] supersymmetric field theories,[2] and supergravity.[3] In previous papers[4] we suggested that they may also be useful in providing a unified classification scheme of nuclear levels. The unification is achieved by treating together, in the same theoretical framework, the collective (bosonic) and single particle (fermionic) excitations. However, the examples discussed so far, both theoretically and experimentally, include only one moderately complex case, that of bosons with SO(6) symmetry and fermions with $j=\frac{3}{2}$,[4,5] and some simple cases in which bosons with SU(5) and SO(6) symmetry are coupled to fermions with $j=\frac{1}{2}$.[6] If only these cases were to exist, supersymmetry in nuclei would be intriguing for its implications to other fields of physics, but would not be a useful tool for analyzing the large variety of observed spectra. In this paper we wish to introduce a new, more complex, class of supersymmetry in nuclei. With the addition of this class, supersymmetry considerations can be extended to many nuclei, thus providing a powerful tool for analyzing nuclear spectra.[7]

In the case discussed up to now, supersymmetries were constructed by combining bosons and fermions into spinor groups (exploiting the accidental isomorphism of some Lie groups) and subsequently by embedding the spinor groups into a supergroup.[4] We now suggest a different way to construct supersymmetries, as follows. Suppose one has fermions with angular momenta $j,j',\ldots,$ and that these angular momenta can be split into a pseudo-orbital part, k, and a spin part, $s=\frac{1}{2}$. The pseudo-orbital part, k, does not necessarily need to coincide with the actual orbital angular momentum, although in some simple cases it may. Suppose now that the bosons can be described by the group chain $G^{(B)}\supset G'^{(B)},\ldots.$ If the pseudo-orbital angular momentum k sits in a representation of $G^{(B)}$ or $G'^{(B)},\ldots,$ then it can be combined with its bosonic counterpart to yield a common boson plus fermion $(B+F)$ group. This common group can be subsequently embedded into a supergroup. In order to make our procedure clear, we shall base our discussion on a description of the collective degrees of freedom of nuclei in terms of interacting bosons[8] and consider the case in which the bosons are described by the symmetry III (Ref. 9)

$$U^{(B)}(6)\supset SO^{(B)}(6)\supset SO^{(B)}(5)\supset SO^{(B)}(3)\supset SO^{(B)}(2)\ , \tag{1}$$

where we have used the same notation as in Ref. 4. We then consider, as an example, the case in which the odd fermions can occupy single particle levels with $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$. These angular momenta can be split into a pseudo-orbital part $k=0,2$ and a spin part $s=\frac{1}{2}$. Since $k=0,2$ sit in the six-dimensional representation of SU(6), [1], and SO(6), (1,0,0), we can, using the procedure described above, form the group chain

$$\begin{aligned}&U(6/12)\supset U^{(B)}(6)\times U^{(F)}(12)\supset U^{(B)}(6)\times U^{(F)}(6)\times U^{(F)}(2)\\&\supset SO^{(B)}(6)\times SO^{(F)}(6)\times SU^{(F)}(2)\supset SO^{(B+F)}(6)\times SU^{(F)}(2)\\&\supset SO^{(B+F)}(5)\times SU^{(F)}(2)\supset SO^{(B+F)}(3)\times SU^{(F)}(2)\\&\supset Spin(3)\supset Spin(2)\ ,\end{aligned} \tag{2}$$

where, once more, we have used the same notation as in Ref. 4 and $G^{(B+F)}$ indicates the common group obtained by the direct product of the corresponding boson and fermion groups. If we now write the Hamiltonian for the combined system of interacting bosons and fermions, $H=H_B+H_F+V_{BF}$, in terms of invariant operators of the chain (2), we can find its eigenvalues in closed form. For the case in which we have N bosons and M fermions, with $\mathcal{N}=N+M$ labeling the totally supersymmetric representations of U(6/12), we have

$$\begin{aligned}&E(\mathcal{N};N;M;\Sigma,(\Sigma_1',\Sigma_2',\Sigma_3');(\sigma_1,\sigma_2,\sigma_3);(\tau_1,\tau_2),\nu_\Delta;L;S;J;M_J)\\&=E_0(\mathcal{N},N)-(A/4)\Sigma(\Sigma+4)-(A'/4)[\Sigma_1'(\Sigma_1'+4)+\Sigma_2'(\Sigma_2'+2)+\Sigma_3'^2]\\&\quad-(A''/4)[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+\sigma_3^2]+(B/6)[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+CL(L+1)\\&\quad+C'S(S+1)+C''J(J+1)\ ,\end{aligned} \tag{3}$$

where we have grouped all terms contributing to binding energies in $E_0(\mathcal{N},N)$. This formula should be compared with Eq. (3.15) of Ref. 4. In order to find the values of the quantum numbers $\Sigma,(\Sigma_1,\Sigma_2,\Sigma_3),\ldots$, appearing in Eq. (3), we have to decompose the representation $[\mathcal{N}\}$ of U(6/12) into representations of its subgroups. This can be done, in general, using techniques similar to those of Ref. 4 and we give here only the results for the special case $M=1$, a situation of particular experimental interest, since it describes the low-lying states of odd-even nuclei. In this case, $\Sigma_1'=1$, $\Sigma_2'=\Sigma_3'=0$, and $S=\frac{1}{2}$ for all states. Thus, the two terms A' and C' in Eq. (3) can be absorbed in E_0 and only five terms A,A'',B,C,C'' are needed to describe the excitation energies. For a given number of bosons N, one has[9] $\Sigma=N,N-2,\ldots$; the values of σ_1, σ_2, and σ_3 can then be obtained by the multiplication rule

$$(\Sigma,0,0)\otimes(1,0,0)$$

$$=(\Sigma+1,0,0)\oplus(\Sigma,1,0)\oplus(\Sigma-1,0,0)\ , \tag{4}$$

except for $\Sigma=0$ when the right-hand side contains only the representation (1,0,0). Thus, each representation $(\Sigma,0,0)$, $\Sigma\geq 1$, splits into three representations when an odd fermion is coupled. When $\sigma_2=0$, the decomposition of the representations $(\sigma_1,\sigma_2,0)$ of $SO^{(B+F)}(6)$ into representations (τ_1,τ_2) of $SO^{(B+F)}(5)$ is the same as that given in Ref. 9. When $\sigma_2=1$, the decomposition is

$$\begin{aligned}(\sigma_1,1,0)=&(\sigma_1,0)+(\sigma_1-1,0)+\cdots+(1,0)\\&+(\sigma_1,1)+(\sigma_1-1,1)+\cdots+(1,1)\ .\end{aligned} \tag{5}$$

Each $SO^{(B+F)}(5)$ representation decomposes further into representations of $SO^{(B+F)}(3)$. For $\tau_2=0$ the decomposition is as in Ref. 9. When $\tau_2=1$, the decomposition gives

$$\begin{aligned}L=&2\tau_1+1,\ldots,3\\&\tau_1,\ldots,1\quad \tau_1\geq 1\\&\tau_1+2,\ldots,5\quad \tau_1\geq 3\\&\tau_1+3,\ldots,7\quad \tau_1\geq 5\\&\cdots\ .\end{aligned} \tag{6}$$

(Note that $\tau_1+1,\ldots,3$ is missing.) Finally, $J=L\pm\frac{1}{2}$.

The U(6/12) symmetry, described above, may, to some extent, be applicable to odd neutron nuclei in the Pt-Os region. The even nuclei in this region can be described, to a good approximation, by an $SO^{(B)}(6)$ symmetry.[9,10] In the odd-even nuclei, the odd proton occupies mostly a level with $j^\pi=\frac{3}{2}^+$ $(d_{3/2})$. The corresponding spectra have been studied by using a U(6/4) symmetry.[4] In the even-odd nuclei, the odd neutron occupies mostly three levels with $j^\pi=\frac{1}{2}^-,\frac{3}{2}^-,\frac{5}{2}^-$ $(p_{1/2},p_{3/2},f_{5/2})$. The angular

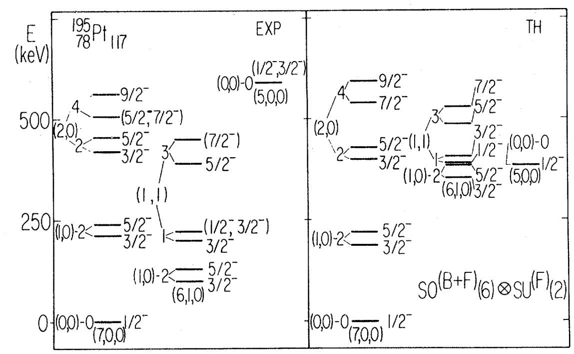

FIG. 1. Comparison between the experimental low-lying spectrum (Ref. 11) of ^{195}Pt and that obtained using Eq. (3) with $(A''/4)=12$ keV, $(B/6)=35$ keV, $C=5$ keV, and $C''=6$ keV. The number of bosons is $N=6$. The term containing the parameter $(A/4)$ is not needed since all states belong to the same representation $\Sigma=6$. Assignments are based on a preliminary analysis of energies, electromagnetic transition rates, and intensities of one nucleon transfer reactions.

momentum content of these three levels is the same as that of the U(6/12) symmetry discussed above. It is intriguing to note that the experimental spectra[11] of ^{195}Pt and ^{197}Pt appear to be approximately described by the energy formula (3), as shown in Fig. 1 for ^{195}Pt. All experimental states have a corresponding theoretical state of the same spin and parity, although the representation (6,1,0) appears to be displaced downward by ~ 250 keV relative to its theoretical counterpart. It is worthwhile noting that other simple theoretical interpretations of the observed spectra, such as the Nilsson model,[12] cannot account for all the observed levels.[11].

The new class of supersymmetry discussed here allows one to construct closed solutions for many other situations which are likely to be encountered in the study of nuclear spectra. Another important case is that of supersymmetries associated with the limit II of the interacting boson model,[13]

$$U^{(B)}(6)\supset SU^{(B)}(3)\supset SO^{(B)}(3)\supset SO^{(B)}(2)\ . \qquad (7)$$

As an example, we mention the case in which fermions with $j=\frac{1}{2},\frac{3}{2}$ are coupled to it. In this case the angular momentum of the fermions can be split into a pseudo-orbital angular momentum $k=1$ and $s=\frac{1}{2}$. Since $k=1$ sits in the three-dimensional representation (1,0) of SU(3), we can, using the procedure described above, form the group chain

$$\begin{aligned}U(6/6)&\supset U^{(B)}(6)\times U^{(F)}(6)\supset SU^{(B)}(3)\times SU^{(F)}(3)\times SU^{(F)}(2)\\&\supset SU^{(B+F)}(3)\times SU^{(F)}(2)\supset SO^{(B+F)}(3)\times SU^{(F)}(2)\\&\supset \mathrm{Spin}(3)\supset \mathrm{Spin}(2)\ . \end{aligned}\qquad (8)$$

The corresponding energy formula is

$$\begin{aligned}&E(\mathcal{N};N,M;(\lambda_B,\mu_B),(\lambda_F,\mu_F);(\lambda,\mu);,K;L;S;J;M_J)\\&\quad=E_0(\mathcal{N},N)+\alpha C(\lambda_B,\mu_B)+\alpha' C(\lambda_F,\mu_F)+\alpha'' C(\lambda,\mu)+\beta L(L+1)+\beta' S(S+1)+\beta'' J(J+1)\ ,\end{aligned}\qquad (9)$$

where

$$C(\lambda,\mu)=\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu\ .$$

Indeed, the new class of supersymmetry discussed here was originally suggested[7] as a possible way to construct analytic solutions in the SU(3) limit of the interacting boson-fermion model. Examples of nuclei described by the chain (8) appear also to have been found in the region of the Er-Tm isotopes.[14]

In conclusion, the introduction of this new class of supersymmetries opens the way for a systematic application of supersymmetry considerations to the study of spectra of odd-even nuclei. This is particularly important since the spectra of odd-even nuclei are very complex, especially in transitional regions. Simple, closed solutions to the eigenvalue problem for the combined system of collective (bosonic) and single particle (fermionic) degrees of freedom can provide a framework within which these spectra can be analyzed. As in the case discussed previously,[4] one can test experimentally several aspects of the classification scheme discussed here. In the first of these, one concentrates (as done in Fig. 1) only on the odd-even (or even-odd) nucleus and studies whether or not the common boson plus fermion symmetry [for example, $SO^{(B+F)}(6)\times SU^{(F)}(2)$] is applicable. In the second aspect, one studies whether or not the parameters $A,A',\ldots,$ in the energy eigenvalue expression, Eq. (3), and in the corresponding expressions for electromagnetic transition rates and other quantities, are the same both for even-even and even-odd nuclei. This provides a test of supersymmetry [for example U(6/12)]. When performing these tests it is of importance to compare not only energy levels but also other properties. We shall present in a future publication the predictions of the U(6/12) symmetry for electromagnetic transition rates.

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