Nuclear structure effects in high momentum transfer reactions

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It is shown that, at large momentum transfers, one can extract nuclear structure information from the response (function) of the nucleus to an external scalar probe by factorizing the on-shell form factor associated to the struck nucleon. This result, derived both for elastic and inelastic scattering, arises from compensations between off-shell effects and exchange current effects generated by the (local) nucleon-nucleon interaction. The corrections to this on-shell factorization are in general found to be small at large momentum transfers. The role of the final state interaction in the nuclear transition form factor is then investigated and it is shown how one can, for instance, correct the orthogonality defect introduced by using a plane wave approximation for the struck nucleon.

[NUCLEAR REACTIONS High momentum transfer reactions, elastic and inelastic scattering, on shell form factor, factorization, orthogonality.]

I. INTRODUCTION

Electron scattering is still presently the best tool to extract information on the nuclear transition densities or on the corresponding form factors. In the Born approximation, which is justified since the electron interaction is weak, the nuclear response function is factorized into an elementary nucleon form factor, not necessarily on shell, and a nuclear transition form factor.[1,2] To obtain information on the internal part of the nuclear densities, it is obvious that one needs data at large $\vec{q}^{\,2}$. The interpretation of such data is, however, obscured by the presence of off shell effects in the nucleon form factor as well as of meson exchange effects which may become important in that regime.[3,4] The motivation of the current work is thus the study of such effects. To gain some insight, we shall simplify the problem and discuss the response of the nucleus to a scalar probe as shown in Fig. 1. In Ref. 5 it has been remarked that for a class of processes contributing to the elastic nuclear response, the off-shell effects in the nucleon form factor cancelled specific exchange effects generated by the local nucleon-nucleon potential. In that framework, one obtains, for large $\vec{q}^{\,2}$, the usual factorization

$$F_A(q)=f_N(q)S_{00}(\vec{q}) , \tag{1}$$

where $f_N(q)$ denotes the on-shell nucleon form factor and $S_{00}(\vec{q})$ the elastic nuclear form factor.

In this work we shall show that the factorized form (1) can be extended to a larger class of processes contributing to the elementary nucleon-scalar probe vertex, on one hand, and to the case of the inelastic response of the nucleus, on the other hand. These results can be derived assuming that the nucleons, which are treated nonrelativistically but have structure, are distinguishable and are assumed to interact via two body local scalar potentials. Therefore the two-body local interactions between the nonrelativistic nucleons are treated here on a com-

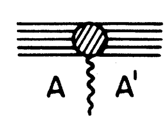

FIG. 1. Diagrammatic representation of the nuclear response to the scalar probe (wavy line).

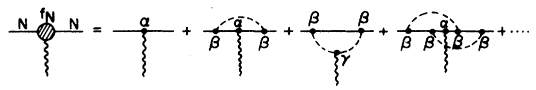

FIG. 2. Diagrammatic representation of the nucleon form factor (the dashed lines represent scalar mesons while the wavy lines represent the scalar probe).

pletely different level than the meson effects which dress the nucleons. The elementary nucleon form factor, $f_N(q)$, is represented diagrammatically in Fig. 2. We stress that the scalar mesons of mass μ are treated relativistically, in contrast to the nucleons. In the following, we shall not be concerned with problems of renormalization since we use the loop integrals formally without attempting any direct evaluation. For this reason, we do not specify the vertex functions α, β, γ which might be functions of the square of the four-momentum transfer but not explicitly of the external energies. The typical regime in which we shall work is characterized by $\mu \lesssim |\vec{q}\,|$, and by nuclear excitation energies less than or of the order of $\vec{q}\,^2/2m$ (where m is the nucleon mass).

In order to calculate the response of the nucleus to a scalar probe (illustrated in Fig. 3), one has to replace the free nucleon propagators by the interacting Green's function

$$G(E)=(E+i\eta-H_A)^{-1} . \tag{2}$$

Here, E represents the energy of the A-body system and H_A is the nuclear Hamiltonian

$$H_A=T+V . \tag{3a}$$

The kinetic energy operator T is defined by

$$T=\sum_i^A \frac{\vec{Q}_i^{\,2}}{2m} , \tag{3b}$$

where m is the nucleon mass and $\vec{Q}_i$ denotes the three-momentum of the ith target nucleon. The potential V in Eq. (3a) is the sum of the two-body (local) nucleon nucleon interactions

$$V=\sum_{i<j} V_{ij} . \tag{3c}$$

The interacting Green's function (2) is shown diagrammatically in Fig. 4, where it is expanded in terms of the free Green's function

$$G_0(E)=(E+i\eta-T)^{-1} \tag{4}$$

according to the relation

$$G(E)=G_0(E)+G_0(E)VG(E) . \tag{5}$$

If we were to approximate the interacting Green's function (2) by the free one (4), we would obtain the usual impulse approximation for the nuclear transition form factor. The calculation involves three dimensional integrals over the Fermi momenta of the target nucleons and, as a result, one obtains a factorized formula similar to (1) where, however, the nucleon form factor is *off shell* due to the initial binding of the struck nucleon. The other terms in the expansion (5) of the interacting Green's function (2), shown in Figs. 4(b)–(d), when inserted in the evaluation of the nuclear response function [see Figs. 3(b) and (c)] describe interaction (or exchange current) effects generated by the nucleon-nucleon interaction in the nuclear transition form factor. In Ref. 5 only the process described by Fig. 3(b) has been considered in the specific case of the elastic scattering on a deuteron target. In this case, it has been shown that, in the limit of large $\vec{q}\,^2$, the nucleon-nucleon interaction effects, generated by Figs. 4(b),(c), . . . , compensated the off-shell effects generated by the impulse approximation term of Fig. 4(a), so that one recovers the on-shell factorization (1). In the current work we extend this result and prove it for elastic scattering, direct inelastic transitions, and quasifree scattering in the case of a many body system of nonrelativistic distinguishable constituents interacting via two body local potentials. Note that similar ideas may be applied to the study of inclusive processes.[6–8]

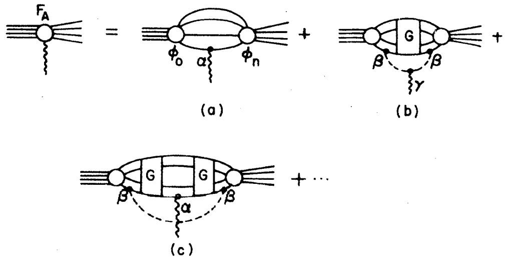

FIG. 3. Diagrammatic expansion of the nuclear response function. The box represents the interacting Green's function.

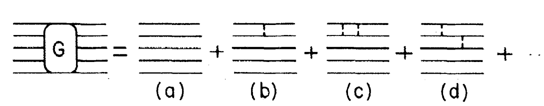

FIG. 4. Green's function representation. (a) corresponds to the impulse approximation. (b), (c), and (d) are generated by the nucleon-nucleon (local) interaction.

An additional difficulty which is met when one attempts to extract nuclear structure information from exclusive processes, like one nucleon knockout reactions, for instance, arises from the treatment of the final state continuum wave function. In the practical evaluation of such a process, one generally introduces a plane wave (or an optical potential distorted wave) approximation for the final state wave function. This inconsistent treatment of the initial and final nuclear states leads to an orthogonality defect.[9-12] In this work, we derive an approximate formula for the transition form factor which corrects for the lack of orthogonality. In this paper we also present results concerning the low momentum transfer behavior of the transition form factor.

The paper is organized as follows. In Sec. II we consider the case of the inelastic nuclear response described in Fig. 3(b), i.e., the scalar probe couples to a virtual meson. Section III is devoted to the study of the inelastic process displayed in Fig. 3(c), where the scalar probe is coupled to the virtual nucleon. Elastic scattering is then easily derived as a particular case. In Sec. IV we show how to correct for the lack of orthogonality in the case of an exclusive excitation process when the continuum final state is treated in a plane wave approximation. The conclusions are presented in Sec. V together with a brief discussion of the relation of our approach to more general meson theories.

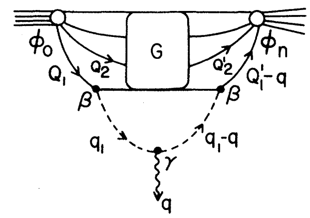

FIG. 5. Kinematics of the scalar probe-virtual meson coupling program.

II. COUPLING OF THE SCALAR PROBE VIA VIRTUAL MESONS

In order to study off shell and interaction (or exchange current) effects in the nuclear response function we have to consider the processes which give rise to an explicit energy dependence in the nucleon form factor. This is obviously not the case of the process described by Fig. 3(a), where no off-shell dependence occurs and the interaction effects are implicitly built-in in the initial and final nuclear wave functions. We therefore first investigate the process, illustrated by Fig. 3(b), where the scalar probe interacts with the target nucleons via virtual mesons. The kinematics of this process is specified in Fig. 5. The formal evaluation of the diagram can be written as

$$F_A^{(0,n)}(q)=\int \phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\langle\vec{Q}_1-\vec{q}_1,\vec{Q}_2,\ldots,|G(\epsilon_0-q_{10})|\vec{Q}'-\vec{q}_1,\vec{Q}_2',\ldots,\rangle \times\frac{\gamma\beta^2 d^4q_1 d^3Q_1 d^3Q_2\cdots d^3Q_1' d^3Q_2'\cdots}{[q_{10}{}^2-\vec{q}\,{}^2-\mu^2][(q_{10}-q_0)^2-(\vec{q}_1-\vec{q})^2-\mu^2]}\phi_n^*(\vec{Q}_1'-\vec{q}_1,\vec{Q}_2',\ldots,)\,, \tag{6}$$

where q is the four-momentum transfer. In Eq. (6), ϕ_0 and ϕ_n are exact ground state and excited nuclear states, respectively, ϵ_0 and ϵ_n being the corresponding energies

$$H_A\phi_0=\epsilon_0\phi_0,\quad H_A\phi_n=\epsilon_n\phi_n\ ,$$

and H_A has been defined in (3). Note that the final state, ϕ_n, may be a continuum state. Finally, μ denotes the meson mass and the various vertex functions may depend upon the corresponding square of the four-momentum transfers q, q_1, and q_1-q.

The many body matrix element of the interacting Green's function in (6) reads explicitly

$$\langle\vec{Q}_1-\vec{q}_1,\vec{Q}_2,\ldots,|G^{-1}(\epsilon_0-q_{10})|\vec{Q}_1'-\vec{q}_1,\vec{Q}_2',\ldots,\rangle$$
$$=\left\{\epsilon_0-q_{10}-\frac{(\vec{Q}_1-\vec{q}_1)^2}{2m}-\sum_{i=2}^{A}\frac{\vec{Q}_i{}^2}{2m}\right\}\prod_{i=1}^{A}\delta(\vec{Q}_i-\vec{Q}_i')$$
$$-\sum_{i<j}V_{ij}(\vec{Q}_i-\vec{Q}_i')\delta(\vec{Q}_i+\vec{Q}_j-\vec{Q}_i'-\vec{Q}_j')\prod_{l\neq i,j}^{A}\delta(\vec{Q}_l-\vec{Q}_l')\,. \tag{7}$$

For simplicity, we ignore here the nuclear center of mass motion effect in this expression. The factorization of $F_A^{on}(q)$, defined by (6), in terms of an on shell nucleon form factor implies the cancellation of the off-shell effects in the impulse approximation [zeroth order term in the expansion of the full Green's function (5), i.e., Fig. 4(a)] with the contribution of interaction effects (higher order terms of the Green's function expansion, i.e., Figs. 4(b), (c), etc.). It can be displayed by introducing an approximation to the amplitude (6) such that the associated corrections become small in the limit of large $\vec{q}\,^2$. It is important to note that the approximation is made on the full expression (6) and not directly at the level of the matrix element (7) of the Green's function (2). It indeed corresponds to an effective approximation, G_a, to the Green's function (2) where the matrix elements depend upon external parameters like the momentum transfer $\vec{q}$. Let us thus make the ansatz[13]

$$\langle \vec{Q}_1-\vec{q}_1,\vec{Q}_2,\ldots,\,|\,G_a{}^{-1}(\epsilon_0-q_{10})\,|\,\vec{Q}_1'-\vec{q}_1,\vec{Q}_2',\ldots,\,\rangle=[\epsilon_0-q_{10}-\bar{\epsilon}(\vec{q}_1,\vec{q})]\prod_{i=1}^{A}\delta(\vec{Q}_i-\vec{Q}_i')\,. \tag{8}$$

The parametric energy $\bar{\epsilon}$ in Eq. (8), which is a function, to be specified later, of both momentum transfers $\vec{q}_1$ and $\vec{q}$, does not depend on the nucleon momenta $\vec{Q}_i,\vec{Q}_i'$. As is clear from (8), the approximate Green's function G_a is defined to be diagonal in the nucleon momenta. We now expand the exact Green's function G in terms of the approximate one G_a, which reads schematically

$$G=G_a+G_a(H_A-\bar{\epsilon})G_a+G_a(H_A-\bar{\epsilon})G_a(H_A-\bar{\epsilon})G_a+\cdots\,, \tag{9}$$

where we have used the definitions (2) and (8). The function $\bar{\epsilon}$ is then defined in a variational way such that the first order correction in the expansion (9) vanishes, leading then to a factorized expression similar to (1) with on-shell nucleon form factor $f_N(q)$.

The matrix element of the first order correction term, $G_a(H_A-\bar{\epsilon})G_a$, gives explicitly

$$\begin{aligned}\Delta^{(1)}=\int\prod_{i=1}^{A}d^3Q_i d^3Q_i'\,&\frac{\gamma\beta^2d^4q_1}{[q_1{}^2-\mu^2][(q_1-q)^2-\mu^2]}\phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\frac{1}{\epsilon_0-q_{10}-\bar{\epsilon}(\vec{q}_1,\vec{q})}\\ &\times\left\{\left[\frac{(\vec{Q}_1-\vec{q}_1)^2}{2m}+\sum_{i=2}^{A}\frac{\vec{Q}_i{}^2}{2m}-\bar{\epsilon}\right]\prod_{i=1}^{A}\delta(\vec{Q}_i-\vec{Q}_i')\right.\\ &\qquad\left.+\sum_{i<j}V_{ij}(\vec{Q}_i-\vec{Q}_i')\delta(\vec{Q}_i+\vec{Q}_j-\vec{Q}_i'-\vec{Q}_j')\prod_{\substack{l=1\\ l\neq i,j}}^{A}\delta(\vec{Q}_l-\vec{Q}_l')\right\}\\ &\times\frac{1}{\epsilon_0-q_{10}-\bar{\epsilon}(\vec{q}_1,\vec{q})}\phi_n^*(\vec{Q}_1'-\vec{q},\vec{Q}_2',\ldots,)\,.\end{aligned} \tag{10}$$

The binding potential, V, in this expression is eliminated by using the many body Schrödinger equation applied either to the initial (ϕ_0) or final (ϕ_n) state. Now requiring this first order correction term to vanish imposes the following relations:

$$0=\int\phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\left[\epsilon_0-\frac{\vec{Q}_1{}^2}{2m}-\bar{\epsilon}(\vec{q},\vec{q})+\frac{(\vec{Q}_1-\vec{q}_1)^2}{2m}\right]\phi_n^*(\vec{Q}_1-\vec{q},\vec{Q}_2,\ldots,)\prod_{i=1}^{A}d^3Q_i \tag{11a}$$

and

$$0=\int\phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\left[\epsilon_n-\frac{(\vec{Q}_1-\vec{q})^2}{2m}-\bar{\epsilon}(\vec{q}_1,\vec{q})+\frac{(\vec{Q}_1-\vec{q}_1)^2}{2m}\right]\phi_n^*(\vec{Q}_1-\vec{q},\vec{Q}_2,\ldots,)\prod_{i=1}^{A}d^3Q_i\,. \tag{11b}$$

In order to arrive at an explicit determination of the function $\bar{\epsilon}(\vec{q}_1,\vec{q})$, we thus have to evaluate the integral

$$I(\vec{q}_1,\vec{q})=\int\phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\frac{\vec{Q}_1\cdot\vec{q}_1}{m}\phi_n^*(\vec{Q}_1-\vec{q},\vec{Q}_2,\ldots,)\prod_{i=1}^{A}d^3Q_i\,. \tag{12}$$

We also have to calculate

$$J(\vec{q})=\int\phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\frac{\vec{Q}_1\cdot\vec{q}}{m}\phi_n^*(\vec{Q}_1-\vec{q},\vec{Q}_2,\ldots,)\prod_{i=1}^{A}d^3Q_i\,, \tag{13a}$$

which, upon subtracting (11b) from (11a), is obtained to be

$$J(\vec{q})=\left[\epsilon_0-\epsilon_n+\frac{\vec{q}^{\,2}}{2m}\right]S_{on}(\vec{q})\ , \tag{13b}$$

where $S_{on}(\vec{q})$ denotes the nuclear transition form factor

$$S_{on}(\vec{q})=\int\prod_{i=1}^{A}d^3Q_i\phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\phi_n^*(\vec{Q}_1-\vec{q}_1,\vec{Q}_2,\ldots,)\ . \tag{14}$$

The integral (12), $I(\vec{q}_1,\vec{q})$, is actually determined by the relations (13). This is seen by considering the scalar product $\vec{Q}_1\cdot\vec{q}_1$ which can be written as

$$\vec{Q}_1\cdot\vec{q}_1=\frac{(\vec{Q}_1\cdot\vec{q})(\vec{q}_1\cdot\vec{q})}{\vec{q}^{\,2}}+\vec{Q}_{1\perp}\cdot\vec{q}_{1\perp}\ , \tag{15}$$

where $\vec{Q}_{1\perp}$ and $\vec{q}_{1\perp}$ are the projections of the vectors $\vec{Q}_1$ and $\vec{q}_1$ on the plane perpendicular to the momentum transfer $\vec{q}$. The second term on the right hand side of (15) will not contribute to expression (10) because of the symmetry of the integrand in the d^4q_1 integral, under the exchange $\vec{q}_{1\perp}\to-\vec{q}_{1\perp}$. The contribution of the integral (12) to the correction (10) is then simply

$$I(\vec{q}_1,\vec{q})=\frac{(\vec{q}_1\cdot\vec{q})}{\vec{q}^{\,2}}\left[\epsilon_0-\epsilon_n+\frac{\vec{q}^{\,2}}{2m}\right]S_{on}(\vec{q})\ . \tag{16}$$

Using these results, we see that the correction term Δ^1, Eq. (10), vanishes provided the function $\bar{\epsilon}(\vec{q}_1,\vec{q})$ is defined by the relation

$$\bar{\epsilon}(\vec{q}_1,\vec{q})=\epsilon_0+\frac{\vec{q}_1^{\,2}}{2m}-\left[\epsilon_0-\epsilon_n+\frac{\vec{q}^{\,2}}{2m}\right]\frac{\vec{q}_1\cdot\vec{q}}{\vec{q}^{\,2}}\ . \tag{17}$$

This choice of the parametric energy corresponds to what has been referred to as the optimal approximation in Ref. 13. We may now replace $\bar{\epsilon}(\vec{q}_1,\vec{q})$ by its value (17) and the Green's function G by G_a in (6). Up to second order corrections, the expected factorized formula then reads

$$F_A^{(0,n)}(q)=f_N^{(1)}(q)S_{on}(\vec{q})\ , \tag{18}$$

where the nuclear transition form factor $S_{on}(q)$ is defined by (14) and the contribution $f_N^{(1)}(q)$ to the elementary nucleon form factor is

$$f_N^{(1)}(q)=\int\frac{\gamma\beta^2d^4q_1}{(q_1^{\,2}-\mu^2)[(q_1-q)^2-\mu^2]\left\{\left[\epsilon_0-\epsilon_n+\frac{\vec{q}^{\,2}}{2m}\right]\frac{\vec{q}_1\cdot\vec{q}}{\vec{q}^{\,2}}-\frac{\vec{q}_1^{\,2}}{2m}-q_{10}\right\}}\ . \tag{19}$$

Indeed, Eq. (19) represents the contribution of the process, illustrated in Fig. 6, to the *on shell* nucleon form factor. This may be readily seen if we rewrite (19) as

$$f_N^{(1)}(q)=\int\frac{\gamma\beta^2d^4q_1}{(q_1^{\,2}-\mu^2)[(q_1-q)^2-\mu^2]}\cdot g(q,q_1)\ , \tag{20a}$$

where the nucleon propagator $g(q,q_1)$ is equal to

$$g(q,q_1)=(E_{\vec{p}}-E_{\vec{p}-\vec{q}_1}-q_{10})^{-1}\ , \tag{20b}$$

or, equivalently,

$$g(q,q_1)=[E_{\vec{p}-\vec{q}}-E_{\vec{p}-\vec{q}_1}-(q_{10}-q_0)]^{-1}\ . \tag{20c}$$

In Eqs. (20b) and (20c), we have introduced the vector

$$\vec{p}=m\left[\epsilon_0-\epsilon_n+\frac{\vec{q}^{\,2}}{2m}\right]\frac{\vec{q}}{\vec{q}^{\,2}} \tag{21a}$$

and the energies $E_{\vec{k}}$ are defined by ($\vec{k}$ an arbitrary vector)

$$E_{\vec{k}}=m+\frac{\vec{k}^2}{2m} \tag{21b}$$

while the energy transfer, q_0, is given by

$$q_0=(\epsilon_0-\epsilon_n)\ . \tag{22}$$

It remains now for us to show that the corrections to the factorized expression (18) of the nuclear response function are small in the limit of large momentum transfers. We therefore consider the first nonvanishing term in expansion (9), i.e., $G_a(H_A-\bar{\epsilon})G_a(H_A-\bar{\epsilon})G_a$, whose matrix element reads

$$\Delta^{(2)}=\int \phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)[H_A-\bar{\epsilon}(\vec{q}_1,\vec{q})][H_A-\bar{\epsilon}(\vec{q}_1,\vec{q})]\phi_n^*(\vec{Q}'_1-\vec{q},\vec{Q}'_2,\ldots,)$$
$$\times\frac{\gamma\beta^2 d^4q_1}{[q_1{}^2-\mu^2][(q_1-q)^2-\mu^2]}g^3(q,q_1)\prod_{i=1}^{A}d^3Q_i d^3Q'_i\ , \tag{23}$$

where $g(q,q_1)$ is defined by (20b).

Using the definition (3) of the Hamiltonian H_A and eliminating the potentials by the repeated use of the many body Schrödinger equation, we arrive at

$$\Delta^{(2)}=\int \phi_0(\vec{Q}_1,\vec{Q}_2,\ldots,)\frac{[(\vec{Q}_1-\vec{p})\cdot\vec{q}_1][(\vec{Q}_1-\vec{p})\cdot(\vec{q}_1-\vec{q})]}{m^2\left[\dfrac{\vec{p}^{\,2}}{2m}-\dfrac{(\vec{p}-\vec{q}_1)^2}{2m}-q_{10}\right]^3}$$
$$\times\phi_n^*(\vec{Q}_1-\vec{q},\vec{Q}_2,\ldots,)\frac{\gamma\beta^2 d^4q_1}{[q_1{}^2-\mu^2][(q_1-q)^2-\mu^2]}\prod_{i=1}^{A}d^3Q_i\ , \tag{24}$$

where the momentum $\vec{p}$ is given by (21a).

In order to estimate the correction term (24) to the main term (18) we separately discuss final bound states and continuum states for quasifree scattering. For bound states (in particular the ground state for elastic scattering) we assume a rather fast falloff of the wave functions ϕ_0 and ϕ_n in momentum space so that the dominant contribution to the d^3Q_1 integral in (24) comes from the region where the momentum $\vec{Q}_1$ has the same order of magnitude as half the momentum transfer $\vec{q}$. Since we work in a region of large momentum transfer such that $\vec{q}^{\,2}/2m$ is much larger than the nuclear excitation energy, $\epsilon_n-\epsilon_0$ (i.e., $|\vec{q}|\gg k_F$), we read from (21a):

$$\vec{p}\sim\frac{\vec{q}}{2}\ .$$

With these assumptions, it is clear that the correction term $\Delta^{(2)}$ of Eq. (24) is at least of the order of $k_F{}^2/\vec{q}^{\,2}$ as compared to the leading term (18).[5]

In the case of a final continuum state, we first consider the limit of a plane wave for the outgoing nucleon which has the asymptotic momentum $\vec{k}$ and the corresponding energy $\epsilon_{\vec{k}}=\vec{k}^2/2m$. The final state wave function peaks then at

$$\vec{k}=\vec{Q}_1-\vec{q}$$

and $|\vec{Q}_1|$ is of the order of the Fermi momentum k_F. The momentum $\vec{p}$ in (21a) is thus given by

$$\vec{p}\simeq\frac{(\vec{Q}_1\cdot\vec{q})}{\vec{q}^{\,2}}q$$

and we have

$$\vec{Q}_1-\vec{p}=\vec{Q}_{1\perp}\ .$$

The correction term $\Delta^{(2)}$ of Eq. (24) is therefore suppressed as compared to the main term [Eqs. (18) and (19)] by the factor

$$\frac{(\vec{Q}_{1\perp}\cdot\vec{q}_1)^2}{\left\{\dfrac{(\vec{Q}_1\cdot\vec{q})(\vec{q}_1\cdot\vec{q})}{\vec{q}^{\,2}}-\dfrac{\vec{q}_1{}^2}{2}-mq_{10}\right\}^2}\ .$$

Since the integral over the four-momentum q_1 in

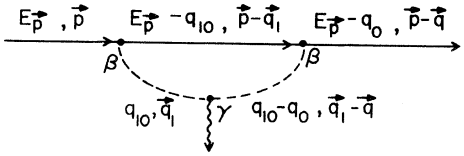

FIG. 6. Contribution to the on shell nucleon form factor corresponding to probe-meson coupling.

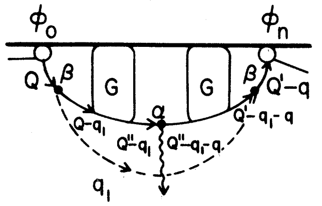

FIG. 7. Kinematics of the scalar probe-virtual nucleon coupling diagram.

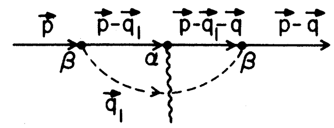

FIG. 8. Contribution to the on shell nucleon form factor corresponding to probe nucleon coupling.

Eq. (24) implies that we have $|\vec{q}_1| \sim |\vec{q}|$ (Ref. 14), the correction term (24) is suppressed by at least a factor of the order of $(k_F/|\vec{q}|)$. In order to complete the proof, we should now show that the exact result differs from the plane wave result by terms of the order of $(k_F/|\vec{q}|)$ for large momentum transfers. This will be discussed in Sec. IV [see Eq. (53)]. We thus have proven, under reasonable assumptions for the nuclear wave functions, the on shell factorization in the case of inelastic transitions.

III. COUPLING OF THE SCALAR PROBE VIA VIRTUAL NUCLEONS

The discussion of the preceding section can be extended to another class of processes, shown in Fig. 7, where the kinematics is specified. We shall not use the many body notation of the latter section, although we stress that the derivation in the following goes through as before, but shall refer, for the sake of simplicity, to a bound nucleon plus a core. The form factor illustrated in Fig. 7 is then given by

$$F_A^{(0,n)}(q)=\int \phi_0(\vec{Q})\frac{\alpha\beta^2 d^3Q d^3Q' d^3Q'' d^4q_1}{q^2-\mu^2}$$
$$\times\langle\vec{Q}-\vec{q}_1|G(\epsilon_0-q_{10})|\vec{Q}''-\vec{q}_1\rangle\langle\vec{Q}''-\vec{q}_1-\vec{q}\,|G(\epsilon_0-q_{10}-q_0)|\vec{Q}'-\vec{q}_1-\vec{q}\rangle\phi_n^*(\vec{Q}'-\vec{q})\,, \tag{25}$$

where ϕ_0 and ϕ_n denote the wave function of the nucleon, interacting with the core, in the initial and final states. The matrix elements of the inverse of the full Green's function then read

$$\langle\vec{Q}-\vec{q}_1|G^{-1}(\epsilon_0-q_{10})|\vec{Q}''-\vec{q}_1\rangle=\left\{\epsilon_0-q_{10}-\frac{(\vec{Q}-\vec{q}_1)^2}{2m}\right\}\delta(\vec{Q}-\vec{Q}'')-V(\vec{Q}-\vec{Q}'') \tag{26a}$$

and

$$\langle\vec{Q}''-\vec{q}_1-\vec{q}\,|G^{-1}(\epsilon_0-q_{10}-q_0)|\vec{Q}'-\vec{q}_1-\vec{q}\rangle=\left\{\epsilon_0-q_{10}-q_0-\frac{(\vec{Q}'-\vec{q}_1-\vec{q})^2}{2m}\right\}\delta(\vec{Q}''-\vec{Q}')-V(\vec{Q}''-\vec{Q}')\,. \tag{26b}$$

In the same spirit as before, we make the double ansatz

$$\langle\vec{Q}-\vec{q}_1|G_{a1}{}^{-1}(\epsilon_0-q_{10})|\vec{Q}''-\vec{q}_1\rangle=\{\epsilon_0-q_{10}-\bar{\epsilon}_1(\vec{q}_1,\vec{q})\}\delta(\vec{Q}-\vec{Q}'')\,, \tag{27a}$$
$$\langle\vec{Q}''-\vec{q}_1-\vec{q}\,|G_{a2}{}^{-1}(\epsilon_0-q_{10}-q_0)|\vec{Q}'-\vec{q}_1-\vec{q}\rangle=\{\epsilon_0-q_{10}-q_0-\bar{\epsilon}_2(\vec{q}_1,\vec{q})\}\delta(\vec{Q}''-\vec{Q}')\,. \tag{27b}$$

In these equations, since we only look for an approximation to expression (25), the parametric energies $\bar{\epsilon}_1$ and

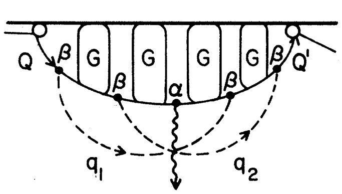

FIG. 9. A more complicated process.

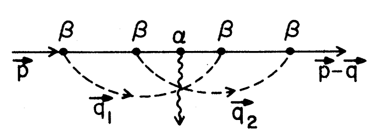

FIG. 10. Contribution to the on shell nucleon form factor corresponding to the process of Fig. 9.

$\bar{\epsilon}_2$ only depend on the external variables $\vec{q}_1$ and $\vec{q}$. These functions will be determined from the condition that the first order corrections to (25) vanish when expanding the interacting Green's functions, G, in terms of the approximate one, G_a. The first condition, which arises from the expansion of the matrix element $\langle \vec{Q}-\vec{q}_1 \,|\, G(\epsilon_0-q_{10}) \,|\, \vec{Q}''-\vec{q}_1 \rangle$, reads

$$\Delta_1 = \int \phi_0(\vec{Q}) \frac{\beta^2 \alpha d^4 q_1}{q_1{}^2-\mu^2} G_{a1} \left\{ \left[\frac{(\vec{Q}-\vec{q}_1)^2}{2m} - \bar{\epsilon}_1(\vec{q}_1,\vec{q}) \right] \delta(\vec{Q}-\vec{Q}') + V(\vec{Q}-\vec{Q}') \right\} ,$$
$$G_{a1} G_{a2} \phi_n^*(\vec{Q}'-\vec{q}) d^3Q d^3Q' = 0 . \tag{28}$$

Using the same techniques as before [see Eqs. (10)—(16)], we arrive at

$$\bar{\epsilon}_1(\vec{q}_1,\vec{q}) = \epsilon_0 + \frac{(\vec{p}-\vec{q}_1)^2}{2m} - \frac{\vec{p}^{\,2}}{2m} \tag{29}$$

and we therefore have

$$\langle \vec{Q}-\vec{q}_1 \,|\, G_{a1}(\epsilon_0-q_{10}) \,|\, \vec{Q}''-\vec{q}_1 \rangle = \frac{\delta(\vec{Q}-\vec{Q}'')}{\dfrac{\vec{p}^{\,2}}{2m} - q_{10} - \dfrac{(\vec{p}-\vec{q}_1)^2}{2m}} = \frac{\delta(\vec{Q}-\vec{Q}'')}{E_{\vec{p}} - E_{\vec{p}-\vec{q}_1} - q_{10}} . \tag{30}$$

In these equations, the momentum $\vec{p}$ and the energy $E_{\vec{p}}$ are defined by Eqs. (21). Similarly the second condition which follows from the expansion of the term

$$\langle \vec{Q}''-\vec{q}_1-\vec{q} \,|\, G(\epsilon_0-q_{10}-q_0) \,|\, \vec{Q}'-\vec{q}_1-\vec{q} \rangle$$

yields

$$\bar{\epsilon}_2(\vec{q}_1,\vec{q}) = \epsilon_n + \frac{\vec{q}_1{}^2}{2m} - \frac{\vec{q}\cdot\vec{q}_1}{\vec{q}^{\,2}} \left[\epsilon_0 - \epsilon_n - \frac{\vec{q}^{\,2}}{2m} \right] = \epsilon_n + \frac{(\vec{p}-\vec{q}-\vec{q}_1)^2}{2m} - \frac{(\vec{p}-\vec{q})^2}{2m} \tag{31}$$

and

$$\langle \vec{Q}''-\vec{q}_1-\vec{q} \,|\, G_{a2}(\epsilon_0-q_{10}-q_0) \,|\, \vec{Q}'-q_1-\vec{q} \rangle = \frac{\delta(\vec{Q}''-\vec{Q}')}{E_{\vec{p}} - E_{\vec{p}-\vec{q}-\vec{q}_1} - q_{10} - q_0} = \frac{\delta(\vec{Q}''-\vec{Q}')}{E_{\vec{p}-\vec{q}} - E_{\vec{p}-\vec{q}-\vec{q}_1} - q_{10}} . \tag{32}$$

Now replacing the exact Green's functions by their approximate counterparts (30) and (32) in expression (25), we obtain the following factorized formula:

$$F_A^{(0,n)}(q) = f_N^{(2)}(q) S_{on}(\vec{q}) , \tag{33}$$

where, now,

$$S_{on}(\vec{q}) = \int \phi_0(\vec{Q}) \phi_n^*(\vec{Q}-\vec{q}) d^3Q , \tag{34}$$

and the associated part $f_N^{(2)}(q)$ of the nucleon form factor is given by

$$f_N^{(2)}(q) = \int \frac{\beta^2 \gamma d^4 q_1}{(q_1{}^2-\mu^2)[E_{\vec{p}} - E_{\vec{p}-\vec{q}_1} - q_{10}][E_{\vec{p}-\vec{q}} - E_{\vec{p}-\vec{q}-\vec{q}_1} - q_{10}]} . \tag{35}$$

It is again easily seen that this is an on-shell contribution to the form factor, Fig. 8, since [cf. Eqs. (21) and (22)]

$$E_{\vec{p}} = m + \frac{\vec{p}^{\,2}}{2m} = m + \frac{(\vec{p}-\vec{q})^2}{2m} + q_0 = E_{\vec{p}-\vec{q}} + q_0 .$$

The result (33) therefore displays once more the cancellation between the exchange current contribution and the off-shell effects generated in the impulse approximation. The evaluation of the corrections to expression (33) proceeds just as before and the conclusions are the same.

The case of elastic scattering is trivially obtained from Eqs. (33), (34), and (35) by noting that the vector $\vec{p}$ [see Eq. (21a)] simply reduced to the vector $\vec{q}/2$.

We can now immediately show that the previous results can be extended to more general processes provided the nucleon is assumed to be nonrelativistic. Consider, for example, the process of Fig. 9. Here we again replace the exact Green's functions by approximate ones, G_a, which as in Eq. (27) do not depend on nucleon momenta in such a way that the first order correction is zero [cf. Eq. (28)]. Eliminating the potential term in $(G^{-1}-G_a{}^{-1})$, by the use of the Schrödinger equation, we can find each of the approximate Green's functions. The essential point is that the local potential commutes with all the approximate Green's functions since they do not depend on nucleon momenta. A procedure similar to that used in going from Eq. (25) to Eq. (33) leads to a factorized formula like (33) where the contribution to the nucleon form factor

$$f_N^{(3)}(q)=\int \frac{\alpha\beta^4 d^4q_1 d^4q_2}{(q_1{}^2-\mu^2)(q_2{}^2-\mu^2)}\frac{1}{E_{\vec{p}}-E_{\vec{p}-\vec{q}_1}-q_{10}} \times\frac{1}{[E_{\vec{p}}-E_{\vec{p}-\vec{q}_1-\vec{q}_2}-q_{10}-q_{20}][E_{\vec{p}}-E_{\vec{p}-\vec{q}_1-\vec{q}_2-\vec{q}}-q_{10}-q_{20}-q_0]} \times\frac{1}{[E_{\vec{p}}-E_{\vec{p}-\vec{q}_2-\vec{q}}-q_{20}-q_0]}\ . \tag{36}$$

In this expression the momentum $\vec{p}$ is again given by Eq. (21a) and it is clear that (36) is the on shell contribution to the form factor defined in Fig. 10.

IV. ROLE OF FINAL STATE INTERACTIONS AND ORTHOGONALITY

In the last two sections we have obtained the on-shell factorization formula for the case of inelastic scattering (from which the elastic scattering case is trivially derived) by considering a large class of exchange current contributions to the nuclear response function. Since our derivation does not only hold for discrete excitations, knockout processes can be treated on the same footing. We note that the quasielastic limit i.e.,

$$\epsilon_n-\epsilon_0=-q_0=q^2/2m\ ,$$

corresponds to $\vec{p}=0$ in Eq. (21a). In this region, however, the resulting cancellation is of comparatively minor importance since off shell effects are, here, much less relevant than, for instance, in elastic scattering at high momentum transfer. Yet, on the other side, in order to extract information on ground state properties like single hole wave functions in a knockout process [e.g., $(e,e'p)$,$(\pi,\pi'N)$, . . . ,], the factorization formulae are certainly insufficient. In fact, in the nuclear form factor $S_{on}(\vec{q})$, defined by Eq. (14), the details of the ground state wave function are still obscured by the final state interaction effects. It is clear that one has, in principle, to treat in a consistent way both the initial state and the final continuum state. In general, though, one approximates the final state wave function by a plane wave, or an optical potential distorted wave; a consequence of this approximation is the lack of orthogonality between the initial and final states.[9–12] In order to eliminate, at least partially, this orthogonality defect, one has to introduce corrections to the plane wave approximation for the struck nucleon. We work in the same framework as before and, for simplicity, refer to a nucleon interacting with an inert core. We first specify a few notations. The nuclear form factor is defined by [see Eq. (34)]

$$S_{0\vec{k}}(\vec{q})=\int d^3Q\psi_{\vec{k}}^{(-)*}(\vec{Q})\phi_0(\vec{Q}+\vec{q})\ , \tag{37}$$

where $\vec{k}$ is the momentum of the ejected nucleon of energy $\epsilon_{\vec{k}}=\vec{k}^2/2m$, $-\vec{q}$ the momentum transfer, ϕ_0 the initial (bound) wave function, and $\psi_{\vec{k}}^{(-)}$ the ingoing continuum final state wave function associated with the energy $\epsilon_{\vec{k}}$. This state is related to the corresponding plane wave state by the formal relation

$$\langle\psi_{\vec{k}}^{(-)}|=\langle\chi_{\vec{k}}|(1+VG)\ , \tag{38}$$

where V is the interaction between the nucleon and the core and G denotes the full Green's function of the nucleon. The first term on the right hand side of (38) (i.e., 1) corresponds to the standard plane wave approximation for the form factor

$$S_{0\vec{k}}^{\rm pw}(\vec{q})=\phi_0(\vec{k}+\vec{q})\ , \tag{39}$$

while the corrections, arising from the VG term, ensure the orthogonality with respect to the initial state.[9] To try to evaluate these corrections, we thus consider the matrix element

$$A_{\vec{k}}(\vec{q})=\langle\chi_{\vec{k}}|VG|\phi_0\rangle =\int d^3Qd^3Q'V(\vec{Q}-\vec{k}) \times\langle\vec{Q}|G|\vec{Q}'\rangle\phi_0(\vec{Q}'+\vec{q})\ . \tag{40}$$

In the same spirit as before we wish to evaluate this integral (40) in the high energy region, i.e., large $\vec{k}$, by introducing an effective Green's function, G_a, such that

$$\langle \vec{Q} \,|\, G_a^{-1}(\epsilon_{\vec{k}}) \,|\, \vec{Q}' \rangle = \delta(\vec{Q}-\vec{Q}')g^{-1}(\epsilon_{\vec{k}},\bar{\epsilon}) , \tag{41a}$$

where

$$g^{-1}(\epsilon_{\vec{k}},\bar{\epsilon}) = \epsilon_{\vec{k}} - \bar{\epsilon}(\vec{k},\vec{q}) + i\eta . \tag{41b}$$

We assume the function $\bar{\epsilon}$ to depend only on external variables like $\vec{k}$ and $\vec{q}$. Now expanding the exact Green's functions in (40) in terms of the approximate one, (41), the correction may be expressed as

$$A_{\vec{k}}(\vec{q}) = \sum_{n=0}^{\infty} A_{\vec{k}}^{(n)}(\vec{q}) \tag{42}$$

when the nth order term, $A_{\vec{k}}^{(n)}(\vec{q})$, contains the approximate Green's function to the power $(n+1)$. The high energy limit, in principle, ensures the convergence of this expansion.

The zeroth order term in (42) (i.e., $n=0$) reads simply

$$A_{\vec{k}}^{(0)}(\vec{q}) = \left\{ \epsilon_0 - \frac{(\vec{k}+\vec{q})^2}{2m} \right\} g(\epsilon_{\vec{k}},\bar{\epsilon})\phi_0(\vec{k}+\vec{q}) . \tag{43}$$

Following the same algebraic manipulations as in the previous sections, we obtain the first order term of the expansion (42):

$$A_{\vec{k}}^{(1)}(\vec{q}) = g^2(\epsilon_{\vec{k}},\bar{\epsilon}) \int d^3p \left\{ \epsilon_0 - \bar{\epsilon}(\vec{k},\vec{q}) + \frac{k^2}{2m} - \frac{(\vec{k}+\vec{q})^2}{2m} - \frac{\vec{q}\cdot\vec{p}}{m} \right\} V(p)\phi_0(\vec{p}+\vec{k}+\vec{q}) . \tag{44}$$

However, in contrast to what we have done previously, we cannot define the parametric energy $\bar{\epsilon}(\vec{k},\vec{q})$ independently of the nuclear potential, since it appears explicitly in (44), and require at the same time that the first order correction (44) vanish. We thus shall follow the work of Ref. 7 and we define $\bar{\epsilon}(\vec{k},\vec{q})$ as

$$\bar{\epsilon}(\vec{k},\vec{q}) = \epsilon_0 + \frac{\vec{k}^2}{2m} - \frac{(\vec{k}+\vec{q})^2}{2m} , \tag{45a}$$

where ϵ_0 ($\epsilon_0 < 0$) is the energy of the nucleon in the initial bound state and $[\vec{k}^2 - (\vec{k}+\vec{q})^2]/2m$ represents the average kinetic energy transferred to the nucleon. Equation (41b) now becomes

$$g^{-1}(\epsilon_{\vec{k}},\bar{\epsilon}) = \frac{(\vec{k}+\vec{q})^2}{2m} - \epsilon_0 + i\eta . \tag{45b}$$

The first order correction (44) reads then

$$A_{\vec{k}}^{(1)}(\vec{q}) = -g^2(\epsilon_{\vec{k}},\bar{\epsilon}) \int d^3p \frac{\vec{q}\cdot\vec{p}}{m} V(\vec{p})\phi_0(\vec{p}+\vec{k}+\vec{q}) . \tag{46}$$

Before we try to evaluate the higher order terms in (42) we study the small momentum transfer behavior in the high energy limit, i.e., $|\vec{q}| \ll |\vec{k}|$. The transition form factor (37), using (42)–(46), is given by

$$S_{0\vec{k}}(\vec{q}) = -\frac{4m}{(k^2-2m\epsilon_0)^2} \int d^3p\, \vec{q}\cdot\vec{p}\, V(p)\phi_0(\vec{p}+\vec{q}) , \tag{47}$$

where we have left out terms of order q^2 and $1/k^6$. Therefore the approximate expression (46) does indeed preserve the orthogonality property

$$S_{0\vec{k}}(\vec{q}=0) = \int d\vec{r}\, \psi_{\vec{k}}^{(-)*}(\vec{r})\phi_0(\vec{r}) = 0 .$$

It is clear that such a linear behavior, related to the scalar character of the probe in the momentum transfer, can only hold for very small values of $\vec{q}$, as compared to the asymptotic momentum $\vec{k}$,[10] as can be seen by inspection of the q^2 corrections. To have some insight into the $\vec{k}$ dependence of the form factor (46) we go to the coordinate space representation and write, assuming a spherically symmetric potential,

$$\begin{aligned} F(\vec{k},\vec{q}) &= \int \vec{q}\cdot\vec{p}\, V(\vec{p})\phi_0(\vec{p}+\vec{k})d^3p \\ &= i \int r^2 dr \frac{d}{dr} V(r) R_{NL}(r) \\ &\quad \times \int d\Omega_r e^{-i\vec{k}\cdot\vec{r}} \frac{\vec{q}\cdot\vec{r}}{r} Y_L^M(\Omega_r) , \end{aligned} \tag{48}$$

where the bound state wave function has been expressed as

$$\phi_0(\vec{r}) = R_{NL}(r) Y_L^M(\Omega_r) .$$

The radial integral is surface dominated because of the presence of the derivative of the potential. The angular integration can be performed straightforwardly and the k dependence only appears through the spherical Bessel functions $j_{L-1}(kr)$ and $j_{L+1}(kr)$. Then using the asymptotic expansion of these functions, expression (48) has the following behavior:

$$F(\vec{k},\vec{q}) \propto \frac{\vec{k}\cdot\vec{q}}{k^2} \int dr \sin\left[kr-(L\pm 1)\frac{\pi}{2}\right] \times R_{NL}(r)\frac{d}{dr}V(r) .$$

Assuming a sharp edge potential, the function $F(\vec{k},\vec{q})$ would display oscillations related to the radius of the potential and a $1/k$ falloff [note that the form factor (47) would then present a $1/k^5$ falloff]. However, the effect of the surface thickness is here essential and it will modify the falloff to an exponential decrease times some inverse power law while preserving an oscillatory structure.[15]

As a further illustration of Eqs. (47) and (48) we make an explicit evaluation in the case of the Coulomb potential for the s wave and obtain, as in Ref. 10 (where the sign of q is the opposite),

$$F(\vec{k},\vec{q}) \underset{\vec{q}\to 0}{\simeq} \pi^2 \frac{\vec{k}\cdot\vec{q}}{k^2} ,$$

which shows a large reduction as compared to the plane wave result [$\phi_0(k)$, Eq. (39)] since we have

$$S_{0\vec{k}}(\vec{q}) \underset{\vec{q}\to 0}{\simeq} -2\frac{\vec{k}\cdot\vec{q}}{k^2} S_{0k}^{\rm pw}(\vec{q}) . \tag{49}$$

As a last remark concerning the small momentum transfer behavior in the high energy limit, we mention that in the case of a dilute system where the spatial range of the wave function (s wave) is large as compared to that of the potential, we again obtain the result (49) because the wave function peaks at $\vec{p}=-\vec{k}$.

We now return to expansion (42) and study the limit of high momentum transfers. The momenta $|\vec{k}|$ and $|\vec{q}|$ are then both large compared to the Fermi momentum. Retaining at each order in the expansion (42) the contribution of leading order in $(\vec{q}\cdot\vec{p}/m)$, thereby neglecting commutators of the potential with lower order terms in $(\vec{q}\cdot\vec{p}/m)$, the nth order correction becomes

$$A_{\vec{k}}^{(n)}(\vec{q}) = g^{n+1}(\epsilon_{\vec{k}},\bar{\epsilon}) \times \int d^3p \left[-\frac{\vec{q}\cdot\vec{p}}{m}\right]^n V(\vec{p})\phi_0(\vec{p}+\vec{k}+\vec{q}) , \tag{50}$$

where $g(\epsilon_{\vec{k}},\bar{\epsilon})$ is given by (45b). Within these approximations the series (42) can be resummed and the form factor reads

$$S_{0\vec{k}}(\vec{q}) = \phi_0(\vec{k}+\vec{q}) + \int d^3p\, V(\vec{p}) \frac{1}{\dfrac{(\vec{k}+\vec{q})^2}{2m} - \epsilon_0 + \dfrac{\vec{p}\cdot\vec{q}}{m}} \times \phi_0(\vec{p}+\vec{k}+\vec{q}) . \tag{51}$$

For $\vec{k}$ and $\vec{q}$ which are both large but otherwise unrelated, it is not easy to find a simple expression for the form factor (51). Although in the case where $|\vec{k}+\vec{q}|$ is small as compared to k_F, the convergence of the series (42) is hard to assess [in the d^3p integral in the correction (50), the dominant contribution comes from the small values of $\vec{p}$], expression (51) has the interesting feature that in this limit one recovers the plane wave result. Indeed, setting $\vec{k}+\vec{q}=\vec{\lambda}$ in (51) with $|\vec{\lambda}| \ll k_F$, we have

$$S_{0\vec{k}}(\vec{q}) = \phi_0(\vec{\lambda}) + \int d^3\vec{p}\, V(\vec{p}) \frac{1}{\dfrac{\lambda^2}{2m} - \epsilon_0 + \dfrac{\vec{q}\cdot\vec{p}}{m} + i\eta} \times \phi_0(\vec{p}+\vec{\lambda}) . \tag{52}$$

Since the integrand peaks at small values of p and using the Schrödinger equation $V\phi_0 = (\epsilon_0 - K)\phi_0$ we find that the integral (52) in the limit $|\vec{\lambda}| \ll k_F, k, q$ is of the order $(k_F/q)\phi_0(\lambda)$. We thus obtain

$$S_{0\vec{k}}(\vec{q}) \underset{|\vec{q}+\vec{k}| \ll k,q}{=} \phi_0(\vec{k}+\vec{q}) \left[1 + 0\left[\frac{k_F}{q}\right]\right] . \tag{53}$$

To summarize, it is important to remark that the correction, in (51), to the plane wave approximation for the final state is expressed in terms of the potential between the nucleon and the core but not of the continuum final state wave function. Although expression (51) for the transition form factor has been derived assuming $|\vec{k}|$ and $|\vec{q}|$ are large, it has built-in the right small momentum transfer behavior. In other words expressions (51) and (47) coincide in the limit $|\vec{q}|$ going to zero to order $\vec{q}^{\,2}$. This leads us to conjecture that (51) has a larger range of validity than indicated from its derivation. We thus expect that expression (51) will be useful for practical calculations.

V. CONCLUSIONS

The study of elastic and inelastic nuclear reactions at large momentum transfer is very important to shed light on high momentum components of the wave function, short range correlations, etc. Hence, one has to seriously consider the possibility of factorizing the on-shell nucleon form factor from the response of the nucleus. We have shown that in the case of a scalar probe, there is indeed the possibility that various effects (i.e., exchange current versus off shell effects), which, in principle, preclude a transparent interpretation of the high momentum transfer data, may cancel each other to a large extent. The uncertainty in extracting nuclear structure information would then be substantially reduced.

As far as possible applications are concerned in the case of knockout reactions where a correct handling of the continuum is not always possible, one can still start in the high momentum transfer limit from a plane wave approximation for the struck nucleon and calculate, in addition, the corrections in (51) which become small in the quasielastic regime. Such corrections take care of what is usually called the orthogonality defect.[9–12] We think that the systematic procedure we have presented may be a useful starting point for a realistic analysis of exclusive and inclusive processes in the domain of large momentum transfer. Within the discussion of the orthogonality problem, we have also presented some general results concerning the low momentum transfer behavior of the transition form factor.

As practical examples we mention two cases. One can, for instance, calculate in quasielastic electron scattering [e.g., $(e,e'p)$] the form factor (37) via Eq. (51) and compare it with the experimental results in order to obtain information on the occupation amplitudes associated with the single particle bound states. The plane wave and correction terms clearly factorize the occupation amplitude and, with some information on the potential from the scattering of the nucleon on the residual nucleus, one may achieve a reliable estimate of the correction term.

Formulae such as (51) may also be useful to analyze the inclusive large angle proton nucleus scattering $p+A\rightarrow p'+x$, thus providing insight into the reaction mechanism. As a matter of fact, two mechanisms have been proposed for that reaction. In the first one, the observed proton comes from the target,[16] which would correspond to an inelastic transition at small momentum transfers, whereas the second assumes that one observes the backward scattered projectile[17] which, in contrast, would correspond to an inelastic transition at large momentum transfers.

The results we have derived in this work are valid for a system of A particles which have structure, are distinguishable, nonrelativistic, and interact via two body local interactions. The nucleons are treated nonrelativistically even in the loop integrals contributing to the elementary nucleon form factor, whereas the mesons are treated relativistically. If one would try to extend these results to include the effects of antisymmetrization, one would have to consider simultaneously, in addition to the processes we have studied, the related processes where the virtual meson flips to another nucleon (i.e., genuine meson exchange effects) together with those where the virtual meson enters as a constituent in the nuclear wave function (i.e., renormalization effects). The framework in which such a calculation would be performed has been outlined in Ref. 18. There, it was shown that the cancellations between recoil terms and wave function reorthonormalization arise as an explicit consequence of the nonrelativistic limit for the nucleons, and this result holds not only for zero momentum transfer but is true for any momentum transfer [see Fig. 4, Eq. (37), and the discussion in Ref. 18]. The cancellation would not, in general, occur if one takes into account relativistic effects (see also the discussion in Ref. 19).

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