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Quark contributions to the $pp \rightleftarrows d\pi^+$ reaction

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A six-quark model is used to estimate short distance contributions to the $pp \rightleftarrows d\pi^+$ amplitude. Substantial corrections to typical calculations are obtained.

[NUCLEAR REACTIONS Calculated cross sections for $\pi^+ d \leftrightarrow pp$. Energies near the (3,3) resonance. Application of quark models to nuclear physics.]

I. INTRODUCTION

The recent successes of quark and quantum chromodynamic concepts have led many physicists (e.g., Refs. 1 — 10) to ask how these new ideas might influence the understanding of atomic nuclei. In particular, it is reasonable to expect that quarks might play a role in high momentum transfer processes involving nuclei.

The $\pi^+ d \rightleftarrows pp$ reaction is an interesting candidate case for which the role of quarks could be explicit. If a pion of kinetic energy 180 MeV is absorbed on a deuteron (see Fig. 1), energy conservation requires that each outgoing nucleon has a momentum of about 550 MeV/c. As the deuteron is a weakly bound system, the $\pi^+ d \rightarrow pp$ process involves a momentum transfer of magnitude 550 MeV/c to each nucleon. Thus the relevant distance scale, as computed from the uncertainty principle, is about 0.4 fm. This is smaller than the radius of a nucleon ($\sim$1 fm), so that nucleons are expected to overlap during the $\pi^+ d \rightarrow pp$ process. If the composite nature of baryons is relevant, it is natural to invoke quark degrees of freedom.

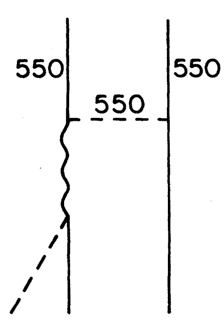

FIG. 1. Kinematics for the $pp \leftrightarrow d\pi^+$ process. The numbers are the momenta (in MeV/c) of the protons or virtual bosons. Solid lines are nucleons, dashed lines are mesons, and the wiggly line is the delta.

A completely fundamental description of the pertinent quark and gluon degrees of freedom of an overlapping two baryon system is very difficult to achieve at present. Therefore, we apply and generalize a phenomenological method developed earlier by one of us.[10] The idea is simple; whenever two baryons overlap treat them as six quarks in a spherical bag of radius r_0, to be determined phenomenologically ($r_0 = 0.8$ fm from Ref. 10). For radial separations greater than r_0, conventional baryon-baryon wave functions are to be used.

On the other hand, it is necessary to recall that calculations in which the momentum transfer is accomplished by the exchange of a virtual π or ρ meson have enjoyed a good deal of success[11,12] in explaining many features of the $\pi d \leftrightarrow pp$ data, especially at energies near the pion-nucleon (3,3) resonance. However, there are large sensitivities to poorly known aspects of the calculations such as virtual meson-nucleon delta coupling constants. Furthermore, at, for example, 600 MeV, the differences between theory and experiment are at about the 20% level and are not easily reduced.[13] There are also some serious theoretical difficulties.[11] Therefore, it is necessary to investigate possible corrections to this conventional approach.

Our aim in this work is to estimate a new short distance contribution to the $\pi^+ d \rightarrow pp$ reaction amplitude. As shown in Fig. 2, the basic feature of our mechanism is that the formation of a Δ occurs at large separations and the momentum transfer is accomplished by a quark-quark interaction which occurs within a six-quark confinement region (bag) formed when two baryons overlap. The entire process is described as follows. A π^+ in its encounter with a nucleon is absorbed and a delta isobar (Δ) is formed. (For simplicity we work in an energy regime in which this Δ formation is the dominant pion-nucleon interaction.) The relevant short-distance part of the Δ-nucleon (N) wave function is described, following Ref. 10, as six quarks in a bag. Two of the quarks interact, changing the six-quark ΔN wave function to the short distance, six-quark part of the proton-proton one. The escape of the two protons is achieved simply by using the exterior, or ordinary, part of the proton wave function, which has proper asymptotic behavior.

The amplitude corresponding to Fig. 2 is found, by itself, to account for about one-third of the experimental cross section. The details of the computation are presented in Sec. II, and numerical results are presented in Sec. III. A brief discussion and summary is also included in Sec. III. The present work serves as an improvement, based on an improved procedure[14] for determining six-quark wave functions, of earlier brief reports.[15]

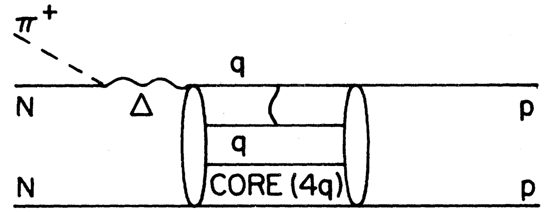

FIG. 2. $\pi^+ d \leftrightarrow pp$ in the hybrid model.

II. FORMALISM

The matrix elements of the transition operator T for the $pp \to d\pi^+$ process are described in a channel spin representation[16] as follows:

$$T_{c'c} \equiv \langle L_\pi J_D; JM \mid T \mid L_p S_p; JM \rangle \ , \tag{1}$$

where the orbital angular momentum and spin of the protons are L_p and S_p (we include no coupled proton-proton states in this calculation); J, M are the total angular momentum and z projection of the channel; L_π is the pion angular momentum; and J_D is the total angular momentum of the deuteron ($J_D = 1$). The cross section for the pion production process, $d\sigma/d\Omega$, is given in terms of the T-matrix elements as

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{v}\rho(E) 4 \sum_{\substack{cc' \\ c_1 c_1'}} T_{c'c} T^*_{c_1' c_1} (2J+1)(2J_1+1)(-1)^{1+S_p} \times \begin{Bmatrix} L_\pi & J & J_D \\ J_1 & L_{\pi_1} & \mathscr{L} \end{Bmatrix} \begin{Bmatrix} L_p & J & S_p \\ J_1 & L_{p_1} & \mathscr{L} \end{Bmatrix} P_{\mathscr{L}}(\cos\theta) \langle \mathscr{L} || Y_{L_\pi} || L_{\pi_1} \rangle \langle \mathscr{L} || Y_{L_p} || L_{p_1} \rangle \ , \tag{2}$$

where v is the relative velocity of the two protons

$$v = \frac{2p}{(p^2 + M^2)^{1/2}} \ , \tag{3}$$

p is the magnitude of the proton's momentum in the center of mass (c.m.) frame, and M is the proton mass. The density of states factor $\rho(E)$ is given by

$$\rho(E) = kE \ , \tag{4}$$

where k is the pion's c.m. momentum and

$$E = \frac{\omega_k E_D(k)}{E_D^{(k)} + \omega_k} \ . \tag{5}$$

The pion (ω_k) and deuteron [$E_D(k)$] total energies are defined in the usual way as the square root of the sum of the squares of k and the appropriate mass. In (2) θ is the angle between the outgoing pion and the incident proton; the reduced matrix elements of the spherical harmonics are found, e.g., on page 443 of Ref. 17. The result (2) follows from a straightforward application of Fermi's golden rule. Our matrix elements, $T_{c'c}$, are related in a simple way to those (P_{lj}) of Niskanen:

$$T_{c'c} = \frac{C}{(2L_p+1)^{1/2}} \frac{1}{(2L_\pi+1)^{1/2}} e^{i(L_p - L_\pi)} p_{lj} \ , \tag{6}$$

where C is a kinematic factor independent of channel quantum numbers.

A simple application of perturbation theory gives an expression for the T-matrix element for the $pp \to d\pi^+$ process of Fig. 2. Since we are concerned with obtaining a simple estimate, Fig. 2 is the only term to be included in $T_{c'c}$. Then

$$T_{c'c}=\sum_{L_\Delta S_\Delta J_\Delta}\int d^3p_\Delta \frac{\langle l_\pi J_D;J\mid V_{\pi N\Delta}\mid \psi_{\Delta N}^{L_\Delta S_\Delta J}(\vec{p}_\Delta)\rangle}{E^+-E_\Delta(p_\Delta)}\mathscr{A}^*_{L_\Delta S_\Delta J}(E_\Delta)\,\langle 6q;\Delta N\mid V\mid 6q;L_pS_pJ_p\rangle \alpha_{L_pS_pJ}(E)\ . \qquad (7)$$

The quantity $\alpha_{L_pS_pJ}(E)$ is the "probability" (see Ref. 14) that two protons of c.m. energy E, with quantum numbers L_p, S_p, and J_p, can fuse into a six-quark spherical bag. The ket

$$\mid 6q;L_pS_pJ_p\rangle$$

is the six-quark part of the proton-proton wave function, normalized to unity. We take the quark-quark interaction of Isgur and Karl[18] as the operator V. The quantity

$$\mathscr{A}_{L_\Delta S_\Delta J}(E_\Delta)\langle 6q,\Delta N\mid$$

is the six-quark wave function describing the short separation distance part ($r<r_0$) of the ΔN system of angular momentum quantum numbers [L_Δ (orbital) and S_Δ (spin)] and energy E_Δ. Here $\mid 6q,\Delta N\rangle$ is the six-quark part of the ΔN system (quantum numbers $L_\Delta S_\Delta$andJ_Δ) normalized to unity. The wave function $\psi_{\Delta N}^{L_\Delta S_\Delta J}(p_\Delta)$ describes the exterior and interior (to r_0) parts of the ΔN system (a plane wave representation is used); $V_{\pi N,\Delta}$ converts a Δ into a nucleon and a pion. (Only the exterior part of $\psi_{\Delta N}$ is used in the $V_{\pi N\Delta}$ matrix element.) Finally $\mid l_\pi J_D;JM\rangle$ is the wave function of a free pion (of angular momentum L_π) times the deuteron wave function, coupled to a total angular momentum J. The remainder of this section contains specific details regarding the quantities appearing in Eq. (7).

A. $\alpha_{L_pS_pJ}(E)$

A formalism invented by Wigner,[19] which he used to study the fusing of two colliding nuclei into one compound nucleus, is applied in Ref. 14 to obtain six-quark probabilities. The basic idea is that for radial separations less than r_0 the ordinary radial wave functions

$$4\pi\frac{U_{LSJ}(kr)}{kr}\phi_i\phi_j\ ,$$

(where $\phi_i\phi_j$ are the product of the internal nucleon wave functions) are replaced by

$$\alpha_{LSJ}(E)\psi_{LSJ}(r)\ ,$$

where r is described in terms of quark coordinates and $\psi_{LSJ}(r)$ is normalized to 1 and is independent of energy. (Only the relative coordinate r is made explicit here.) The assumption of the continuity of the wave function and its derivative across the boundary for a theory based on the Schrödinger equation (or continuity of current for a theory based on the Dirac equation) leads to constraints on $\alpha_{LSJ}(E)$. Because $\alpha_{LSJ}(E)$ has units of $\mathrm{fm}^{3/2}$, it is useful to define a probability $P_{LSJ}(E)$ via

$$P_{LSJ}(E)=\frac{\mid\alpha_{LSJ}\mid^2}{\Omega}\ , \qquad (8)$$

where Ω is some chosen volume. Thus $P_{LSJ}(\epsilon)$ represents the relative probability for the six-quark wave function compared with that of a plane wave in a volume Ω. The values of $P_{LSJ}(E)$ determined in Ref. 14 are shown in Fig. 3. In obtaining those results the Reid soft core wave function[20] is used to obtain $U_{LSJ}(kr)$, and

$$\Omega=\frac{4\pi}{3}R^3$$

with $R=1.4$ fm (a pion Compton wavelength) and $r_0=0.8$ fm. The physical interpretation of these results is discussed in Ref. 14.

B. Delta-nucleon wave function

It is convenient to define a quantity $X_{L_\Delta S_\Delta J,E}(\vec{r})$ by

$$X_{L_\Delta S_\Delta J,E}(\vec{r})=\int d^3p_\Delta\frac{\langle\vec{r}\mid\psi_{\Delta N}^{L_\Delta S_\Delta J}(p_\Delta)\rangle}{E^+-E_\Delta(p_\Delta)}\times\mathscr{A}^*_{L_\Delta S_\Delta J_\Delta}(E_\Delta)\ . \qquad (9)$$

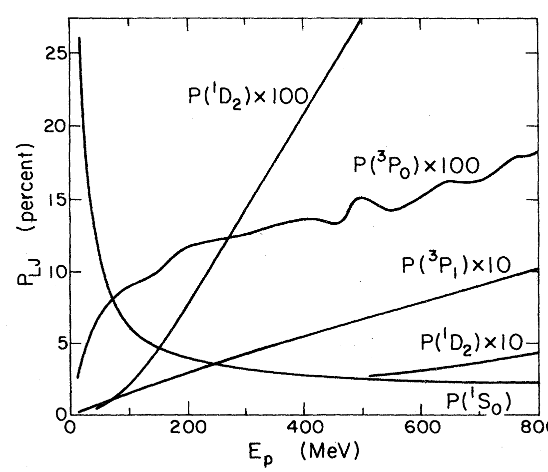

FIG. 3. Six-quark probabilities versus proton-proton center of mass energy.

We wish to simplify Eq. (9) so that it can be easily used in Eq. (7). To do this we first note that

$\mathscr{A}^*_{L_\Delta S_\Delta J_\Delta}(E_\Delta)\langle 6q,\Delta N|$

is the six-quark wave function describing the short-distance part of the ΔN system. From this definition [or directly from Eq. (7)] we may write

$$\mathscr{A}^*_{L_\Delta S_\Delta J_\Delta}(E_\Delta)=\int d^3r\langle\psi_{\Delta N}^{L_\Delta S_\Delta J}|\vec{r}\rangle\langle\vec{r}|6q;\Delta N\rangle \tag{10a}$$

$$=\int_{<r_0} d^3r\langle\psi_{\Delta N}^{L_\Delta S_\Delta J}|\vec{r}\rangle\tilde{\alpha}^*_{L_\Delta S_\Delta J_\Delta}\phi_{L_\Delta S_\Delta J_\Delta}(\vec{r})\,, \tag{10b}$$

where $\phi_{L_\Delta S_\Delta J_\Delta}(\vec{r})$ is the wave function describing the separation between quark clusters in the internal region $r<r_0$, and $\tilde{\alpha}_{L_\Delta S_\Delta J_\Delta}$ is the six-quark probability amplitude of the ΔN system as described below. Note that we shall not need any detailed properties of $\phi(r)$ in this paper, only its existence. In this regard it is important to keep in mind that our hybrid model can be defined *entirely in quark coordinates,* and that the relative variable $\vec{r}$ is introduced only for convenience.

The use of (10b) in (9) gives

$$X_{L_\Delta S_\Delta J}(\vec{r})=\tilde{\alpha}^*_{L_\Delta S_\Delta J}\int d^3r' G_{\Delta N}^{L_\Delta S_\Delta J}(\vec{r},\vec{r}\,')\phi_{L_\Delta S_\Delta J_\Delta}(\vec{r}\,')\,, \tag{11}$$

where $G_{\Delta N}^{L_\Delta S_\Delta J}(\vec{r},\vec{r}\,')$ is the Green's function (for the given quantum numbers) of the nonoverlapping ΔN system. To obtain $G_{\Delta N}$ we neglect ΔN nucleon interactions so that

$$G_{\Delta N}(\vec{r},\vec{r}\,')=\sum_{S_\Delta M_\Delta}\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')}\frac{|S\Delta M_S\rangle\langle S_\Delta M_S|}{E_{\pi N}-E_R+\dfrac{i\Gamma(E)}{2}-\dfrac{k^2k^2}{2\overline{M}_\Delta}}\,, \tag{12}$$

where $\overline{M}_\Delta$ is a renormalized Δ mass given by

$$\overline{M}_\Delta=M_\Delta\Big/\left[1+\frac{i}{2}\frac{\partial T}{\partial E}\right]. \tag{13}$$

The mass and width parameters M_Δ and $\Gamma(E)$ are given in Ref. 21. The quantity $E_{\pi N}$ is

$$E_{\pi N}=\omega(k)+(k/2)^2/2M\,. \tag{14}$$

A simple contour integration transforms (12) into

$$G_{\Delta N}(\vec{r},\vec{r}\,')=\frac{-2\overline{M}_\Delta i}{\hbar^2}Q\sum_{L_\Delta S_\Delta JM}\mathscr{Y}_{L_\Delta S_\Delta JM}(\hat{r})j_{L_\Delta}(Qr_<)h_L^{(1)}(Qr_>)\mathscr{Y}^\dagger_{L_\Delta S_\Delta JM}(\hat{r}\,')\,, \tag{15}$$

where $j_L(x)$ and $h_L^{(1)}(x)$ are spherical Bessel functions, $\mathscr{Y}$ is the usual spin-angle function, and the Δ-N relative momentum, Q is

$$Q^2=\frac{2\overline{M}_\Delta}{\hbar^2}(E_{\pi N}-E_R+i\Gamma(E_{\pi N})/2)\,. \tag{16}$$

The quantity Q varies rapidly with energy. Observe that $G_{\Delta N}^{L_\Delta S_\Delta J}(\vec{r},\vec{r}\,')$ is obtained by applying the relevant angular momentum projection operators on $G_{\Delta N}(\vec{r},\vec{r}\,')$ of Eq. (15).

From (11) we realize that the pertinent values of r and r' obey $r>r'$ since r' refers to variables within the six-quark bag. The use of (15) in (11) then gives

$$X_{L_\Delta S_\Delta J}(\vec{r})=\tilde{\alpha}^*_{L_\Delta S_\Delta J}\left[\frac{-2\overline{M}_\Delta iQ}{\hbar^2}\right]\mathscr{Y}^\dagger_{L_\Delta S_\Delta JM}h_L^{(1)}(Qr)j_L(Qr_0)\mathscr{M}_{L_\Delta S_\Delta J'}\,, \tag{17}$$

where

$$\mathscr{M}_{L_\Delta S_\Delta J}=\int d^3r'\frac{j_L(Qr')}{j_L(Qr_0)}\mathscr{Y}^\dagger_{L_\Delta S_\Delta JM}(\hat{r}\,')\phi_{L_\Delta S_\Delta J_\Delta}(\vec{r}\,')\,. \tag{18}$$

The quantity $1 = j_L(Qr_0)/j_L(Qr_0)$ is inserted in (17) to render the d^3r' matrix element less dependent on energy. [Since r_0 is small (~ 0.8 fm) $j_L(Qr')j_L(Qr_0)$ is expected to vary slowly with energy for $r' < r_0$.]

As a final step define

$$\alpha^*_{L_\Delta S_\Delta J} = \alpha^*_{L_\Delta S_\Delta J} \mathscr{M}_{L_\Delta S_\Delta J} , \tag{19}$$

so that

$$X_{L_\Delta S_\Delta J}(\vec{r}) = \alpha^*_{L_\Delta S_\Delta J} \left[\frac{-2\overline{M}_\Delta iQ}{\hbar^2} \right] j_L(Qr_0) \mathscr{Y}^\dagger_{L_\Delta S_\Delta JM}(\hat{r}) h_L^{(1)}(Qr) . \tag{20}$$

This is the simplification of (9) that we seek. For the ΔN system, the quantity $\alpha_{L_\Delta S_\Delta J}$ plays the role that $\alpha_{L_p S_p J_p}$ does for the pp system, and could be obtained from calculations analogous to Ref. 14. However, it is possible to use the pp results to constrain $\alpha_{L_\Delta S_\Delta J}$, and this is sufficient for the present estimate. Calculations show (see below) that terms of $T_{cc'}$ with $L_\Delta = 0$ give the dominant contributions. For the pp system $P(^1S_0)$ (Fig. 3) is independent of energy except, of course, for energies near the virtual state. Because no such state is known to exist for the $L_\Delta = 0$, ΔN system we take $\alpha_{L_\Delta S_\Delta J}$ to be independent of energy. Indeed we assume that $P_\Delta(L_\Delta = 0) = |\alpha_{L_\Delta S_\Delta J}|^2/\Omega$ is 0.10. This is about equal to (but somewhat larger than) the value of P for the $^1S_0$ pp system for energies greater than about 100 MeV.

The use of (9) and (19) in (7) leads to

$$T_{c'c} = \sum_{L_\Delta S_\Delta J_\Delta} C \langle l_\pi J_D ; J \mid V_{\pi N \Delta} \mid \chi_{\Delta N}^{L_\Delta S_\Delta J} \rangle \alpha^*_{L_\Delta S_\Delta J} \alpha_{L_p S_p J}(E) \langle 6q ; \Delta N \mid V \mid 6q ; L_p S_p ; J \rangle , \tag{21a}$$

where

$$C \equiv \frac{-2\overline{M}_\Delta iQ}{\hbar^2} j_L(Qr_0) \tag{21b}$$

and

$$\langle \vec{r} \mid \chi_{\Delta N}^{L_\Delta S_\Delta J} \rangle \equiv h_L^{(1)}(Qr) \mathscr{Y}_{L_\Delta S_\Delta JM}(\hat{r}) . \tag{21c}$$

Equation (20) is the main result of this subsection.

C. Interior matrix element: $\langle 6q ; \Delta N \mid V \mid 6q ; NN(L_p S_p ; J) \rangle$

Although the "probability amplitudes" $\alpha_{L_\Delta S_\Delta J}$ and $\alpha_{L_p S_p J}$ can be constrained by applying Wigner's ideas of 1946, the normalized wave functions $\psi_{6q}(\Delta N)$ and $\Psi_{6q}(NN, L_p S_p J)$ are not. We employ the following model for the $^1D_2$ state, which is most important[17] for the $pp \leftrightarrow \pi d$ reaction: Configurations with a core of six quarks in the $S_{1/2}$ state and two valence quarks are used. The minimum number of configurations to represent orthogonal NN and ΔN states is two. However, there is also a spurious state representing pure center-of-mass motion. For this reason we use three configurations, and write

$$| 6q ; NN, {}^1D_2 \rangle = N_1 [\, | (p^2)_2 S^4 \rangle + a_1 \, | (sd)_2 S^4 \rangle + a_2 \, | (d^2)_2 S^4 \rangle] , \tag{22}$$

where $(p^2)_2$, $(sd)_2$, and $(d^2)_2$ represent a quark pair of color $\overline{3}$, coupled to spin 0 and angular momentum 2. The labels s, d, and p represent the lowest-energy single-quark state of the given angular momentum. The factor S^4 is our notation for four quarks coupled to orbital and spin angular momentum of zero.

In order to approximately account for the redundant center-of-mass variable, we project out of the first two configurations in Eq. (22) the spurious state corresponding to center-of-mass motion. This is done by making use of the fact that our potential, defined in the relative coordinates, will not couple the spurious center-of-mass state to the physical state. Thus if we write

$$| a \rangle = \frac{1}{(1+\epsilon^2)^{1/2}} [\, | (sd) S^4 \rangle + \epsilon \, | (p^2)_2 S^4 \rangle] , \tag{23a}$$

$$| b \rangle = \frac{1}{(1+\epsilon^2)^{1/2}} [\, | (p^2)_2 S^4 \rangle - \epsilon \, | (sd) S^4 \rangle] , \tag{23b}$$

and calculate $\langle a \mid V \mid b \rangle$ using the Isgur-Karl interaction,[18] we find that the matrix element coupling states $1a\rangle$ and $1b\rangle$ vanishes for $\epsilon = -1/\sqrt{2}$, i.e.,

$$\langle b \mid V \mid a \rangle \mid_{\epsilon = -1/\sqrt{2}} = 0 , \tag{24}$$

with the matrix elements leading to this result given in Eq. (27). Thus the spurious state is determined to be

$$|{}^1D_2,\text{ spurious}\rangle=\frac{1}{\sqrt{3/2}}\left[\left|(sd)_2S^4\rangle-\frac{1}{\sqrt{2}}\right|(p^2)_2S^4\rangle\right] . \tag{25}$$

From this we choose $a_1=1/\sqrt{2}$ in Eq. (22). The effect of center-of-mass motion is thereby greatly reduced for the wave function (22).

The wave function for the ΔN state is written as

$$|6q;\Delta N,{}^1D_2\rangle=N_2\,|1(p^2)_2S^4\rangle+a_1\,|(sd)_2S^4\rangle+a_2'\,|(d^2)_2S^4\rangle , \tag{26}$$

with the orthogonality condition $a_2a_2'=-(1+a_1{}^2)$. For spin singlet states only the spin-spin part (V_{ss}) of the Isgur-Karl force contributes:

$$V_{ss}(\vec{r})=\frac{2\alpha_s}{3m^2}\frac{8\pi}{3}\vec{S}_1\cdot\vec{S}_2\delta(\vec{r}) , \tag{27a}$$

where

$$\frac{4\alpha_s}{3\sqrt{2\pi}m^2}\frac{1}{b^2}=300\text{ MeV} . \tag{27b}$$

Note that only the $S=0$ component of the $6q$-ΔN wave function (allowed in our model) contributes with this force for the ${}^1D_2$ state. When one computes the matrix elements of (27a) the term on the left-hand side of (27b) appears as the only dimensional factor, so that the quark mass, m, and the oscillator parameter, b, need not be specified. The matrix element of V in the pure configurations are

$$\langle (sd)_2|V|(sd)_2\rangle=-37.5\text{ MeV} ,\quad \langle (d^2)_2|V|(d^2)_2\rangle=-56.25\text{ MeV} ,$$
$$\langle (p^2)_2|V|(p^2)_2\rangle=-75\text{ MeV} ,\quad \langle (d^2)_2|V|(p^2)_2\rangle=57.5\text{ MeV} ,$$
$$\langle (sd)_2|V|(p^2)_2\rangle=-53\text{ MeV},\quad \langle (d^2)_2|V|(sd)_2\rangle=41\text{ MeV} .$$

From (22), (24), and (27) we find

$$\langle 6q;\Delta N|V|6q;NN;{}^1D_2\rangle=[-84.4+(a_2+a_{2'})86.5]/[(\tfrac{3}{2}+a_{2'}{}^2)]^{1/2} , \tag{28}$$

where $a_1=1/\sqrt{2}$ and $a_2a_{2'}=-\frac{3}{2}$ has been used. Taking $a_2=0.9$ and $a_{2'}=-1.667$ we obtain

$$\langle 6q;\Delta N|V|6q;NN;{}^1D_2\rangle=48\text{ MeV} . \tag{29}$$

Note that the result (29) would also be obtained by using $|a\rangle$ and $|b\rangle$ of Eq. (23) as $|NN\rangle$ and $|\Delta N\rangle$ states with $\epsilon=0.25$. Using this simpler form for ${}^1S_0$ and ${}^3P_1$ channels, which are less important in the calculation, we find

$$\langle 6q\Delta N|V|6q;NN;{}^1S_0\rangle=59.9\text{ MeV} , \tag{30a}$$

$$\langle 6q\Delta N|V|6q;NN;{}^3P_1\rangle=38.9\text{ MeV} . \tag{30b}$$

The matrix elements (29) and (30) are the only ones we include in the sum of Eq. (2).

D. Exterior matrix elements: $\langle l_\pi J_D;J\,|\,V_{\pi N\Delta}\,|\,\phi_\Delta^{L_\Delta S_\Delta J}\rangle$

The matrix element for pion absorption on a deuteron leading to a ΔN state has been computed many times (e.g., Ref. 17). The only difference between our computation and others is that the radial integral starts at $r_0=0.8$ fm, not at the origin. In the interest of completeness we display the relevant formulae.

The $\Delta\to N\pi$ operator is given by

$$V_{\pi N\Delta}=\frac{g}{m_\pi}u(k)(\vec{S}_1\cdot\vec{\nabla}_\pi\phi_\phi^{(+)}(r_1)\vec{T}_1+\vec{S}_2\cdot\nabla_\pi\phi_\pi^{(+)}(\vec{r}_2)\vec{T}_2) , \tag{31}$$

where g is the $\pi N\Delta$ coupling constant $[g^2=\frac{75}{25}(4\pi)(0.08)]$; $\vec{S}_n$ and $\vec{T}_n$ convert the nth Δ to a nucleon by converting spin (isospin) $\frac{3}{2}$ to spin (isospin) $\frac{1}{2}$; $\phi_\pi^{(+)}(\vec{r}_n)$ is that part of the pion field operator that creates a pion field at $\vec{r}_n$; and, $u(k)$ is the form factor. We follow the cloudy bag model[4] and use

$$u(k)=\frac{3j_1(kR)}{kR} , \tag{32}$$

with bag radius $R=0.8$ fm.

The use of (31) and (21c) leads to the result

$$\langle l_\pi J_D;J \mid V_{\pi N\Delta} \mid \phi_{\Delta N}^{L_\Delta S_\Delta J} \rangle = \frac{gu(k)}{m_\pi\sqrt{2\omega_k}}(1+(+)^{L_\Delta+S_\Delta+J}) \times \sum_{\substack{L=l_\pi\pm1 \\ L_D=0,2}} C_L(-1)^{l_\pi-J}\int_{r_0}^{\infty} r\,dr\, j_L\left[\frac{kr}{2}\right]u_{L_D}(r)h_{L_\Delta}^{(1)}(Qr) \times \hat{l}_\pi \hat{S}_\Delta \left[\frac{3}{\pi}\right]^{1/2} \times \begin{Bmatrix} L_D & 1 & 1 \\ L & 1 & l_\pi \\ L_D & S_\Delta & J \end{Bmatrix} \hat{L}_D\hat{L} \times \langle L_D 0 L 0 \mid L_\Delta 0\rangle \begin{Bmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ 1 & \frac{1}{2} & S_\Delta \end{Bmatrix}, \quad (33)$$

where

$$C_L = -\left[\frac{l_\pi+1}{2l_\pi+1}\right]^{1/2} \text{ if } L=l_\pi+1$$
$$= \left[\frac{l_\pi}{2l_\pi+1}\right]^{1/2} \text{ if } L=l_\pi-1\,, \quad (34)$$

and $\hat{J}\equiv(2J+1)^{1/2}$. The quantities $u_{0,2}(r)$ stand for the radial deuteron wave functions of orbital angular momentum 0 and 2. In making our computations the Reid soft core potential is used [but only the s state is kept since that contribution is known to dominate the low momentum transfer integral of (33)].

The formulae of the present section, along with the various stated approximations, completely specify the calculation. Key results are (2), (7), (20), (28), and (33).

III. NUMERICAL RESULTS AND DISCUSSION

Our results are shown in Figs. 4 and 5. The energy dependence of the total cross section as well as the shape of the angular distribution are fairly well reproduced. However, the cross section results are too small by a factor of about 3. This undershoot is not surprising, since the one-pion exchange term (Fig. 1) can give a large contribution for proton-proton separation distances greater than $r=r_0$ ($=0.8$ fm).

Even though the computed cross sections are too small, the six-quark contributions are not at all negligible. A factor of 3 in the cross section is a factor of 1.7 in the amplitude. Thus if the one pion amplitude squared were of the size of the data σ_{exp}, the coherent inclusion of the quark contributions could, depending on relative phases, make the full result anything between $0.18\sigma_{\text{exp}}$ and $2.48\sigma_{\text{exp}}$.

That short distance contributions have large influences on computed $pp\rightarrow d\pi^+$ cross sections is well known in the conventional picture of color zero hadron interactions. Rho meson exchanges,[22] E/h_0 off-shell field-theoretic corrections,[23] as well as form factors arising from the relative time dependence of the relativistic deuteron wave function[24] all yield, in appropriate contexts, sizable short-distance corrections, all of the order of the six-quark amplitudes which we have calculated. In our view the hybrid model gives a better representation of the short-distance nuclear behavior. Moreover, it is more economical to compute all short-distance effects in terms of quark degrees of freedom. The present

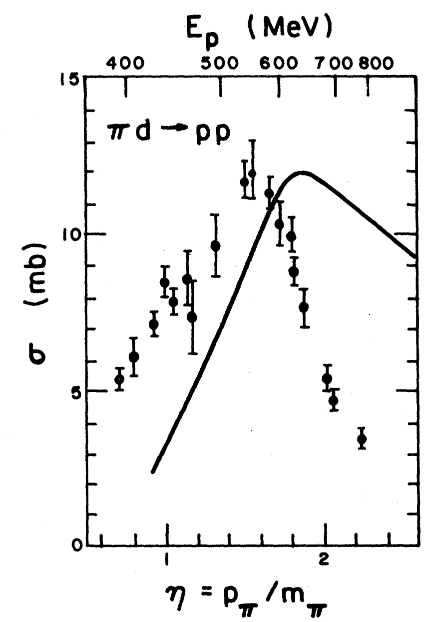

FIG. 4. Computer total cross sections for $\pi^+d\rightarrow pp$. Our theoretical result is multiplied by a factor of 4.

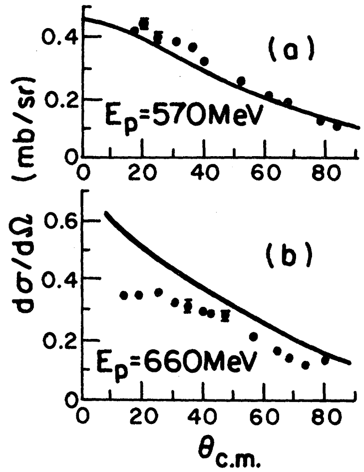

FIG. 5. Angular distributions for $pp \rightarrow d\pi$. E_p is the proton laboratory energy.

work represents the first such attempt.

Important improvements upon this work are needed. Methods for computing the parameters $\alpha_{L_\Delta S_\Delta J'}$ and ϵ must be developed. The quark amplitude must be added coherently to the pion-exchange contribution. If these steps could be taken reliably one might be able to sort out some of the puzzles in understanding the $pp \rightarrow d\pi^+$ process.

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[1]M. Harvey, Nucl. Phys. A352, 326 (1981).

[2]D. P. Stanley and D. Robson, Phys. Rev. Lett. 45, 235 (1980); D. Robson in Proceedings of Nuclear Structure with Intermediate Energy Probes, Los Alamos, New Mexico, 1980; in *International School of Nuclear Physics*, 1981, Erice, Italy, 5th Course: Quarks and the Nucleus (Pergamon, New York, to be published); Nucl. Phys. A308, 381 (1978).

[3]L. S. Celenza, W. S. Pong, and C. M. Shakin, Brooklyn College Report BCINT 81/061/109, 1981 (unpublished).

[4]G. A. Miller, A. W. Thomas, and S. Théberge, Phys. Lett. 91B, 192 (1980); S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D 22, 2383 (1980); A. W. Thomas, S. Théberge, and G. A. Miller, *ibid.* 24, 216 (1981); S. Théberge, G. A. Miller, and A. W. Thomas, Can. J. Phys. 60, 59 (1982); G. A. Miller, S. Théberge, and A. W. Thomas, Comments Nucl. Part. Phys. 10, 101 (1981).

[5]H. J. Weber, Z. Phys. A 297, 261 (1980); 301, 141 (1981); B. L. G. Bakker, M. Bozoian, J. N. Maslow, and H. J. Weber, Phys. Rev. C 25, 1134 (1982).

[6]R. A. Freedman, W-Y. P. Hwang, and L. Wilets, Phys. Rev. D 23, 1103 (1981).

[7]H. J. Pirner and J. P. Vary, Phys. Rev. Lett. 46, 1376 (1981).

[8]C. W. Wong, in *International School of Nuclear Physics*, 1981, Erice, Italy, 5th Course: Quarks and the Nucleus (Pergamon, New York, to be published); K. F. Liu and C. W. Wong, Phys. Rev. D 21, 1350 (1980).

[9]C. de Tar, Phys. Rev. D 24, 752 (1981); 24, 762 (1981).

[10]L. S. Kisslinger, Phys. Lett. 112B, 307 (1982).

[11]M. Betz, B. Blankleider, J. A. Niskanen, and A. W. Thomas, in *Pion Production and Absorption in Nuclei—1981 (Indiana University Cyclotron Facility)*, Proceedings of the Conference on Pion Production and Absorption in Nuclei, AIP Conf. Proc. No. 79, edited by Robert D. Bont (AIP, New York, 1982), p. 65.

[12]D. F. Measday and G. A. Miller, Annu. Rev. Nucl. Part. Sci. 29, 121 (1979).

[13]J. A. Niskanen, see Ref. 11, p. 101.

[14]G. A. Miller, L. S. Kisslinger, and E. M. Henley, University of Washington report (unpublished).

[15]G. A. Miller, see Ref. 11, p. 47; L. S. Kisslinger, see Ref. 11, p. 243.

[16]A. K. Kerman, Los Alamos Laboratory Report LAMS-2740 (unpublished).

[17]J. A. Niskanen, Nucl. Phys. A298, 417 (1978).

[18]N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978).

[19]E. P. Wigner, Phys. Rev. 70, 15 (1946).

[20]R. V. Reid, Jr., Ann. Phys. (N.Y.) 50, 411 (1968).

[21]R. A. Freedman, G. A. Miller, and E. M. Henley, Phys. Lett. 103B, 937 (1981); Nucl. Phys. (to be published).

[22]M. Brack, D. O. Riska, and W. Weise, Nucl. Phys. A287, 425 (1977).

[23]M. Alberg, E. M. Henley, and G. A. Miller, Nucl. Phys. A306, 447 (1978); G. A. Miller in *Meson-Nuclear Physics—1979 (Houston)*, Proceedings of the 2nd International Topical Conference on Meson-Nuclear Physics, AIP Conf. Proc. No. 54, edited by E. V. Hungerford III (AIP, New York, 1979), p. 561.

[24]B. K. Keister and L. S. Kisslinger, Nucl. Phys. A326, 445 (1979).