PHYSICAL REVIEW C VOLUME 27, NUMBER 4 APRIL 1983

Stretched excitations and the spin-dependent part of the pion-nucleon interaction

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(Received 20 August 1982)

The p-wave form of the spin-orbit operator provides the basis for a single scattering model for the excitation of unnatural parity levels in pion-nucleus scattering in the region of the Δ resonance. The model is found to give a reasonable description of the π^+ and π^- experimental cross sections for the excitation of stretched 4^- and 6^- states in ^{16}O and ^{28}Si. In the calculations the interaction strength was fixed from the free pion-nucleon data and the spin transition densities were determined from recent (e,e') and (p,p') data. A schematic discussion of possible corrections to the model is also presented.

NUCLEAR REACTIONS Distorted wave calculation of 162 MeV (π,π') cross sections, ^{28}Si(6^-) and isospin mixed ^{16}O(4^-) stretched states. Impulse approximation for pion-nucleus spin-orbit interaction, energy dependence of t matrix.

I. INTRODUCTION

In this paper we present a simple single scattering model for describing the excitation of unnatural parity levels in nuclei via inelastic pion scattering in the Δ-resonance region. The model is based on the use of the short range limit, or p wave form of the spin-orbit operator[1] to represent the spin-dependent part of the effective pion-nucleus interaction. With this assumption, the spin dependent part of the effective pion-nucleus interaction is characterized by a single isospin and energy dependent strength parameter. Making the impulse approximation[2] and fixing the strength parameter from the free pion-nucleon phase shifts,[3] we apply the model, within the framework of the distorted wave approximation, to the recnt experimental differential cross section data[4,5] for the excitation of stretched 4^- and 6^- levels in ^{16}O and ^{28}Si taken at $E_\pi = 162$ MeV. Information about the spin transition densities required for these calculations is taken from the other recent (e,e') (Refs. 6 and 7) and (p,p') (Refs. 8–10) studies of these same excitations. We also discuss the energy dependence of the spin-dependent part of the effective pion-nucleus interaction as given by the impulse approximation and reiterate the important role of stretched excitations as a means for isolating this energy dependence. The latter point is of added significance in view of recent discussions of possible Fermi-motion current corrections[11,12] and Δ-hole capture contributions[13] to the transition matrix elements for unnatural parity transitions.

II. THE MODEL

The p wave form of the spin-orbit operator is given in coordinate space by

$$t_{\pi N}(\vec{r}) = t^{LS}\vec{\nabla}_r\delta(\vec{r})\times\vec{p}\cdot\vec{\sigma}_N, \tag{1}$$

where $\vec{r}$ is the relative coordinate, $\vec{p}$ is the momentum of either particle in the center of momentum frame (c.m.), $\vec{\sigma}_N$ is the nucleon Pauli spin operator, and t^{LS} is the strength parameter of interest with units of MeV fm^5. This form is minimally nonlocal and its use is analogous to the Kisslinger prescription[14] for treating the spin-independent part of the pion-nucleon interaction. Its main advantage is that it provides a more transparent picture of the spin dependent effects in pion-nucleus scattering than is obtained from the more conventional nonlocal models.[12,15,16] The impulse approximation estimate of t^{LS} is easily obtained by forming the momentum space matrix element of Eq. (1) in the π-N c.m.

$$t_{\pi N}=\langle \vec{k}' \,|\, t_{\pi N}(\vec{r}) \,|\, \vec{k} \rangle$$
$$=-it^{LS}\vec{\sigma}_N\cdot\vec{k}\times\vec{k}', \tag{2}$$

employing the usual relationship between the t matrix and the scattering amplitudes f

$$t=-\frac{2\pi\hbar^2}{\omega_c}f, \tag{3}$$

and making use of standard representations of f (Refs. 2, 3, and 17) to obtain:

$$t^{LS}=t_0^{LS}+t_1^{LS}\vec{t}_\pi\cdot\vec{\tau}_N, \tag{4a}$$

$$t_0^{LS}k_c{}^2=\frac{2\pi\hbar^2}{\omega_c}\tfrac{1}{3}[-\alpha_{11}-2\alpha_{31}+\alpha_{13}+2\alpha_{33}], \tag{4b}$$

$$t_1^{LS}k_c{}^2=\frac{2\pi\hbar^2}{\omega_c}\tfrac{1}{3}[\alpha_{11}-\alpha_{31}-\alpha_{13}+\alpha_{33}], \tag{4c}$$

$$\alpha=\frac{e^{i\sigma}\sin\delta}{k},\quad \delta=\delta_{2T,2J}. \tag{4d}$$

Here k_c and ω_c are the wave number and reduced energy in the π-N c.m., $\vec{t}_\pi$ is the pion isospin, and $\vec{\tau}_N$ is the nuclear Pauli isospin operator. We evaluate Eq. (4) using the phase shifts of Ref. 3. Estimates of t^{LS} from other dynamical models such as the intermediate Δ-hole approach[18] would also be useful. This might require that t^{LS} be density dependent.

The incorporation of the interaction of Eq. (1) into a folding model description of inelastic pion-nucleus scattering is straightforward. The problem is simply one of taking a matrix element between the initial and final states of the target nucleus to obtain the scattering potential

$$U_{\pi A}=\gamma\langle f \,|\, \sum_N t_{\pi N}(\vec{r}) \,|\, i \rangle. \tag{5}$$

Here $\gamma\approx k\omega_c/k_c\omega$ is the usual factor required for transformation from the π-N c.m. to the pion-nucleus (π-A) c.m.,[17] with k and ω representing the wave number and reduced energy in the latter frame. The contribution to the pion-nucleus optical potential from Eq. (5) for targets with spin-unsaturated shells was discussed recently.[19] Only the contribution for unnatural parity transitions is of interest here.

For this case the integration in Eq. (5) is most easily carried out by applying the discretized momentum techniques of Ref. 20 and methods for expanding the spin-orbit operator described in Refs. 21 and 22. The final result for a $0^+\to J^\pi$ unnatural parity transition is

$$U_{\pi A}=\bar{U}_J(r_\pi)[Y_J(\hat{r}_\pi)\times\nabla_\pi]^J, \tag{6}$$

where $\bar{U}_J(r_\pi)$ is the reduced radial transition potential which is essentially the form factor required for a distorted wave calculation. This is given by

$$\bar{U}_J(r_\pi)=\alpha_\pi\frac{2}{\pi}\sum_{\substack{n\\T}}k_n{}^2\Delta k_n j_J(k_n r_\pi)\times\epsilon^T t_T^{LS}k_n{}^2\rho_J^{sT}(k_n), \tag{7}$$

where k_n and Δk_n are the momentum eigenvalues on a finite sphere and associated differences,[20] j_J is a spherical Bessel function, $\alpha_\pi\approx k_c/k$ results from the decomposition of

$$\vec{p}=\alpha_\pi\vec{p}_\pi-\alpha_N\vec{p}_N$$

in the π-A c.m.,[23] and ϵ^T is 1 for $T=0$ and ∓1 for $T=1$ for $\pi^\pm$ scattering which assumes $t_z=+\frac{1}{2}$ and $+1$ for n and π^-, respectively. The quantity $\rho_J^{sT}(k_n)$ in Eq. (7) is the transverse spin transition density defined by

$$\rho_J^{sT}(k_n)=\left[\frac{J+1}{2J+1}\right]^{1/2}\rho_{J,J-1}^{sT}(k_n)-\left[\frac{J}{2J+1}\right]^{1/2}\rho_{J,J+1}^{sT}(k_n), \tag{8a}$$

$$\rho_{JL}^{sT}(k_n)=\langle f\,||\sum_N j_L(k_n r_N)[Y_L(\hat{r}_N)\times\vec{\sigma}_N]^J\tau_N^T||\,i\rangle, \tag{8b}$$

$$\tau_N^0=1,\quad \tau_N{}^1=\vec{\tau}_N, \tag{8c}$$

where the matrix element in Eq. (8b) is reduced in coordinate space only according to the convention of Ref. 24. These results show clearly that, at this level of approximation, pion-inelastic scattering provides a unique measure of the transverse spin transition density for an unnatural parity transition. Transitions to states of stretched configuration, i.e., $[j_p j_h^{-1}]^J$ with $j_p=l_p+\frac{1}{2}$, $j_h=l_h+\frac{1}{2}$, and $J=j_p+j_h$, are a special class of transitions where the current transition densities vanish and $\rho_{J,J+1}^{sT}$ in Eq. (8a) vanishes so that the (e,e'), (p,p'), and (π,π') reactions are related through a common spin transition density $\rho_{J,J-1}^{sT}$. This has been discussed explicitly in Refs. 1, 10, and 25 and works cited therein. Modifications in this picture due to the effects described in Refs. 11–13 will be discussed below.

III. APPLICATION

The experimental data of Ref. 4 consist of full π^+ and π^- differential cross sections for the excitation of three levels in ^{16}O at $E_x = 17.79$, 18.98, and 19.80 MeV which are attributed to the stretched $[1d_{5/2}-1p_{3/2}{}^{-1}]4^-$ configuration. Differences in the magnitudes of the π^+ and π^- cross sections for these levels were interpreted in terms of Coulomb mixing[4] and the mixing matrix elements and relative particle-hole strengths for the three levels were deduced from the π^+ and π^- cross section ratios assuming Δ dominance $[t_0^{LS}=2t_1^{LS}$ in Eq. (4)]. It was found that the 18.98 MeV level was nearly a pure $T=1$ excitation while the 17.79 and 19.80 MeV levels were predominantly $T=0$ excitations with $T=1$ admixtures making the lower state more protonlike and the upper state more neutronlike in character. It was also noted that the total $T=0$ particle-hole strength observed was only about 50% that of the $T=1$ strength. No model was applied to calculate the absolute inelastic pion cross sections in this work so no estimate was made of the total particle-hole strength observed. Subsequent work[1,6,26] based on the (e,e') and (p,p') data for these same levels[6,8] generally corroborates the results of Ref. 4 and further indicates that only about 40% of the $T=1$ particle-hole strength is being observed. Additional information about the fragmentation of the stretched particle-hole strength in ^{16}O via 3p-3h configurations is available from studies of the ^{13}C(^{6}Li,t)^{16}O* reaction.[27] A more complete discussion of these transitions will be given in a forthcoming paper.[6]

The experimental data of Ref. 5 consist of full π^+ and partial π^- differential cross sections for the stretched $[1f_{7/2}-1d_{5/2}{}^{-1}]6^-$ $T=0$ and $T=1$ levels in ^{28}Si at $E_x=11.58$ and 14.36 MeV, respectively. These were the first stretched levels observed via pion inelastic scattering. The two levels here are well separated and Coulomb mixing is not a factor. In Ref. 5 it was concluded on the basis of a calculation using the model of Ref. 16 that about 15% and 38% of the total $T=0$ and $T=1$ particle-hole strength was being observed. As in the ^{16}O case results from (e,e') and (p,p') studies[1,7,9,10] were found to be consistent with the pion results. Additional supporting evidence is available from a recent low energy resonant (p,p) experiment.[28]

The primary concern of this paper is to obtain an absolute estimate of the inelastic pion scattering differential cross sections of Refs. 4 and 5 based on the interaction of Eq. (1). To this end we have assumed a harmonic oscillator form for the isovector spin transition density $\rho^{s1}_{J,J-1}(k_n)$ and fixed the size and strength parameters from the most recent electron scattering data.[6,7] We neglect possible shape differences between the isoscalar and isovector spin transition densities[29] and assume that $\rho^{s0}_{J,J-1}(k_n)$ is proportional to $\rho^{s1}_{J,J-1}(k_n)$. With this assumption the strengths and mixing amplitudes for the spin transition densities for the $T\approx 0$ excitations in ^{16}O were determined by the same fitting procedure used in Ref. 4 except that we use the t^{LS} given by Eq. (4) rather than that given by the Δ-dominance assumption. This results in $T=0$ densities about 10% larger than those given in Ref. 4. The isoscalar density for ^{28}Si was fixed from the ratio of $T=0$ to $T=1$ strength deduced from the (p,p') data in Ref. 10. The complete spin transition densities are summarized in Table I in terms of appropriate particle-hole spectroscopic amplitudes.[30]

The results of our distorted wave calculations are compared with the experimental data of Refs. 4 and 5 in Figs. 1 and 2. These calculations were made

TABLE I. Wave functions used in present calculations.

Target	E_x (MeV)	$[j_p j_h^{-1}]J^\pi$	α (fm^{-1})[a]	$Z(T=0)$[b]	$Z(T=1)$[b]
	17.79	$[1d_{5/2}-1p_{3/2}{}^{-1}]4^-$	0.618	0.330	−0.077
^{16}O	18.98	$[1d_{5/2}-1p_{3/2}{}^{-1}]4^-$	0.618	0.001	−0.620
	19.80	$[1d_{5/2}-1p_{3/2}{}^{-1}]4^-$	0.618	0.348	0.075
^{28}Si	11.58	$[1f_{7/2}-1d_{5/2}{}^{-1}]6^-$	0.567	0.310	
	14.36	$[1f_{7/2}-1d_{5/2}{}^{-1}]6^-$	0.567		0.528

[a]Harmonic oscillator size parameter $\alpha=[M\omega/\hbar]^{1/2}$ which has been obtained with the standard shell model center of mass correction included. This correction has been excluded in a consistent fashion in the work of Refs. 1, 10, and 25.

[b]Spectroscopic amplitudes first introduced by Schaeffer and Raynal and defined explicitly in Ref. 30. These take the value of unity for a pure particle-hole excitation and are the same as S used in Ref. 1 and $\sqrt{2}$ smaller than S_{ph} used in Refs. 10 and 25.

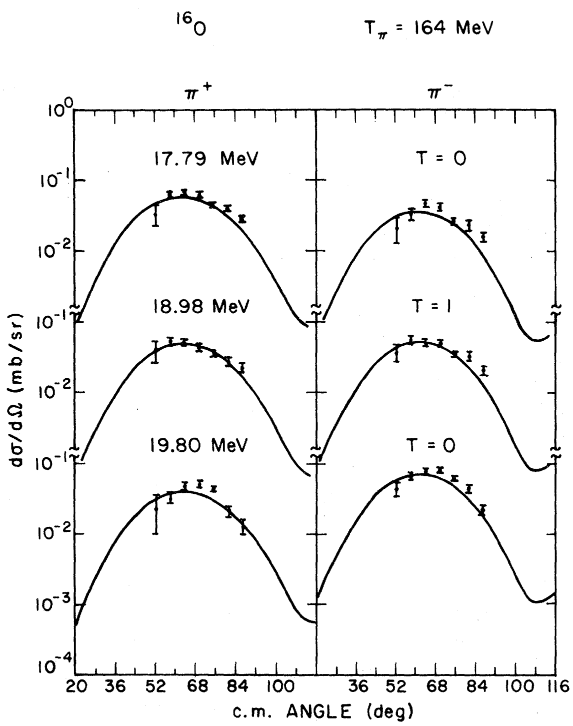

FIG. 1. Theoretical results ($\times$ 1.30) for inelastic scattering of π^+ (left) and π^- (right) from the three isospin mixed 4^- states in ^{16}O. The 162 MeV data are from Ref. 4.

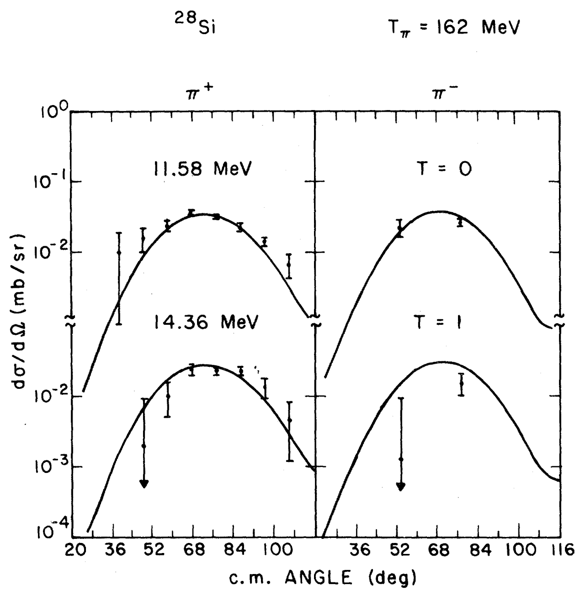

FIG. 2. Theoretical results ($\times$ 1.15) for inelastic scattering of π^+ (left) and π^- (right) from the $T=0$ (top) and $T=1$ (bottom) 6^- states in ^{28}Si. The 162 MeV data are from Ref. 5.

with the general inelastic scattering potential code ALLWRLD (Ref. 31) which generates the reduced transition potential defined in Eq. (7) and the pion distorted wave code MSUDWPI (Ref. 32) which can handle the derivative operator in Eq. (6). Two different optical potentials were considered. One was based on the Stricker, McManus, and Carr (SMC) form[33] with parameters taken from the phase shifts[3] and simple estimates of the absorption.[34] The other used the Kisslinger form[14] with parameters evaluated from the phase shifts[3] at an energy 30 MeV below the beam energy.[35] The two potentials gave equivalent fits to the elastic scattering data and produced essentially identical results. The results shown in Figs. 1 and 2 can be assumed to come from either prescription. We also note that the distortion produced only about 3% differences in the π^+ and π^- cross sections, justifying the assumptions often made when extracting spectroscopic information from π^+/π^- ratios.

All in all the agreement between the theoretical results and the experimental data is quite good. Most important is the overall magnitude of the theoretical cross sections since the relative cross sections (for ^{16}O and ^{28}Si, separately) have been essentially fixed by the choice of densities. A slight upward renormalization of t^{LS} (15% for ^{16}O and 7% for ^{28}Si) was required to produce the agreement shown. The present results are also in good agreement with the results of the nonlocal calculations given in Ref. 5. There are some small systematic differences in the shapes of the experimental ^{16}O differential cross sections that are not reproduced by the present theoretical calculations. These might reflect inadequacies in the assumption that $\rho^{s1}_{J,J-1}$ and $\rho^{s0}_{J,J-1}$ have identical shapes. The largest discrepancy between theory and experiment occurs for the case of π^- excitation of the 6^- $T=1$ state in ^{28}Si. The data are rather sparse here and the experimental results that $\sigma_{\pi^-} < \sigma_{\pi^+}$ is difficult to understand in light of the arguments for π^- enhancement advanced in Ref. 29.

IV. DISCUSSION

The energy dependence of the impulse approximation estimate of t^{LS} is displayed in Fig. 3. The points shown with the curves represent the values deduced from the experimental data above. We show these curves in the hope of encouraging experimenters with energy dependent data on stretched excitations to make similar comparisons. To understand the importance of this as a means of gaining an independent estimate of the properties of the spin-dependent part of the effective pion-nucleus interaction one must note the full complexity of the

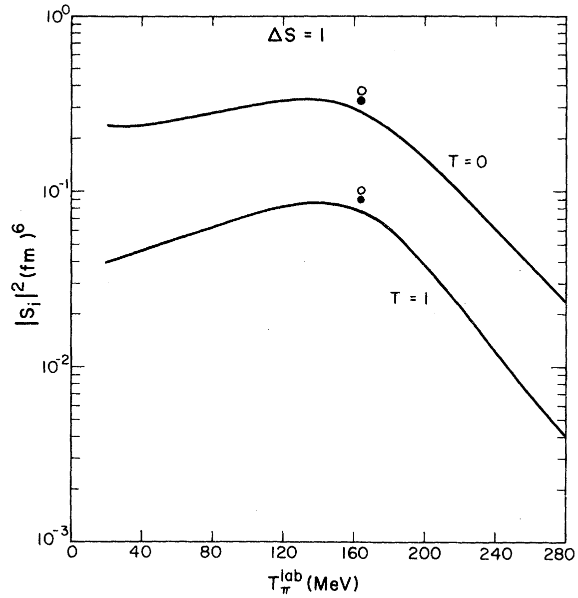

FIG. 3. Squared modulus of the target independent strength parameter $S_T=(\omega_c/2\pi\hbar^2)t_T^{LS}$ as a function of energy. The points represent the values required to fit the ^{16}O (open) and ^{28}Si (solid) experimental data.

pion-nucleus scattering potential for unnatural parity transitions that is implied by the considerations of Refs. 11—13. This may be written in schematic form as

$$U_{\pi A} \sim t^{LS}\rho_J^s + \frac{dt^{LS}}{dE}\rho_J^{ls} + \frac{dt^C}{dE}\rho_J^l + t_{\pi\Delta}\rho_J^\Delta, \qquad (9)$$

where the first term is the coupling to the transverse spin density under discussion here. The next two terms represent the coupling to the spin-current and current densities through the spin-orbit and central components of the pion-nucleon interaction which results from the Fermi motion corrections of Refs. 11 and 12. The last term represents the possible coupling to explicit Δ-hole components in the final state nuclear wave functions discussed in Ref. 13. The latter is effective only for isovector transitions. For stretched excitations the last two terms do not contribute because $\rho_J^l=0$ and the Δ-hole admixtures in isovector states of high spin are negligible.[36] The remaining terms serve to determine the spin-orbit component of the pion-nucleon interaction provided a reliable estimate of ρ_J^{ls} can be made. The simple structure of the stretched excitations will be useful in this regard although the full details of the second term in Eq. (9) are not yet available. In any event comparisons of theory and experiment for stretched excitations like that made above will contain useful information on the energy dependence of t^{LS} and the spin-current coupling for unnatural parity transitions in inelastic pion scattering.

In summary a simple model has been presented for incorporating the spin-dependent part of the pion-nucleon interaction in distorted wave calculations for unnatural parity excitations via inelastic pion scattering in the Δ-resonance region. Impulse approximation estimates of the inelastic differential cross sections for stretched excitations in ^{16}O and ^{28}Si, based on this model, were found to be in reasonable agreement with the available experimental data taken on resonance. These calculations serve mainly as an illustration of the suggested parametrization of the spin-dependent part of the pion-nucleon interaction which, we believe, provides a convenient basis for summarizing the features of the experimental data and discussing the spin dependence in various dynamical models for the pion-nucleon interaction. The importance of extending this work to study the energy dependence of t^{LS} has been stressed.

ACKNOWLEDGMENTS

The authors wish to thank C. Hyde for supplying us with the ^{16}O(e,e') data prior to publication, A. W. Carpenter for providing the fits to the (e,e') data, and H. McManus for useful discussions. This work was supported in part by the National Science Foundation and the Department of Energy.

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