Single and double scattering contributions to K^+ -deuteron vector analyzing power at 1.5 GeV/c

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The vector analyzing power for K^+ ²H elastic scattering at 1.5 GeV/c is predicted based on the Glauber model. The calculation, including the single and double KN scattering processes, uses the KN amplitudes obtained by an original phase-shift analysis. Results are also compared with those calculated by another KN analysis. It is shown that the deuteron analyzing power is sensitive to the structure of the KN amplitudes and is strongly affected by the KN polarization.

NUCLEAR REACTIONS ²H(K⁺, K⁺)²H, P_L = 1.5 GeV/c; calculate $iT_{11}(\theta)$, Glauber model

I. INTRODUCTION

There have been many attempts to investigate kaon-nucleus scattering to understand more on the kaon-nucleon interactions.¹ Particular interest has been in studying nuclear reactions induced by the K^+ meson since the interaction in the strangeness 1 system is expected to be quite different from that induced by π or \overline{K} mesons. Also the possibility of $S = 1$ baryon states makes the K^+ -nucleus reaction interesting, especially in the intermediate energy region.

In the phenomenological study of K^+N scattering, much work has been devoted to K^+p scattering experiments and analyses. However, the neutron part of the KN interactions, which includes the isosinglet channel, has to be investigated by using nuclear targets. The simplest choice among them is the deuteron.

The study of the K^+ ²H channel has been concentrated on the deuteron breakup reactions, $²$ for which</sup> the impulse approximation gives a reasonable framework to deduce the K^+n elastic and charge exchange reaction parameters.³ On the other hand, there have been a few measurements of differential cross sections for K^+ ²H elastic (coherent) scattering.^{4,5} Although there is difficulty using K^+ ²H coherent scattering data in KN amplitude analyses, owing to the complexity of the deuteron recombination mechanism, these data have an advantage to reveal interference effects between the K^+p and K^+n amplitudes. In this sense an experiment with a spin polarized deuteron target would provide particular information which is unavailable ip unpolarized cross sections owing to a strong effect of the deuteron structure.

The first measurement of the vector analyzing power iT_{11} for K^+ ²H elastic scattering at 1.5 GeV/c is under preparation by the KEK National Laboratory for High Energy Physics-University of California at Los Angeles (KEK-UCLA) collaboration.⁶ In this paper we present a prediction for that quantity. The calculation is based on the multiple scattering formalism (Glauber model) including the single and double scattering processes. Although this model has a certain limitation in its applicability, the result will provide a basis on which we can discuss the entire reaction mechanism. The KN amplitudes, which are the physical basis of the calculation, have been obtained by an original phase-shift analysis, $\frac{7}{7}$ and another recent analysis by Martin and Oades 8 is also used for comparison.

The formulas employed are briefly reviewed in Sec. II, where some problems of the Glauber model are discussed. In Sec. III results are shown and discussed within the framework of the Glauber model. Section IV is devoted to conclusions.

II. GLAUBER MODEL

In calculating the $K²H$ amplitude by the Glauber model, we basically follow the prescriptions given by Bertocchi and Capela,⁹ and Alberi and Bertocchi.¹⁰ Their work includes general extensions from the original eikonal model 11 by employing the proper relativistic kinematics and the spin dependence. This section does not describe the Glauber formulas

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FIG. 1. The Breit frame and the coordinate system.

in full detail but states some remarks on approximations employed in the model and on additional assumptions made in the present calculation.

The calculation is done in the Breit frame which is shown in Fig. 1. In this frame the magnitude of the deuteron momenta in the initial and final states, $\frac{1}{2}d$, measures the applicability of the model $\left(\begin{array}{c} t \\ t \end{array} \right) = d^2$). The validity of the Glauber model is, as is well known, limited to low momentum transfer scattering. One of the reasons for this limitation is that the model treats the deuteron as a static object whose inner structure is described well in terms of the low energy NN interaction.

The single scattering diagram is shown in Fig. 2(a), where we assume that the noninteracting nucleon is on its mass shell (the spectator assumption).⁹ The K ²H amplitude can be expressed as follows: (We suppress the spin indices for simplicity.)

$$
T = 2 \int d^3k \, \psi(-\frac{1}{4}\vec{d} - \vec{k}) T_N \psi(\frac{1}{4}\vec{d} - \vec{k}), \qquad (1)
$$

where ψ is the deuteron wave function in momentum space and T_N is the KN amplitude. This has to be considered as a sum of two contributions from the proton and the neutron, that is,

FIG. 2. Diagrams for (a) single and (b) double scattering. Solid and dashed lines represent the nucleon and kaon, respectively. Crossed lines are on-shell particles.

$$
T_N = T_p + T_n = \frac{1}{2}(3T_1 + T_0),\tag{2}
$$

where T_I denotes the isospin I amplitude in the KN system.

An idea of the model is to take T_N outside of the k integration and to evaluate it at a certain value of \overline{k} at which the rest of the integrand is maximum. This approximation is based on the fact that the KN amplitude is a slowly varying function of k compared to the deuteron part. The maxima occur
at $\vec{k} \pm \frac{1}{4} \vec{d}$, or approximately at $k = 0$.¹⁰ We choose the $k = 0$ approximation since, in this case, the corresponding KN amplitude is closer to the physical domain than for the $\vec{k} = \pm \frac{1}{4} \vec{d}$ choice. T_N in Eq. (1) is necessarily unphysical by the very nature of the model. Our knowledge of the KN amplitude is limited to the physical kinematics region unless we can extrapolate it into the unphysical domain without ambiguity. Therefore, we substitute T_N by the approximate physical amplitude. For the $k=0$ approximation the interacting nucleon has a mass of

$$
m^{'2} = m^2 - |t| / 8,
$$

while

$$
m^2=m^2-|t|/2
$$

for $\vec{k} = \pm \frac{1}{4} \vec{d}$, where *m* is the physical nucleon mass approximately half of the deuteron mass M . We actually calculate T_N by simply setting $m' = m = \frac{1}{2}M$.

The energy available in the KN interaction is weakly dependent on the momentum transfer for the $k = 0$ approximation.¹⁰ The present calculation, however, neglects the energy dependence of T_N since it is ill determined, especially for the $I=0$ states.

Equation (1) is then expressible as the KN amplitude times the deuteron form factor:

$$
T = 2T_N \int d^3k \, \psi(-\frac{1}{4}\vec{d} - \vec{k}) \psi(\frac{1}{4}\vec{d} - \vec{k}) = 2T_N \phi(\frac{1}{2}\vec{d}),
$$
 (3)

where

$$
\phi(\vec{\mathbf{q}}) = \int d^3r \, |\Phi(\vec{r})|^2 e^{i\vec{\mathbf{q}} \cdot \vec{r}}.
$$
 (4)

 $\Phi(\vec{r})$ is the deuteron wave function in coordinate space. In Eq. (3) the structure of the deuteron can be considered separately from the KN dynamics for single scattering.

We proceed to the discussion of the double scattering contribution. Figure 2(b) shows the diagram where intermediate particles are assumed to be on the mass shells. The corresponding K^2H amplitude is

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$$
T = \frac{i}{16\pi^2} \int d^3p \, d^3q \frac{\delta(\omega + E - \omega' - E' - E'')}{\omega' \sqrt{E' E''}} \psi(-\frac{1}{4}\vec{d} - \frac{1}{2}\vec{p} - \vec{q}) T^{(1)} T^{(2)} \psi(\frac{1}{4}\vec{d} - \frac{1}{2}\vec{p} + \vec{q}),\tag{5}
$$

where

$$
\omega = (n^2 + \frac{1}{4}d^2 + \mu^2)^{1/2}, \quad \omega' = [(\vec{n} - \vec{p})^2 + \mu^2]^{1/2},
$$

\n
$$
E = (\frac{1}{4}d^2 + M^2)^{1/2}, \quad E' = [(\frac{1}{2}\vec{p} + \vec{q})^2 + m^2]^{1/2},
$$

\n
$$
E'' = [(\frac{1}{2}\vec{p} - \vec{q})^2 + m^2]^{1/2}.
$$
\n(6)

 $T^{(1)}T^{(2)}$ is a product of the KN amplitudes corresponding to the successive interactions of the kaon with the nucleons. This must be considered as a sum of all possible combinations of the KN scattering channels, which yields

$$
T^{(1)}T^{(2)} = T_p^{(1)}T_n^{(2)} + T_n^{(1)}T_p^{(2)} - T_{cx}^{(1)}T_{cx}^{(2)}
$$

= $\frac{1}{4}[3T_1^{(1)}T_1^{(2)} + 3T_1^{(1)}T_0^{(2)}$
+ $3T_0^{(1)}T_1^{(2)} - T_0^{(1)}T_0^{(2)}],$ (7)

where T_{cx} is the KN charge exchange amplitude. The minus sign for the charge exchange term in Eq. (7) comes from the fact that the charge exchange diagram is obtained by interchanging the two nucleons at the deuteron vertex, where they make an isosinglet state.¹² Equation (7) contains only the elastic \cancel{KN} channels. Possible inelastic processes, however, would not make sizable contributions.¹³

The procedure to make the sixfold integration in Eq. (5) calculable is the same as was used for the single scattering term. Since strong dependence of the integrand on the momentum \vec{q} is expected only in the deuteron wave function part, the rest of the integrand can be taken out of the q integration, evaluated at an appropriate value of \vec{q} that makes the deuteron part maximum. This is $\pm \frac{1}{2} \vec{p} - \frac{1}{4} \vec{d}$. Then Eq. (5) yields

$$
T = \frac{i}{16\pi^2} \int d^3 p \frac{\delta(\omega + E - \omega' - E' - E'')}{\omega' \sqrt{E' E''}} T^{(1)} T^{(2)} \int d^3 q \psi (-\frac{1}{4} \vec{d} - \frac{1}{2} \vec{p} - \vec{q}) \psi (\frac{1}{4} \vec{d} - \frac{1}{2} \vec{p} + \vec{q})
$$

=
$$
\frac{i}{16\pi^2} \int d^3 p \frac{\delta(\omega + E - \omega' - E' - E'')}{\omega' \sqrt{E' E''}} T^{(1)} T^{(2)} \phi(\vec{p}),
$$
(8)

where the energies of the internal nucleons are redefined as

$$
E' = [(\vec{p} - \frac{1}{4}\vec{d})^2 + m^2]^{1/2},
$$

\n
$$
E'' = [(\frac{1}{4}d)^2 + m^2]^{1/2} = \frac{1}{2}E.
$$
\n(9)

Unlike in the single scattering problem, the offshell nature of the KN amplitude is unimportant in the present case. If we choose

$$
\vec{q} = +\frac{1}{2}\vec{p} - \frac{1}{4}\vec{d}
$$

to determine the KN kinematics, then $T^{(2)}$ has four on-shell legs and remains in the physical region unless p is large. The other amplitude, $T^{(1)}$, has one off-shell nucleon leg, but this is on shell when $p = 0$. Since the form factor $\phi(\vec{p})$ decreases typically like p^{-4} , main contributions of the integration in Eq. (8) come from the region near $p = 0$; hence little effect by the unphysical kinematics problem would take place.

A further step deals with the delta function in Eq. (8), which states the energy conservation between the initial and intermediate states. This is substituted in the literature of the Glauber model by the constrain of $\vec{n} \cdot \vec{p} = 0$, ^{9,10} where \vec{n} is a component of the kaon momentum vertical to the momentum transfer (Fig. 1). This substitution yields the final expression for

T in the same form as the original impact parameter representation of the shadow effect.¹¹ The preser representation of the shadow effect.¹¹ The presen calculation, however, treats the constraint exactly owing to the delta function since otherwise we cannot have a reasonable boundary of $|p|$. The integration in Eq. (8) is then taken over a threedimensional finite domain in p space, instead of the infinite plane perpendicular to \vec{n} . The approximate constraint $\vec{n} \cdot \vec{p} = 0$ is, of course, included in the limit of $|t| \rightarrow 0$.

III. RESULTS AND DISCUSSIONS

Based on the formalism reviewed in the last section, the K^2H amplitudes can be calculated with the use of the deuteron form factor and the KN amplitude. The deuteron form factor, defined in Eq. (4), has been integrated from Humberston's revision¹⁴ of the Hamada-Johnston deuteron wave function¹⁵ and Moravcsik's fit to the Gartenhaus wave function.¹⁶ These two choices, however, make little difference in the final results, so we discuss the result using the former option. The KN amplitude including the $I = 1$ and $I = 0$ states is calculated based on a single energy partial wave analysis⁷ (KH), whose phase shifts are listed in Table I for 1.5 GeV/c laboratory momentum. Also the calculation is made with the partial wave amplitudes given by Martin and Oades (MO) for comparison.

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$L_{1,L-1/2}$ ^b		$L_{1,L+1/2}$		$L_{0,L-1/2}$		$L_{0,L+1/2}$	
δ	η		η				
		$-68,067$	0.2949			-23.517	1.0000
-40.790	0.9480	-4.343	0.8044	28.848	0.3510	-11.064	0.9108
-11.376	0.9285	6.024	0.8992	12.330	0.5956	-6.746	0.6806
-1.177	0.9453	-1.050	0.9317	10.954	1.0000	-7.229	0.9575
-0.415	1.0000	0.343	1.0000	4.379	1.0000	1.433	1.0000

TABLE I. KN phase shifts at 1.5 GeV/ c .²

 δ is the real phase shift in degrees, and η is the absorption parameter.

^bThe partial waves are denoted by $L_{I,J}$.

We follow the Madison Convention¹⁷ for the description of spin-1 polarization. If we choose the coordinate system in the Breit frame as shown in Fig. 1, the K^2H spin amplitudes $T_{M'M}$, where M' and M are the deuteron spin projection, take a matrix form as

$$
T_{M'M} = \begin{pmatrix} A & B & D \\ B & C & B \\ D & B & A \end{pmatrix} \tag{10}
$$

by parity conservation and time reversal invariance. The vector analyzing power is expressed by these four independent amplitudes, A , B , C , and D , as

$$
iT_{11} = \frac{\sqrt{6}}{\Sigma} \text{Re}[B^*(A + C + D)], \tag{11}
$$

FIG. 3. iT_{11} for K^+ ²H scattering at 1.5 GeV/c. Predictions using the KX amplitude from Ref. 7 are denoted by KH and from Ref. 8 by MO. Dashed lines denote the single scattering contributions. The single-plus-double scattering contributions are shown by solid lines. A dashed-dotted line represents a multiplicative effect of the deuteron structure given in the second factor in Eq. (16).

where

$$
\Sigma = \operatorname{Tr}(TT^{\dagger}) = 2 |A|^2 + 4 |B|^2 + |C|^2 + 2 |D|^2.
$$
\n(12)

The resulting iT_{11} is given in Fig. 3, where the single and single-plus-double scattering contributions are denoted by dashed and solid lines, respectively. We notice that there are rich structures. We discuss these structures in terms of structures in the KN amplitude and the deuteron within the framework of the Glauber model.

The K^2H differential cross section by single scattering can be written as

$$
\frac{d\sigma}{d\Omega} \propto \Sigma \propto |F|^2 (\phi_S^2 + \phi_Q^2) + \frac{2}{3} |G|^2 \phi_M^2,
$$
\n(13)

where F and G correspond to the KN spin nonflip and flip amplitudes, respectively, and ϕ_S , ϕ _O, and ϕ _M are the deuteron form factors, called the spherical, quadrupole, and magnetic form factors, respectively. These are defined as follows:

$$
\phi_S = \phi_a(\frac{1}{2}d) + \phi_b(\frac{1}{2}d),
$$

\n
$$
\phi_Q = 2\phi_c(\frac{1}{2}d) - \frac{1}{2}\sqrt{2}\phi_d(\frac{1}{2}d),
$$

and

$$
\phi_M = \phi_a(\frac{1}{2}d) - \frac{1}{2}\phi_b(\frac{1}{2}d) \n+ \frac{1}{2}\sqrt{2}\phi_c(\frac{1}{2}d) + \frac{1}{2}\phi_d(\frac{1}{2}d),
$$
\n(14)

where

$$
\phi_a(q) = \int_0^\infty dr \, j_0(qr) u^2(r),
$$

\n
$$
\phi_b(q) = \int_0^\infty dr \, j_0(qr) w^2(r),
$$

\n
$$
\phi_c(q) = \int_0^\infty dr \, j_2(qr) u(r) w(r),
$$

and

$$
\phi_d(q) = \int_0^\infty dr \, j_2(qr) w^2(r). \tag{15}
$$

FIG. 4. Polarizations for (a) K^+p and (b) K^+n scattering at 1.5 GeV/ c . The predictions by the KH and MO solutions are denoted by solid lines with the signs KH and MO in the figure. The experimental (\bullet) data points are from Ref. 19, and the (\times) data points are from Ref. 20.

Here $u(r)$ and $w(r)$ are the radial deuteron wave function of the S and D wave parts, respectively. Since the spin flip amplitude G is small for forward directions and ϕ_M becomes negligible at large momentum transfers (see Fig. 2 in Michael and Wil- kin^{18}), the differential cross section in Eq. (13) is largely determined by the first term. The single scattering contribution to iT_{11} is then

$$
i T_{11} \sim \frac{\text{Re} F G^*}{|F|^2} \frac{\phi_M (\phi_S + \frac{1}{4} \sqrt{2} \phi_Q)}{\phi_S^2 + \phi_Q^2}.
$$
 (16)

Apart from possible structures of the KN amplitudes F and G , the form factor part of Eq. (16) produces an angular dependence. This is plotted in Fig. 3 by a dashed-dotted line appropriately normalized. We notice that it decreases rapidly from $\theta_{\rm cm} = 40^{\circ}$ to 70', and it changes sign between 70' and 90. The single scattering curves are strongly affected by this kinematical structure both for the KH and MO solutions. On the other hand, an effect by the KN amplitude can be seen below 40' since the form factor is almost constant.

There is a significant difference between the KH and MO solutions below 40'. This may be analyzedas follows: As stated in Eq. (2), the amplitudes F and G in Eq. (16) include the corresponding amplitudes for K^+p and K^+n scattering. Therefore, the deuteron vector analyzing power, which is proportional to the left-right asymmetry of the cross section, is composed of the asymmetry by the proton and neutron and their interferences in the case of single scattering. Since the K^+p polarization is positive at the present energy¹⁹ as shown in Fig. 4, the negative contribution to the MO prediction stems from the K^+n polarization and/or possible interferences. In fact, the MO amplitudes predict large negative polarization for K^+n scattering at forward angles [Fig. 4(b)], while the experimental data²⁰ as well as the KH fit to them show it is negative but smaller [Fig. 4(b)]. In terms of the partial waves, the two dip structure of the $K_{\tau}^{+}n$ polarization indicates that there is a large negative contribution from the interference term between the P_{03} and D_{03} waves. Another sizable contribution from the interference between the P_{11} , P_{01} , and P_{03} waves enlarges the dip in the forward region. The large negative polarization by the MO amplitudes can easily be seen to be owing to the strong attractive behavior of the P_{01} wave. On the contrary, in the KH phase shifts the P_{01} wave becomes inelastic and small very rapidly above 1.2 GeV/ c . We can, therefore, conclude that the behavior of partial waves, especially of the P_{01} wave, is clearly reflected in iT_{11} and that this quantity should be positive around 40', if we consider the consistency between the K^+p and K^+n polarization.

Figure 3 shows negative value for iT_{11} at around 120'—130'. Both analyses show dips in spite of the fact that the form factor part is positive and smooth. These dips correspond to the large negative polarization in the K^+n channel at 120°, as shown in Fig. 4(b), since the K^+p polarization is positive [Fig. 4(a)]. This feature of K^+n polarization is [Fig. 4(a)]. This feature of K^+ common to other KN analyses,²¹ n^{22} mainly due to the interference between P_{03} and D_{03} waves.

The double scattering processes produce the large positive enhancement of iT_{11} around 90° in the KH prediction and behaves quite differently from that using MO amplitudes. Since K^2H observables contain the fourth power of the KN amplitudes through the double scattering term, a crucial test of the KN amplitude analyses will be provided by the measurement of iT_{11} .

IV. CONCLUSION

The vector analyzing power for K^{+2} H scattering at 1.S GeV/c has been predicted within the framework of the multiple scattering approximation. The calculation includes the single and double KN elastic scattering processes. As in Fig. 3, the result shows that the rich angular dependence of iT_{11} is partly due to the deuteron structure and partly due to the KN scattering amplitudes. Especially the effect by the KN amplitudes can be seen in the region below 40° through the single K^+p and K^+n processes and their interferences. A peak appears at around 40', attributed to the positive K^+p polarization and the smaller negative K^+n polarization, which is consistent with the weak attractive nature of the P_{01} wave at 1.5 GeV/c. The significance of the double scattering contributions can be seen around 90'. The $K²H$ vector analyzing power is quite sensitive to the detailed structure of the KN amplitudes. The measurement of the $K²H$ vector analyzing power will provide a good test for available EN scattering analyses.

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