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Supermultiplet symmetry in nuclear reactions: Application to ^{3}He(p,^{2}H)2p and ^{9}Be(^{3}H, ^{6}He)^{6}Li*

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A detailed analysis of the extension of the Barshay-Temmer theorem to reaction product pairs that are members of the same SU(4) multiplet (Wigner supermultiplet) is presented. Nonzero values of the total orbital angular momentum within fragments in the presence of residual spin orbit splitting is shown to destroy symmetry of the reaction products around 90° except in a sum rule sense. Departures from reaction symmetry are related to the SU(4) breaking parameter in the effective nucleon-nucleon potential and it is shown that the asymmetry should decrease with increasing A. A comparison of the ^{3}He(p,^{2}H)2p and ^{9}Be(^{3}H, ^{6}He)^{6}Li angular distributions illustrates this point. Finally, a prediction is made for the ratio of analyzing powers for the ^{6}He+^{6}Li and ^{6}He+^{6}Li*(3.56) final states.

NUCLEAR REACTIONS Antisymmetric scattering theory. Symmetry around 90° of $\sigma(\theta)$ for $a(b,c)c'$ reactions derived for SU(4) invariant potentials. Angular symmetry breakdown related to size of SU(4) breaking potential through DWBA. Discussion of ^{3}He(p,d)2p, ^{9}Be(t,^{6}He)^{6}Li(^{6}Li*), and ^{9}Be($\vec{t}$,^{6}He)^{6}Li(^{6}Li*) reactions.

I. INTRODUCTION

The differential cross section for producing a pair of identical particles in a reaction must be symmetric around 90° because the particles are indistinguishable. Even in the reaction $b(a,c)c'$, where c and c' are not identical but are members of the same isospin multiplet, Barshay and Temmer[1] proved that symmetry around 90° of the final fragments is to be expected provided that isospin is conserved by nuclear forces and that a unique value of isospin is present in the initial channel. Examples of such reactions where a high degree of symmetry around 90° has been observed are ^{4}He(^{2}H,^{3}H)^{3}He (Ref. 2) and ^{9}Be(^{3}H,^{6}He)^{6}Li* (3.56) (Ref. 3). Using arguments similar to those of Barshay and Temmer, Simonius[4] showed that in the latter reaction one should expect an analyzing power that is antisymmetric around 90°. Experiment[5] is in accord with this expectation.

Robson and Richter[6] generalized the arguments further and pointed out that cross sections symmetric around 90° are to be expected whenever the final fragments c and c' are related by raising and lowering operators constructed from generators of a group which commutes with the nuclear Hamiltonian. The group SU(4) for light nuclei was suggested by them to be a possible candidate. In fact, the cross section for ^{9}Be(^{3}H,^{6}He)^{6}Li has been observed[3] to be nearly symmetric around 90° and is nearly of an identical shape as the ^{9}Be(^{3}H,^{6}He)^{6}Li* cross section. *A priori* one could argue that ^{6}He and ^{6}Li are members of the same Wigner supermultiplet (in spectroscopic notation designated as $^{31}S_0$ and $^{13}S_1$, respectively, where we use the notation $^{2T+1\,2S+1}X_J$) and, assuming the same degree of SU(4) symmetry in the initial ^{9}Be and ^{3}H nuclei, a unique 12 nucleon product representation of SU(4) is present in the initial channel. What is so surprising is that the isospin-spin, the tensor, and the spin-orbit components of the nucleon-nucleon force are so ineffectual in mixing other 12 nucleon SU(4) representations into the final channel.

In contrast, in the simpler $A=4$ system symmetry of reaction products around 90° is very far from being obtained in an analogous reaction. The reaction ^{3}He(p,^{2}H)$2p$, with the two protons recoiling in a $^{31}S_0$ state, is identical, with respect to supermultiplicities, to ^{9}Be(^{3}H,^{6}Li)^{6}He. The experimental differential cross section[7] displays only a residual trace of symmetry around 90°.

In the next section an antisymmetrized reaction theory is used to study in detail the relation of reaction symmetry around 90° to SU(4) invariance. The important role of internal orbital angular momentum in the final fragments is stressed and it is shown that, in general, symmetry around 90° is not obtained except as a sum rule over all fragments with the same L and S but different J. The consequences of the assumption of perfect SU(4) symmetry for the ^{9}Be(^{3}H,^{6}He)^{6}Li reaction are presented for both polarized and unpolarized triton beams. In particular, the analyzing power is related to that observed for ^{9}Be(^{3}H,^{6}He)^{6}Li*(3.56).

Finally, the magnitude of the departure from reaction symmetry in the $A=4$ system is related to the strength of the SU(4) breaking part of the nucleon-nucleon potential. A parameter which characterizes the magnitude of this term in the nuclear Hamiltonian is[8]

$$\alpha = V_{\text{Heisenberg}}/V_{\text{Majorana}} \simeq ({}^3V_c^+ - {}^1V_c^+)/({}^3V_c^+ + {}^1V_c^+) = 0.4-0.5\ .$$

Our analysis reveals that the ratio of the forward and backward differential cross section for ^{3}He(p,^{2}H)$2p$ should be

$$(1+\alpha/2)^2/(1-\alpha/2)^2\ ,$$

which is in good agreement with the experimental value.[7] We present arguments that the relevant parameter characterizing SU(4) breaking for the 12 nucleon system is not $\alpha/2$ but $\alpha/8$, so that the greater reaction symmetry in the latter case is understandable.

II. SCATTERING AMPLITUDES INCLUDING EXCHANGE SYMMETRY

A. Complete SU(4) invariance

Throughout this paper we deal with scattering amplitudes which parametrize reactions of the general form $b(a,c)c'$, where the final two fragments c and c' contain the same number of nucleons, n. Define a solution of the $2n$ body time independent Schrödinger equation which for large fragment separation satisfies the asymptotic condition

$$\Psi_{ba} \underset{R_{ab}\to\infty}{\longrightarrow} \psi_b(1\cdots n')\psi_a(n'+1\cdots 2n)e^{i\vec{k}_i\cdot\vec{R}_{ab}} + \text{outgoing waves}\ , \tag{1}$$

where ψ_a and ψ_b are the antisymmetric wave functions for the separated fragments, $\vec{R}_{ab}$ is the relative displacement vector of the two fragments, and $\vec{k}_i$ is the initial relative momentum of the fragments. In a similar way, a noninteracting two cluster wave function for the final channel can be introduced;

$$\psi_{c'c} = \psi_{c'}(1\cdots n)\psi_c(n+1\cdots 2n)e^{i\vec{k}_f\cdot\vec{R}_{cc'}}\ , \tag{2}$$

where $\psi_{c'}$ and ψ_c are the antisymmetric fragment wave functions, $\vec{R}_{cc'}$ is the relative displacement vector, and $\vec{k}_f$ is the final relative momentum vector. Then the transition amplitude for scattering from the physical channel ab to the physical channel cc' is[9]

$$\mathscr{T}_{c'c,ba} = N_{ba}^{1/2}N_{c'c}^{1/2}\left[\Psi_{c'c}, \sum_{i\le n, k\ge n+1} V_{ik}\mathscr{A}\Psi_{ba}\right]\ , \tag{3}$$

where $N_{c'c}=(2n)!/(n!n!)$, $N_{ba}=(2n)!/(n_b!n_a!)$, and

$$\mathscr{A} = 1/N_{ba}\sum(-1)^{\sigma(P)}P_{ba}\ ,$$

where P_{ba} is a general interfragment permutation operator and the parity of the permutation is $\sigma(P)$. For convenience, channel couplings have been suppressed for the moment.

We next introduce the permutation operator $P_{cc'}$ which is distinct from the permutation operator defined above in that it induces the exchange of particle 1 with $n+1$, 2 with $n+2,\ldots,$ k with $n+k$. Then the transition amplitude can be written as

$$\begin{aligned}\mathscr{T}_{c'c,ba} &= N_{c'c}^{1/2}\tfrac{1}{2}\left[[1+P_{cc'}{}^2]\Psi_{c'c}, \sum V_{ik}\mathscr{A}\Psi_{ba}\right]N_{ba}^{1/2}\\ &= N_{c'c}^{1/2}\tfrac{1}{2}\left[[1+(-1)^nP_{cc'}]\Psi_{c'c}, \sum V_{ik}\mathscr{A}\Psi_{ba}\right]N_{ba}^{1/2}\ ,\end{aligned} \tag{4}$$

where use has been made of the fact that $P_{cc'}$ commutes with the channel coupling potential $\sum V_{ik}$ and that the ket is antisymmetric with respect to pair exchanges

$$P_{cc'}\mathscr{A}\Psi_{ba}=(-1)^n\mathscr{A}\Psi_{ba} \ . \tag{5}$$

If the nucleon-nucleon potential were a simple isospin and spin independent central potential, any bound nuclear state could be characterized by definite values of T, L, the orbital angular momentum, and S. The scattering amplitude would describe the transitions from an initial set of magnetic quantum numbers to a final set,

$$[t_b t_a M_b M_a s_b s_a]\rightarrow[t_{c'} t_c M_{c'} M_c s_{c'} s_c] \ .$$

Through an obvious transformation these amplitudes can be related to amplitudes in a representation in which the orbital and spin angular momenta of the fragment pairs are separately coupled to orbital and spin "channel spins." These amplitudes can be expanded in partial wave expansions in which T-matrix elements allowing the transfer of orbital angular momentum appear,

$$A_{L'M'Ss,LMSs}(\hat{k}_f)=i\sqrt{\pi}/k_i \sum_{T\mathscr{L}Sl'l}(2l+1)^{1/2}(LMl0\,|\,\mathscr{L}M)(L'M'l'M-M'\,|\,\mathscr{L}M)$$
$$\times(T_{c'}t_{c'}T_c t_c\,|\,Tt)(T_b t_b T_a t_a\,|\,Tt)$$
$$\times T_{L'l',Ll}(T\mathscr{L}S)Y_{l'}^{M-M'}(\hat{k}_f) \ , \tag{6}$$

where quantities conserved during the interaction are written within the parentheses of the T-matrix elements. The total orbital angular momentum $\mathscr{L}$ comes from the coupling of the orbital channel spin to the angular momentum of the fragment relative motion.

We must examine the SU(4) character of the wave function which describes the bound state of any fragment. The generators of the group are the isospin and spin operators τ_a, σ_b, $\tau_a\sigma_b$, $a,b=1,2,3$, so that the basis states for any SU(4) representation are N-body isospin-spin functions. These basis states are simultaneously basis states for the symmetry group S_N which consists of all the possible permutations of the isospin and spin coordinates of the N nucleons. Thus a representation of SU(4) can be designated by the partition $[f]$, and the Pauli principle requires that the total wave function for the fragment be a sum of products of isospin-spin partners[10] in the S_N representation and space wave functions which are partners in the adjoint S_N representation, $[\bar{f}]$. Thus, the energies of nuclear states are split because the expectation values of the kinetic energy and potential operators depend on the number of symmetric space pairs present in the wave function. A completely antisymmetric wave function is then written as[11]

$$\Psi_{TLS}^{tMs}=1/(n[f\,])^{1/2}\sum_r \psi_L^M([\bar{f}]r,1\cdots N)X_{TS}^{ts}([f\,]r,1\cdots N) \ , \tag{7}$$

where $n[f\,]$ is the dimensionality of the matrices representing the $N!$ permutations, $[f\,]r$ refers to the basis state in the rth row of the representation $[f\,]$, and $[\bar{f}]r$ is the corresponding basis state in the adjoint representation.

The partial wave T-matrix elements in Eq. (6) can be written in the form of the right-hand side (rhs) of Eq. (4) by taking appropriate projections onto states of definite T, $\mathscr{L}$, and S. Since the antisymmetrizing operators for $2n$ particles is acting on Ψ_{ba}, it can be expanded in terms of completely antisymmetric states of fixed $T\mathscr{L}S$ but, in general, varying spatial symmetry,

$$\mathscr{A}\Psi_{ba}=\sum_{[h]}\text{coeff}([h])\Psi(\,[[L_b\times L_a]\times L\,]\mathscr{L}M,[\bar{f}_b]\times[\bar{f}_a]\,|\,[\bar{h}]\ ;$$
$$[T_b S_b\times T_a S_a]TtSs,[f_b]\times[f_a]\,|\,[h]) \ . \tag{8}$$

The representation $[h]$ which occur are those contained in the outer direct product of $[f_b]$ and $[f_a]$. The orbital angular momentum of fragments b and a are coupled to the orbital channel spin L, which, in turn, is coupled to the angular momentum of relative motion l to form a state of total orbital angular momentum $\mathscr{L}$. The isospins and spins are coupled in an obvious way.

The final channel wave function must be treated in a more detailed way. We recall that the wave function for each fragment, c' or c, is according to Eq. (7) written as the sum over the row indices of the same representation $[f\,]$. This summation can be formally dropped by introducing the antisymmetrizer for the n particles in each fragment,[10]

$$\Psi_c = n([f])^{1/2}\mathscr{A}_c \psi_{L_c}^{M_c}([\bar{f}]r,1\cdots n)X_{T_cS_c}^{ts}([f]r,1\cdots n)\;, \tag{9}$$

where

$$\mathscr{A}_c = \frac{1}{n!}\sum(-1)^{\sigma(P)}P_c\;.$$

Since both $\mathscr{A}_{c'}$ and $\mathscr{A}_c$ commute with the coupling potential and since the fragment antisymmetrizers normalized as above satisfy $\mathscr{A}_c\mathscr{A}=\mathscr{A}$, the partial wave T matrix can be written as

$$\begin{aligned}T_{L'l',Ll}(T\mathscr{L}S) = K\Bigg\langle &\{1+(-1)^nP_{cc'}\}\{[[\psi_{L_c}([\bar{f}]s,1\cdots n)\otimes\psi_{L_c}([\bar{f}]s,n+1\cdots 2n)]_{L'}\\ &\otimes j_{l'}(k_fR_{cc'})Y_{l'}(\hat{R}_{cc'})]_{\mathscr{L}}^{M}\\ &\times[X_{T_{c'}S_{c'}}([f]s,1\cdots n)\otimes X_{T_cS_c}([f]s,n+1\cdots 2n)]_{Ts}^{ts}\}\;,\\ &\times\sum_{i,k}V_{ik}\sum_{[h]}\text{coeff}([h])\Psi(\mathscr{L}M[\bar{h}];TtSs[h])\Bigg\rangle\;,\end{aligned} \tag{10}$$

where the constant K, depending on the channel reduced masses and relative momenta, is

$$K = N_{c'c}^{1/2}N_{ba}^{1/2}n([f])[m_cm_{c'}m_bm_ak_ik_f/(m_c+m_{c'})(m_b+m_a)]^{1/2}i^{l-l'}/2\;.$$

In Eq. (10) we have succeeded in representing each of the final fragments as a simple product of a space and an isospin-spin wave function, analogous to writing a member of an isospin multiplet as a simple product of a wave function for space and spin and an isospin wave function.

Just as in the proof for the usual Barshay-Temmer theorem we must examine the action of the permutation operator $P_{cc'}$ on the terms representing $\Psi_{c'c}$ in the expression for the partial wave T-matrix element. Since the orbital wave functions of fragments c' and c are identical, the action of $P_{cc'}$ on the orbital part of $\Psi_{c'c}$ is equivalent to an exchange of the orbital angular momentum coupling order; in addition, $P_{cc'}\vec{R}_{cc'}=-\vec{R}_{cc'}$. As a result, $P_{cc'}$ operating on the orbital product wave function is equivalent to multiplication by $(-1)^{2L_c-L'+l'}$. A detailed examination of the action of $P_{cc'}$ on the isospin-spin product wave function of $\Psi_{c'c}$ has not been reported in the earlier papers, so it will be presented below.

The product of the pair of isospin-spin functions can be expanded in terms of functions belonging to representations of S_{2n} by means of fractional parentage coefficients[11] (which implicitly depend on T_c, S_c, etc.).

$$[X_{T_{c'}S_{c'}}([f]r,1\cdots n)\otimes X_{T_cS_c}([f]r,n+1\cdots 2n)]_{TS}^{ts} = \sum_{[g],w}([f]r[f]r\mid[g]w)X_{TS}^{ts}([g]w,1\cdots 2n)\;. \tag{11}$$

The product isospin-spin function on the left can be expanded in terms of two independent functions of $2n$ coordinates, one even under $P_{cc'}$, the other odd. Under the operation of any of the $(2n)!$ permutation operators of S_{2n} an even function is transformed into another function that is even under the exchange of the coordinates of the two clusters (now with different labels, of course) and likewise for the original odd function. It follows that the even and odd functions are partners or linear combinations of partners in two distinct representations (which may be reducible) and the sum over the partners, denoted by the sum over w in the rhs of Eq. (11) will either be even or odd under $P_{cc'}$ for a given $[g]$. We can, therefore, write

$$P_{cc'}[X_{T_{c'}S_{c'}}\otimes X_{T_cS_c}]_{TS}^{ts} = \sum_{[g]}(-1)^{\beta([g])}\sum_{w}([f]r[f]r\mid[g]w)X_{TS}^{ts}([g]w,1\cdots 2n)\;, \tag{12}$$

where $\beta([g])$ is an even or odd integer.

The sufficient condition for the symmetry around 90° of the reaction cross section is now apparent; the matrix element of Eq. (10) must vanish for all but one $[g]$ in the summation of the rhs of Eq. (12). This will occur if (1) the outer direct product of the isospin-spin functions of fragments b and a contain one and only one representation in common with the outer direct product of fragments c' and c and (2) the generators of SU(4) commute with the nucleon-nucleon potential. Assuming the conditions are satisfied and denoting the one representation in common by $[g]$, all the phase factors arising from $P_{cc'}$ can be combined to give

$$T_{L'l',Ll}(T\mathscr{L}S[g]) = [1+(-1)^{n+l'+L'+\beta([g])}]/2T_{L'l',Ll}(T\mathscr{L}S[g])\;. \tag{13}$$

For a given L' either odd or even values of l' vanish from the partial wave sum for

$$A_{L'M'Ss,LMSs}(\hat{k}_f)$$

given in Eq. (6) so that the reaction cross section which is a weighted sum over the squares of the channel spin amplitudes,

$$|A_{L'M'Ss,LMSs}(\hat{k}_f)|^2 ,$$

is symmetric around 90°.

At this point it may be useful to specialize to the

$$^9\mathrm{Be}(^3\mathrm{H},^6\mathrm{He})^6\mathrm{Li}^*(0^+,T=1)$$

and

$$^9\mathrm{Be}(^3\mathrm{H},^6\mathrm{He})^6\mathrm{Li}(1^+,T=0)$$

reactions. The ^{6}He nucleus and the excited lithium state are assumed to be pure $^{31}S_0$ states while the lithium ground state is assumed to be a pure $^{13}S_1$ state. The group theoretic considerations are shown in Fig. 1(a), where on the left the identical SU(4) representations of the two final fragments are symbolically multiplied and on the right the 12 nucleon product representations are shown. The standard Young tables have been introduced and the isospin-spin multiplicities of the fragments as well as the multiplicities of the product representations are shown below the tables. Finally, the value of $(-1)^\beta$ to be inserted into Eq. (13) is also given on the right of the figure. The sign of $(-1)^\beta$ is obtained from the consideration of the product of fragment multiplets with $T_c = T_{c'}$ and $S_c = S_{c'}$, e.g.,

$$(3,1)\times(3,1)=(1,1)+(3,1)+(5,1) . \tag{14}$$

Then our knowledge of the symmetry of the SU(2) Clebsch-Gordan coefficients tells us that the supermultiplet of which (3,1) is a member is characterized by an odd value of $(-1)^\beta$, while the supermultiplets which contain (1,1) and (5,1) are even under the action of $P_{cc'}$. This result is in disagreement with Eq. (23) of Ref. 3 which uses the factor $(-1)^{S+T}$ in place of our factor $(-1)^{\beta([g])}$ so that the fragment exchange symmetry varies within a supermultiplet, in violation of our result above.

The direct product of the isospin functions of ^{9}Be and ^{3}H is expanded in terms of SU(4) representations of 12 particles in Fig. 1(b). Since $T=1$ only the multiplets (3,1) and (3,3) occur, so the unique product representation is [4332]. From the direct product of ^{6}He and ^{6}Li*(3.56) one can form only one multiplet in common with the initial channel, (3,1), and this multiplet belongs uniquely to the [4332] representation. From the direct product of the isospin-spin functions of ^{6}He and ^{6}Li one can form only the (3,3) multiplet which does not belong uniquely to an SU(4) representation. Only odd values of the relative orbital angular momentum are allowed for two final fragments in the [4332] representation, according to Eq. (13) and Fig. 1(a). A relaxation of SU(4) invariance permits an admixture of the [4422] representation to appear in the final state of the ^{6}He+^{6}Li, which has even values of the orbital angular momentum associated with it. This admixture would be reflected in an asymmetry around 90° of the cross section which would otherwise be the same shape as the ^{9}Be(^{3}H,^{6}He)^{6}Li*. In the limit of SU(4) invariance other observables such as the vector analyzing power should have the same symmetry in both reactions.

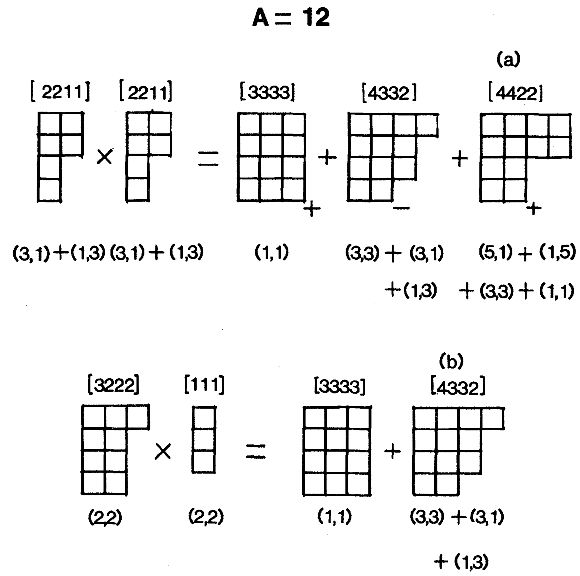

FIG. 1(a). The SU(4) representations assigned to ^{6}He and ^{6}Li(^{6}Li*) are shown on the left and the 12 nucleon outer direct product representations are shown on the right. The standard labels for the Young tables appear above them and the $(2T+1,2S+1)$ associated multiplets are shown below each table. Note that (3,3) appears in two distinct representations. The ± signs denote the exchange symmetry of the final channel wave function under cluster exchange when the clusters are coupled to the representation in question. (b) The SU(4) representations assigned to ^{9}Be and ^{3}H are shown on the left and the direct product of these representations are shown on the right. The representation labels and the iospin-spin multiplicities associated with each representation are indicated.

B. Spin-orbit splitting within supermultiplets

It is the case in ^{6}Li, and perhaps other nuclei, spin orbit splitting separates members of the supermultiplet as to J but does not change the symmetry of the isospin-spin functions. The ${}^{13}D_{3,2,1}$ states of ^{6}Li illustrate this point. However, in the present work we find that this departure from degeneracy in the final fragments with respect to J is sufficient to destroy symmetry around 90° even if the coupling potential does not mix SU(4) representations. When the J degeneracy is removed it is convenient to switch to the standard channel spin representation. The transformation between this representation and the one of Eq. (6) is

$$A_{j'm',jm}(\hat{k}_f)=\sum_{L,L',S}\hat{j}_{c'}\hat{j}_c\hat{j}_b\hat{j}_a\hat{L}\hat{L}'\hat{S}^2\begin{Bmatrix}L_{c'} & S_{c'} & j_{c'}\\ L_c & S_c & j_c\\ L' & S' & j'\end{Bmatrix}\begin{Bmatrix}L_b & S_b & j_b\\ L_a & S_a & j_a\\ L & S & j\end{Bmatrix}\times\sum_s (L'm'-sSs\mid j'm')(Lm-sSs\mid jm)A_{L'm'-sSs,Lm-sSs}(\hat{k}_f)\ , \tag{15}$$

where the total spin S is still a conserved quantity. The degeneracy which has been removed is that with respect to the values of j_c and $j_{c'}$ which are allowed by the coupling of L_c and S_c and $L_{c'}$ and $S_{c'}$, respectively. This prevents us from taking a sum over j_c and $j_{c'}$ in evaluating $|A_{j'm'jm}(\hat{k}_f)|^2$ which, in turn, prevents us from making use of the orthogonality relation

$$\sum_{j_c,j_{c'}}\hat{j}_{c'}^2\hat{j}_c^2\hat{L}'\hat{\bar{L}}'\hat{S}\hat{\bar{S}}\begin{Bmatrix}L_{c'} & S_{c'} & j_{c'}\\ L_c & S_c & j_c\\ L' & S & j'\end{Bmatrix}\begin{Bmatrix}L_{c'} & S_{c'} & j_{c'}\\ L_c & S_c & j_c\\ \bar{L}' & \bar{S} & j'\end{Bmatrix}=\delta_{\bar{L}',L'}\delta_{\bar{S},S} \tag{16}$$

to obtain an incoherent sum with respect to L' of the squares of the amplitudes on the rhs of Eq. (15). In general, cross terms between even and odd L' will destroy symmetry around 90°.

It is hard to find real examples where the above "sum rule" symmetry around 90° should be obtained. As a hypothetical case, suppose ${}^{6}\mathrm{He}^*(2^+)$ were a pure ${}^{31}D_2$ state. Then the reaction ${}^{9}\mathrm{Be}({}^{3}H,{}^{6}\mathrm{He}(2^+)){}^{6}\mathrm{Li}(3^+)$ would not be expected to be symmetric around 90°, but if one were to add the cross sections for production of ${}^{6}\mathrm{Li}(2^+)$ and ${}^{6}\mathrm{Li}(1^+)$ which complete the ${}^{13}D_J$ multiplet, symmetry would be restored. When the final fragments have $L_c=L_{c'}=0$ no such problem arises. Under this condition the final channel spin is identical to the intrinsic spin and the amplitude transformation is

$$A_{Ss,jm}(\hat{k}_f)=\sum_L\hat{j}_b\hat{j}_a\hat{L}\hat{S}\begin{Bmatrix}L_b & S_b & j_b\\ L_a & S_a & j_a\\ L & S & j\end{Bmatrix}(Lm-sSs\mid jm)A_{00Ss,Lm-sSs}(\hat{k}_f)\ . \tag{17}$$

The multiplicative factor in Eq. (13) reduces to $(1+(-1))^{n+\beta+l'}$ and either even or odd l' alone contributes.

C. Relation between the analyzing power of ^{9}Be(^{3}H,^{6}He)^{6}Li* and of ^{9}Be(^{3}H,^{6}He)^{6}Li

Since SU(4) invariance is being assumed, the starting point for our calculation is the expression for the reaction amplitude in the TLS representation of Eq. (6). We are making the *ansatz* $l_a=l_c=l_{c'}=0$, $l_b=1$, $L=1$, $L'=0$, so that $M-M'=M=\pm1$. Parity and angular momentum considerations thus limit l to $l=l'\pm1$, and $\mathscr{L}=l'$. It then turns out that there are but two independent sets of amplitudes, and from Eq. (6) we get

$$\begin{aligned}A_{Ss,1\pm1Ss}(\hat{k}_f)&=i\sqrt{\pi}/k_i\ \ (T_{c'}t_{c'}T_ct_c\mid Tt)\sum_{l'\ \mathrm{odd}}\left[\left[\frac{l'+1}{2}\right]^{1/2}T_{l',l'-1}(Tl'S)+\left[\frac{l'}{2}\right]^{1/2}T_{l',l'+1}(Tl'S)\right]Y_{l'}^{\mp1}(\hat{k}_f)\ ,\\ A_{Ss,10Ss}(\hat{k}_f)&=i\sqrt{\pi}/k_i\ \ (T_{c'}t_{c'}T_ct_c\mid Tt)\sum_{l'\ \mathrm{odd}}[\sqrt{l'}T_{l,l'+1}(Tl'S)-\sqrt{l'+1}T_{l',l'+1}(Tl'S)]Y_{l'}^0(\hat{k}_f)\ .\end{aligned} \tag{18}$$

Note that in consequence of our arguments in the previous section, only odd orbital angular momenta appear and that the T matrices are independent of T and S within a given supermultiplet.

In order to calculate the analyzing powers in the two reactions of interest it is convenient to introduce the scattering amplitude in space of the spin states of each of the reactants,

$$A_{m_{c'}m_c,m_bm_a}(\hat{k}_f)=\sum_{j,j'}(j_{c'}m_{c'}j_cm_c\mid j'm')(j_bm_bj_am_a\mid jm)A_{j'm',jm}(\hat{k}_f)\ . \tag{19}$$

These amplitudes are related to the two independent amplitudes in Eq. (18) through Eq. (15) [more specifically, Eq. (17)]. Conservation of the z projection of angular momentum means that

$$M-M'=m-m'=m_b+m_a-m_{c'}-m_c$$

and this relation simplifies to $M=m_b+m_a-s$ because the subscript c refers to the ^{6}He nucleus in both cases. The analyzing power[12] is defined as the difference between the $\phi=0$ differential cross sections for tritons polarized along the $+y$ and $-y$ directions, respectively, divided by the sum of the two cross sections. The product of the analyzing power and the differential cross section is then given by

$$A_y(\theta)d\sigma/d\Omega=\mathrm{tr}[A\ 1\otimes\sigma_y\ \tilde{A}]\ , \qquad (20)$$

where 1 refers to the initial density matrix for the unpolarized ^{9}Be and σ_y is in the spin space of the triton. In terms of the amplitude elements this yields

$$A_y(\theta)d\sigma/d\Omega=\tfrac{1}{4}\sum_{s,m_b}\mathrm{Im}[A_{s,m_b+1/2}A^*_{s,m_b-1/2}]=\tfrac{1}{2}\sum_{m_b+1/2-s=+1}\mathrm{Im}[A_{s,m_b+1/2}A^*_{s,m_b-1/2}]$$

$$=\mathrm{coeff}\,\mathrm{Im}[A_{+1}(\theta)A_0^*(\theta)]\ . \qquad (21)$$

A_{+1} and A_0 refer in an obvious way to the amplitudes in Eq. (18). The property

$$A_{+1}(\theta,0)=-A_{-1}(\theta,0)$$

was used in the development above. It is clear that the angular parts of the interference terms in Eq. (21) are of the form

$$J_{ll'}(\theta)=P_l{}^1(\cos\theta)P_{l'}(\cos\theta)\quad l,l'\ \mathrm{odd} \qquad (22)$$

with the property

$$J_{ll'}(\pi-\theta)=-J_{ll'}(\theta)\ ,$$

so that the analyzing power is indeed antisymmetric around $\theta=\pi/2$ as proved earlier in Refs. 4 and 5. However, our derivation allows us to relate the analyzing powers for the two reactions. Including both the isospin Clebsch-Gordan coefficients and the SU(4) coefficients [see Eqs. (11) and (12)] the differential cross sections and analyzing powers are

^{6}He+^{6}Li* :

$$d\sigma/d\Omega=\tfrac{1}{24}[2\,|A_{+1}|^2+|A_0|^2]\ ,$$

$$A_y d\sigma/d\Omega=-\tfrac{1}{12}\sqrt{2}\,\mathrm{Im}[A_{+1}A_0^*]\ , \qquad (23)$$

^{6}He+^{6}Li :

$$d\sigma/d\Omega=+\tfrac{1}{8}[2\,|A_{+1}|^2+|A_0{}^2]\ ,$$

$$A_y d\sigma/d\Omega=+\tfrac{1}{12}\sqrt{2}\,\mathrm{Im}[A_{+1}A_0^*]\ .$$

We can thus make the prediction that the cross section and analyzing power ratios between the ^{6}Li* and ^{6}Li channels are

$$d\sigma/d\Omega(^6\mathrm{Li})=3d\sigma/d\Omega(^6\mathrm{Li}^*)\ ,$$
$$A_y(^6\mathrm{Li})=-\tfrac{1}{3}A_y(^6\mathrm{Li}^*)\ . \qquad (24)$$

In the following discussion and comparison of the above relations with experimental data we will refer to the final channels as the ^{6}Li and ^{6}Li* channels, respectively. The most complete and accurate measurements of the differential cross sections for the two channels appear in Ref. 3, where the incoming triton energy was 23.5 MeV. A comparison of the two angular distributions shows that both cross sections are nearly symmetric around 90° although the forward cross section for ^{6}He is definitely larger than the backward in the ^{6}Li channel. The maxima and minima occur at the same angles, although the minima are deeper in the ^{6}Li* channel, but the shapes are not identical: At the positions of the most forward and backward maxima the ^{6}Li:^{6}Li* ratio is about 8:1, which drops to about 4.4:1 for the three central maxima. The ratio of the integrated cross section is also quoted as 4.4. The theoretical ratio of 3 obtained in Eq. (24) omits phase space differences in the final channels, the ^{6}Li phase space being larger by a factor of 1.19 at the energy in question. At a lower incident triton energy of 21.5 MeV the data for the ^{6}Li* channel are very incomplete,[3] especially at back angles, but it appears that the two cross sections are of a similar symmetric shape with the ^{6}Li:^{6}Li* ratio being of the order of 7:1. At this energy the phase space ratio is 1.23:1.

The analyzing power measurements were made at $E_t=17$ MeV and are reported in Ref. 5. The differential cross section for the ^{6}Li channel is now only roughly symmetric around 90°; it appears as if a symmetric cross section were shifted by about 10° toward backward angles so there is a definite admixture of even partial waves in the cross section. The cross section for the Li* channel is not published, but the analyzing power for this channel is plotted and, while incomplete as to angle, a strong tendency toward an antisymmetric $A_y(\theta)$ is revealed. The analyzing power data for the ^{6}Li channel offer a big surprise since they are decidedly *symmetric* around 90°. This is particularly puzzling because the predicted ratio of analyzing powers, $-\frac{1}{3}$, is in good agreement with the data in the forward hemisphere.

It is to be expected that the analyzing powers are a more stringent test of SU(4) invariance than are the reaction cross sections. For example, a detailed study of the 3×8 amplitude matrix for the reaction producing ^{6}Li reveals that under our assumption only the $m_c=0$ spin state of the ^{6}Li ion contributes to the analyzing power. This happens because particular matrix elements are identically zero. If the difference in the cross sections for spin up and down tritons were large in the $m_c=\pm 1$ states due to contributions from tensor and spin orbit forces, the effect could possibly "swamp" the antisymmetric contribution from the $m_c=0$ substrate. Further investigations, both experimental and theoretical, are necessary to resolve the problem of symmetrical $d\sigma/d\Omega$ *and* $A_y(\theta)$ for the ^{6}Li channel.

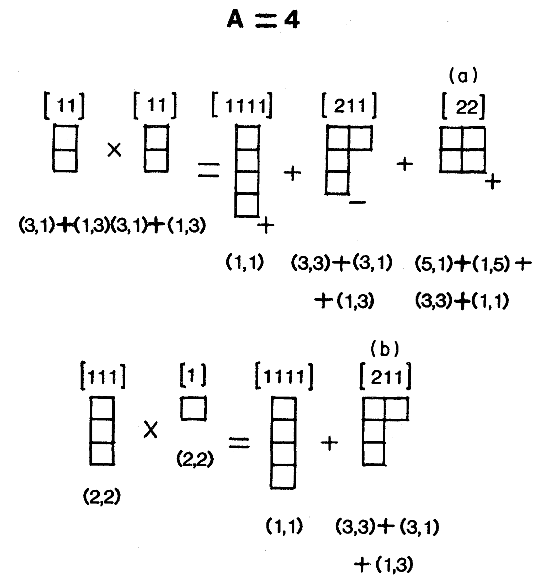

FIG. 2(a). Same as Fig. 1(a) with $^6\text{He}\to{}^2\text{H}$, $^6\text{Li}\to 2p(^{31}S_0)$. (b) Same as Fig. 1(b) with $^9\text{Be}\to{}^3\text{He}$, $^3\text{H}\to p$.

III. SYMMETRY BREAKING

A. $^3\text{He}(p,d)2p$

The reaction He$(p,d)2p$ is analogous to the $^9\text{Be}(^3\text{H},^6\text{Li})^6\text{He}$ reaction when the three body phase space is restricted so that the pair of protons is in a $^{31}S_0$ state. In a kinematically complete experiment it is possible to separate out those scattering events where the protons recoil with little relative energy so that the condition is satisfied and one can measure a differential cross section as if a two body final state were present. Wielinga *et al.* have performed precisely this experiment.[7] If the simplifying assumption is made that all fragments are describable by S-state wave functions, the SU(4) relations are straightforward. In Fig. 2 the four particle SU(4) representations that are contained in the direct products of the initial fragment and final fragment isospin-spin functions, respectively, are displayed. The isospin and spin multiplicities of the separated fragments and of the four body states are shown below the Young graphs symbolizing the representations. Note that the (3,3) multiplet characterizing the final $d+2p$ isospin-spin state occurs in a single representation, the [211], contained in the initial fragment spinor product, but in two representations, the [211] and the [22], that occur in the final fragment spinor product. The admixing of the [22] representation during the collision process is responsible for the marked departure from symmetry around 90° of the measured[7] cross section.

In this section we wish to relate the degree of asymmetry of the cross section around 90° to the SU(4) breaking term in the effective central nucleon-nucleon potential. Consistent with the notation of Sec. II A the initial and final channel wave functions are introduced,

$$\Psi_b\Psi_a = 1/\sqrt{3}\sum_r \psi_l([31]r,1234) \times X_{11}([211]r,1234)\,,$$
$$\Psi_{c'}\Psi_c = \psi_l(12,34)[X_{10}([11]a,12) \otimes X_{01}([11]a,34)]\,. \quad (25)$$

The spatial four body wave functions are obtained by operating with a Young operator for the specified symmetry on the initial channel product wave function $\psi_l(123,4)$ that is completely symmetric with respect to the coordinates of the first three nucleons which are bound in ^{3}He. The final channel spatial wave function is not given a special symmetry with respect to S_4 but, as the notation suggests, is symmetric under the space exchange operators P_{12} and P_{34}. The letter a inside the two particle spinors indicates that they are antisymmetric with respect to exchange of their arguments.

In order to expand the product of the two particle spinors in terms of representations of S_4 the fractional parentage expansion of the four particle states[11] in terms of 2×2 is inverted. One finds that

$$[X_{10}\otimes X_{01}]=-1/\sqrt{2}[X_{11}([211]aa,1234)+X_{11}([22]aa,1234)] . \tag{26a}$$

The operator $P_{cc'}$ is equivalent to the two successive pair exchanges $P_{13}P_{24}$. The result of its operation on the particular partners in the two distinct representation is

$$P_{13}P_{24}X_{11}([211]aa)=-X_{11}([211]aa) ,$$
$$P_{13}P_{24}X_{11}([22]aa)=+X_{11}([22]aa) . \tag{26b}$$

Following the notation of Eq. (10), the partial wave T matrix for the $A=4$ case becomes

$$T_{0l,0l}(1l1[211])=\frac{-K}{2\sqrt{6}}\left[\psi_l(12,34)\{[1-(-1)^l]X_{11}^{1s}([211]aa)+[1+(-1)^l]X_{11}^{1s}([22]aa)\},(V_{13}+V_{14}+V_{23}+V_{24})\times\sum_r \psi_l([31]r,1234)X_{11}^{1s}([211]r,1234)\right] . \tag{27}$$

The effective central potential is written as

$$V_{ij}=v(r_{ij})[W+MP_x(ij)+BP_s(ij)+HP_t(ij)] . \tag{28}$$

The SU(4) breaking parameter α defined in the Introduction is expressed in terms of the potential strengths through

$$\alpha=(B-H)/(W+M) .$$

Evaluation of Eq. (27) for the T-matrix elements yields

$$T_l=\frac{K}{2\sqrt{3}}\{[(1-(-1)^l)(2W+2M)+(1+(-1)^l)(B-H)]I_1(l)+[(1-(-1)^l)(2W-2M+2B+2H)-(1+(-1)^l)(B-H)]\}I_2(l) . \tag{29}$$

The first integral, $I_1(l)$, is of the form of a stripping approximation matrix element evaluated in the "post" form[13]

$$I_1(l)=\left[\psi_l(12,34),(v(r_{13})+v(r_{23}))\psi_l(123,4)\right] , \tag{30a}$$

while the second integral, $I_2(l)$,

$$I_2(l)=\left[\psi_l(12,34),(v(r_{14})+v(r_{24}))\psi_l(123,4)\right] , \tag{30b}$$

arises because of complete antisymmetrization. Note that the potentials are acting between pairs, and one member of each pair, particle 4, is not in the original ^{3}He cluster.

Consideration of our simplified model for $A=4$ has allowed us to derive Eq. (29) for the T-matrix element. The expression reveals (1) that in the case $\alpha=0$ the vanishing of the elements for even l is clearly exhibited and (2) if only the stripping type integral is retained the even partial waves have a strength relative to the odd particle waves of $\alpha/2$. The size of α can be estimated directly from the reaction data. The analysis by van Oers[14] of the ^{3}H$(p,d)d$ reaction suggets that except for very low partial waves, the T-matrix elements vary slowly with l, which in turn implies that the stripping integral varies slowly with l. Owing to the difference in sign of the odd and even Legendre polynomials at 180° the forward and backward amplitudes for ^{3}He$(p,d)2p$ are related by

$$\frac{|A_{1s,1s}(0°)|}{|A_{1s,1s}(180°)|}=\frac{1+\alpha/2}{1-\alpha/2}=1.5\text{—}1.7 . \tag{31}$$

The experimental center of mass (c.m.) cross section[7] has a forward minimum at 35°, near the minimum in ^{3}He$(n,d)d$ at about the same relative energy,[14] and a deep minimum at 100°. With perfect SU(4) symmetry there would be a zero at 90°. A tendency toward a minimum at 150° appears also. Extrapolating the cross sections to 0° and 180° for the cross section measured when the relative energy of the recoiling proton pair, T_{pp}, is restricted to be less than 1.5 MeV, we find the forward backward ratio to be $3.4/1.2=2.8$, in good agreement with Eq. (31). In Ref. 7 the data were analyzed in terms of real, incoherent neutron and proton amplitudes. We suggest a new attempt to fit the data with complex, coherent amplitudes as outlined above so as to obtain a measurement of the effective SU(4) mixing parameter.

B. Symmetry breaking in the stripping approximation

A nuclear rearrangement of the type $b(a,c)c'$ is often conceived of as occurring through the exchange of a cluster of nucleons, call it x, between b and a. This means that c' and x can be coupled to a system with the same quantum numbers as b and that a and x can be coupled to a system with the same quantum numbers as c. The perturbing potential in the post representation can be written[13]

$$V=V_{c'x}(r_{c'x})+V_{c'a}(r_{c'a})-V_{c'c}(R_{c'c}) , \quad (32)$$

and the transition matrix element is evaluated between states that are positive energy solutions to the Schrödinger equation with complex optical potentials. It is generally assumed that the optical potential in Eq. (32) largely cancels out the second term, $V_{c'a}$, so that the first term alone is retained as the coupling potential between the initial and final channels.

The DWBA stripping approximation predicts a strong forward peaking of the production cross section for c, but in the typical light ion reaction, backward peaking is also a prominent feature. This situation is also consistent with the stripping approximation, provided there exists a cluster of nucleons x' that can be coupled to fragment c to form a system with the same quantum numbers as b and that x' and a can be coupled to form a system with the same quantum numbers as c'. Imposing the requirement of overall antisymmetry on the $2n$ nucleon wave function necessitates the adding of the two distinct stripping matrix elements coherently. If the potentials $V_{c'x}$ and $V_{cx'}$ are identical because of a symmetry property of the nucleon-nucleon force, either the odd or even parity partial waves can vanish and symmetry around 90° of the reaction cross section is obtained.

This approach was adopted by von Oertzen and Flynn[3] to explain their ^{9}Be(^{3}H,^{6}He)^{6}Li experiment. Since exchange of either a ^{3}H cluster or a ^{3}He cluster between the target ^{9}Be and the incoming ^{3}H both forward and backward peaking of the production cross section for ^{6}He is understood in terms of the coherent addition of the two amplitudes. It is a powerful method because exisiting DWBA codes can be used to separately calculate the interfering amplitudes. The ^{13}C(^{15}N,^{14}C)^{14}N reaction, a reaction where the ^{14}C(0^+) and the ^{14}N(1^+) are *a priori* not members of the same SU(4) multiplet, also displays near symmetry around 90°. This reaction has been analyzed[15] by taking the coherent sum of diagrams for either n or p exchange. In this analysis the n-^{14}N and p-^{14}C potentials were obtained from a microscopic structure calculation of the nuclei in question. Thus, a prediction was made as to the departure of the cross section from 90° symmetry that was quite successful. In this section we are pointing out that in systems exhibiting approximate SU(4) symmetry the microscopic calculation can be carried out and the difference in the two amplitudes can be related linearly to the size of the symmetry breaking parameter α.

Chwieroth *et al.*[16] have stressed that the various particle exchanges possible in a rearrangement collision are derived naturally if one starts with a T-matrix expression which incorporates antisymmetry. An example of such an expression is Eq. (3) in the present paper. Introducing a DWBA approximation[17] for the initial and final channel wave functions, the initial state is restricted to the Hilbert space ba and the plane wave factor in Eq. (2) is replaced by a distorted wave with incoming boundary conditions. Then the T matrix is

$$\mathscr{T}_{c'c}=N_{cc'}^{1/2}\left[\Psi_{c'c},\ \sum_{\substack{i\le n\\ j\ge n+1}} V_{ij}\frac{(-1)^{\sigma(P)}P_{ba}}{N_{ba}^{1/2}}\Psi_{ba}\right] . \quad (33)$$

For $A=4$, $n_b=3$, and $n_a=1$, it is trivial to show that

$$\sum_{\substack{i\le 2\\ j\ge 3}} V_{ij}\frac{(1-P_{14}-P_{24}-P_{34})}{2}\to(1+P_{13}P_{24})\sum_{\substack{i\le 2\\ j\ge 3}} V_{ij}\equiv(1+P_{cc'})\sum_{\substack{i\le 2\\ j\ge 3}} V_{ij} . \quad (34)$$

If the unity term represents a neutron exchange in ^{3}He$(p,d)2p$, the final cluster exchange term represents proton exchange. When there is more than one nucleon in cluster a, the result cannot be written as the coherent sum of exchange terms representing the exchange of a single fragment. For example, in the $A=12$ case the analog of Eq. (34) is

$$\sum_{\substack{i\le 6\\ j\ge 7}} V_{ij} \sum_{P} \frac{(-1)^{\sigma(P)}P_{ba}}{N_{ba}^{1/2}} \rightarrow \left[\frac{20}{11}\right]^{1/2} (1+P_{cc'}) \sum_{\substack{i\ge 6\\ j\ge 7}} V_{ij}(1-\tfrac{9}{2}P_{110}) \ . \qquad (35)$$

The permutation operator P_{110} can be interpreted as generating the exchange of a particle-hole pair in addition to the exchange of a three body fragment between the target ^{9}Be and the incoming ^{3}H. Following the convention of von Oertzen and Flynn[3] the unity term represents ^{3}H exchange and the $P_{cc'}$ term ^{3}He exchange.

A DWBA approximation of the type already employed by the authors cited[3,15] to describe light ion reactions is obtained by dropping permutation operators to the right of the potentials in Eq. (35) and by dropping the potentials between nucleons if $j>n_b$. At this point the cluster wave function $\Psi_c(\Psi_{c'})$ is decomposed into products of cluster wave functions such as $\Psi_x\Psi_a(\Psi_{x'}\Psi_a)$, and the fragment wave function $\Psi_x(\Psi_{x'})$ is coupled to the other final fragment wave function $\Psi_{c'}(\Psi_c)$ to form a state with the same quantum numbers as initial cluster b. The final step is to decompose the wave function for initial fragment b into products such as $\Psi_{c'}\Psi_x(\Psi_c\Psi_{x'})$. The terms in parentheses correspond to the $P_{cc'}$ part of Eq. (35). It is the fractional parentage coefficient from the last decomposition which can contribute a phase $(-1)^{\beta}$ into the partial wave T matrix which is also written as the coherent sum of two terms,

$$T_{l',Ll}(TS)[(1+A\alpha)+(1-A\alpha)(-1)^{n+l'+S_c+S_{c'}-S+T_c+T_{c'}-T+\beta}]T_{l',Ll}(\text{DWBA}) \ ,$$

where the terms proportional to α come from the SU(4) breaking part of the nucleon-nucleon potential. The two terms have opposite signs because of the relation

$$P_s(ij)=-P_t(ij)P_x(ij)=-P_t(ij) \ ,$$

forces in negative parity states being neglected, and the fact that the products $\Psi_{c'}\Psi_x(\Psi_c\Psi_{x'})$ differ only by the interchange of the isospin and spin quantum numbers.

The rigorous calculation of the coefficient A will be described in a future publication. Here we present a simplified way of estimating its size. Let us presume that the amplitude for forward (backward) peaks becomes quite small in the backward (forward) direction. Then the ratio of the height of the first cross section peak at or after 0° to the height of the last peak at or before 180° should be determined by the ratio of the strengths of the potentials of the two exchange terms,

$$\frac{A(0°)}{A(180°)}=\frac{V_{c'x}}{V_{cx'}} \ . \qquad (36)$$

The potentials describe the interaction of a pair of fragments which are initially bound inside nucleus b whose wave function, like the wave functions of the final fragments, contains a spatial wave function which belongs to a particular partition of the permutation group. Following the methods outlined in Blatt and Weisskopf,[18] the number of even parity pairs in the clusters b, c, c', x, and x' are calculated for the appropriate partitions of the spatial wave functions. All "filled rows," i.e., those containing two protons and two neutrons, contribute to the internal energy of each cluster term proportional to V^+. A single unfilled row with either one (e.g., ^{9}Be) or three nucleons (^{3}H,^{3}He) also contributes terms proportional to V^+. A single unfilled row with a pair of nucleons (e.g., d, $2p$, ^{6}He, ^{6}Li) contributes a term with α dependence since the pair is either in a spin triplet or singlet state,

$$^{13}V^+=(1+\alpha)V^+, {}^{31}V=(1-\alpha)V^+ \ . \qquad (37)$$

Our *ansatz* is that the strength of the intercluster potential is proportional to the difference in the number of even space pairs in the initial bound system and in the final fragments,

$$\frac{V_{c'x}}{V_{cx'}}=\frac{(N_b^+-N_{c'}^+-N_x^+)\pm\alpha}{(N_b^+-N_c^+-N_{x'}^+)\mp\alpha} \ , \qquad (38)$$

where the choice of sign for the α term depends on the identification of c' and c. In the cases of interest we find that for $A=4$, with $c\equiv d$, $c'\equiv 2p$, and $b\equiv {}^3$He,

$$\frac{V_{c'x}}{V_{cx'}}=\frac{(3-1-0)+\alpha}{(3-1-0)-\alpha}=\frac{1+\alpha/2}{1-\alpha/2} \ , \qquad (39a)$$

and the coefficient $A=+\frac{1}{2}$, in complete agreement with the coefficient of $I_1(l)$ in Eq. (29). For $A=12$, with $c\equiv {}^6$He, $c'\equiv {}^6$Li, and $b\equiv {}^9$Be, the ratio is

$$\frac{V_{c'x}}{V_{cx'}}=\frac{(21-10-3)-\alpha}{(21-10-3)+\alpha}=\frac{1-\alpha/8}{1+\alpha/8} \ , \qquad (39b)$$

and the coefficient $A=-\frac{1}{8}$. We conclude that with increasing nucleon number the departures from symmetry of the reaction cross section around 90° should decrease, which is in qualitative agreement with the data.[7,3] Unfortunately, at an incoming triton energy of 23.5 MeV the data of ^{9}Be(^{3}H,^{3}He)^{6}Li seem to favor $A=+\frac{1}{8}$. However, one should redo the DWBA calculations of von Oertzen and Flynn[3] with differing strengths for the two interfering amplitudes to properly take into account the summation of complex amplitudes. Likewise, the $A=4$ reaction amplitude cross section should be analyzed in terms of interfering complex amplitudes of differing strength to obtain an experimental value for A. Finally, it should be noted that similar but numerically different values of A can be derived from using the light ion stripping approximation in the "pre"form so that further experiments and analysis may provide insight into the differences in the two approximations when consistent potentials and optical parameters are used.

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