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Prediction of shape-isomeric bands in light nuclei

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Candidates for shape-isomeric rotational bands, relevant to the controversy over anomalous events in relativistic heavy ion collisions, are specified in the spectra of ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, and ^{40}Ca. No such band is found in ^{12}C, ^{20}Ne, and ^{36}Ar.

[NUCLEAR STRUCTURE Predicted shape-isomeric rotational bands in *p*, *sd* shell.]

Shape-isomeric nuclear states have played an important role in the recent controversy about anomalous events in nuclear emulsion tracks from high-energy cosmic-ray and Bevalac exposures (cf. Ref. 1 and references therein). Bayman *et al.*[2] have pointed out that the extremely long lifetimes and high deformations characteristic of shape-isomeric states could quite naturally explain the observed anomalous emulsion tracks that had been attributed to unknown exotic nuclear or quark phenomena.

In this paper we study whether any evidence for sub-barrier shape-isomeric rotational bands (similar to those observed in actinide fission) can be extracted from the experimental spectra in the *p* and *sd* shell, and attempt to identify candidates for future experimental verification.

In actinide nuclei shape-isomeric states have been studied for many years, and their properties are well known (cf., e.g., Ref. 3). Their extreme deformation (with an axis ratio of 2:1:1) and correspondingly small overlap with the ground state hinders the gamma decay of shape-isomeric states to such an extent that fission (with half-lives up to milliseconds) is their dominant decay mode in actinides.

Although shape-isomeric states have so far been observed only in a narrow region of heavy elements ($235 \leq A \leq 246$) it is conceivable that similar states exist also in light nuclei. The properties of such isomeric states in light nuclei should be completely analogous to those of the known fission isomers, except that they do not fission. Thus they should be characterized by extremely long gamma half-lives, large quadrupole moments, and long rotational bands that have large moments of inertia and a very accurate $J(J+1)$ level spacing. Shape isomers should be observable in light nuclei through such a characteristic band.

The experimental search for shape-isomeric bands is difficult in light nuclei because the very selective fission-coincidence techniques of the conventional isomer spectroscopy are not applicable in discriminating shape-isomeric events from a much larger background. Therefore, the experimental identification of shape isomers in light nuclei will necessarily have to rely on some conjectured level sequence whose band properties can then be verified or rejected.

Recently, we have tentatively identified the 8.507, 9.065, and 10.276 MeV levels in ^{32}S as the 0^+, 2^+, and 4^+ members of a shape-isomeric rotational band.[4] The identification is based on a series of variational calculations after $0^+, 2^+, 4^+, \ldots, 10^+$ projection for the Brink-Boeker B_1 nucleon-nucleon interaction.[5] The resulting shape-isomeric state $\phi(\vec{x}_1, \ldots, \vec{x}_A)$ has, in fact, a density distribution

$$\rho(\vec{r}) = \langle\phi|\sum_{i=1}^{A}\delta(\vec{r}-\vec{x}_i)|\phi\rangle \tag{1}$$

with a much wider range than the ground state (see Fig. 1). It would be tempting to repeat such a microscopic calculation for neighboring nuclei, in particular, because the experimental spectrum of ^{32}S is rather sparse as compared with other nuclei in the *sd* shell. Unfortunately, the enormous computer time required for such a calculation will make this difficult, and more approximative calculations, such as

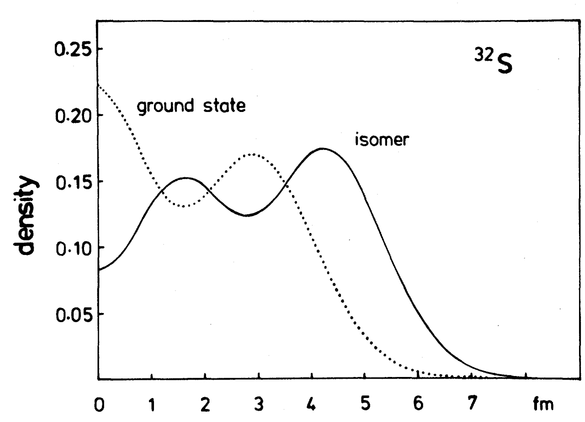

FIG. 1. Calculated intrinsic density distributions of the ground state and the shape isomer in ^{32}S. Each plot is along the axis of maximum symmetry for the intrinsic state of minimum 0^+ energy.

variation without or before projection, have been shown[4] to lead to spurious results.

It turns out, however, that the predicted[4] shape-isomeric state in ^{32}S has a deformation that is similar to the observed deformations in actinides. This may not be surprising since the 2:1:1 deformation at which shape isomerism occurs is usually attributed[6] to the particular degeneracy of the deformed harmonic oscillator with frequencies 1:2:2. In heavy nuclei the pure harmonic-oscillator single-particle spectrum is considerably affected by spin-orbit coupling, and the degeneracy is partly removed. Nevertheless, the remaining effect is sufficiently pronounced to lead to spectacular experimental consequences. In light nuclei the disordering effect of spin-orbit coupling is much weaker, and the harmonic oscillator 2:1:1 degeneracy should even be more important.

In fact, the moment of inertia θ that results from the many-body calculation in ^{32}S turns out to be rather close to what one would predict from simply scaling the observed θ values of shape-isomeric bands in actinides down to the size of a light nucleus. In the conventional models[7] used for calculating the moment of inertia (such as the spheroidal rigid-body or the irrotational-flow model) the classical value θ_0 for a rigid sphere

$$\theta_0 = \tfrac{2}{5} M R^2 \sim A^{5/3} \tag{2}$$

can be split off, and a pure shape factor $f(\epsilon)$ remains,

$$\theta(A,\epsilon) = \theta_0(A) f(\epsilon) \; . \tag{3}$$

Thus, if shape isomerism occurs only at a certain deformation ϵ_0 the moments of inertia of shape isomers in different nuclei are related by

$$\theta(A_1,\epsilon_0)/\theta(A_2,\epsilon_0) = A_1^{5/3}/A_2^{5/3} \; . \tag{4}$$

The rotational constant $\alpha = \hbar^2/2\theta$ of a hypothetical shape-isomeric rotational band

$$E(J^+) = E(0^+) + \alpha J(J+1) \tag{5}$$

in a nucleus of mass A should then be in the vicinity of

$$\alpha(A) = 30\,985\ \mathrm{keV}/A^{5/3} \; . \tag{6}$$

Here the numerical constant is chosen such that it matches the experimental α value of the shape isomer in ^{240}Pu, the one that is presently known with the highest precision,[3]

$$\alpha(A=240) = 3.343\ \mathrm{keV} \; . \tag{7}$$

In the case of ^{32}S the rotational constant derived from the many-body calculation is $\alpha = 90.7$ keV, whereas the extrapolation (6) from $A = 240$ to $A = 32$ yields $\alpha = 96.1$ keV, i.e., an agreement within 6%.

The structure of the intrinsic state of shape-isomeric bands should indicate a certain degree of fission-type clustering or even substantial fragmentation. Therefore, our interest here is centered on the doubly even $N = Z$ nuclei from ^{12}C to ^{40}Ca where some information about the cluster structure of excited bands is available from cluster-model, generator-coordinate or Hartree-Fock calculations.

In our search for shape isomers in light nuclei we have considered all known 0^+ states up to about 10 MeV. For each 0^+ level we have determined all sequences of levels in the experimental spectrum that can possibly be arranged to a "band" with rotational $J(J+1)$ spacing. In spite of the large density of known levels in some nuclei it turns out that the density of level *sequences* with 0^+, 2^+, 4^+, ... assignments and approximate $J(J+1)$ spacing is rather sparse. In each nucleus (except ^{40}Ca, as discussed below) at most one hypothetical band assignment has a rotational constant α in the vicinity of the value of Eq. (6). The results are given in Tables I and II.

In ^{12}C the large spread of 29% is certainly incompatible with the particularly stable rotational pattern that shape-isomeric bands are expected to have. Therefore, the ^{12}C band of Table I has to be ruled out. As all other "bands" in ^{12}C (up to 10.3 MeV) differ from the extrapolated moment of inertia by more than 20%, no candidate remains.

The ^{16}O search leads to a sequence of four levels with definite 2^+, 4^+, and 6^+ assignments. The spacing fits into the rotational $J(J+1)$ scheme within 5%, and their moment of inertia of 310 keV is close to the 305 keV estimate for mass 16. However, this may be an accidental coincidence as the 11.26 MeV bandhead is uncertain,[9] and has a very large width of 2.5 MeV in contrast to the particular stability that a shape isomer is expected to have. Unfortunately, K values or gamma transitions are not available for the conjectured band.

In ^{20}Ne none of the bands that can be associated with the 0^+ levels up to 10.97 MeV has a rotational constant α compatible with the extrapolation (6). Even the best case differs by 49%, and has large deviations (19%) from a rotational spacing.

For ^{24}Mg we obtain the already known second rotational $K^\pi = 0^+$ band starting with the 6.432 MeV 0^+ and 7.348 MeV 2^+ levels. The assignments of the higher members of this band are not unique in the literature. Our assignments are rather close to the calculation of Kato and Bando[12] using the orthogonality condition model for a $^{16}O + 2\alpha$ cluster structure. The band has also been explained[13] in terms of a simpler $^{16}O + {}^{8}Be$ cluster model.

In ^{28}Si our conjectured shape-isomeric band coincides with the prolate rotational band observed recently.[11] The measured quadrupole moment[11] is 876 mb. This corresponds to a calculated rms radius[14] of 3.1 to 3.4 fm depending on the nucleon-nucleon interaction. The cluster structure of the intrinsic state is well known from alpha-cluster calculations. It has

TABLE I. Rotational constants $\alpha = \hbar^2/2\theta = [E(2^+) - E(0^+)]/6$ of shape-isomeric bands extrapolated from actinides (α_{calc}) and the values α_{expt} resulting from the search in the experimental spectrum (Refs. 8–11). The error denotes the maximum deviation within the band. The number of levels and spin assignments (approximate assignments in parentheses) are also given for each band. The corresponding evaluation for ^{240}Pu is included for comparison.

Nucleus	α_{calc} (keV)	α_{expt} (keV)	Number of levels, J^π
^{12}C	493	453 ± 29%	4 2(+1)
^{16}O	305	310 ± 5%	4 3(+1)
^{20}Ne	210	144 ± 19%	5 3(+2)
^{24}Mg	155	149 ± 3%	5 3(+1)
^{28}Si	120	119 ± 4%	4 3(+1)
^{32}S	96.1	90.7 ± 2%	3 2(+1)
^{36}Ar	78.9	102 ± 1%	4 2(+1)
^{40}Ca	66.2	68.0 ± 2%	5 3
^{240}Pu	3.343	3.33 ± 0.2%	5 5

D_{3h} point symmetry consisting of six alphas that form two parallel triangles and one alpha in the center.[14] Thus the structure of the state differs from a binary fragmentation by the central alpha cluster. The latter is also obtained in Hartree-Fock calculations, and is therefore not a particularity of the alpha-cluster variational space (which sometimes overestimates the degree of alpha clustering). The prolate band in ^{28}Si also differs from the known fission isomeric bands in that no other prolate minimum at lower excitation and smaller deformation appears to exist in ^{28}Si. The lower 0_2^+ state at 4.979 MeV has been attributed[15] to a vibrational excitation in the (oblate) ground-state well.

For ^{32}S the isomeric band resulting from our search coincides with the assignment made earlier[4] on the

TABLE II. Predicted shape-isomeric rotational bands and their J^π; T assignments (Refs. 9–11). The band in ^{28}Si has already been observed through gamma transition measurements (Ref. 11).

Nucleus		0^+	2^+	4^+	6^+	8^+
^{16}O	E (MeV)	11.26	13.02	17.784	23.879	
	$J^\pi;T$	$(0^+;0)$	2^+	4^+	6^+	
^{24}Mg	E (MeV)	6.4322	7.3479	9.4558	12.859	16.850
	$J^\pi;T$	0^+	2^+	$(2\text{–}4)^+$	. . .	8^+
^{28}Si	E (MeV)	6.6914	7.3807	9.1639	11.509	
	$J^\pi;T$	0^+	2^+	$(3^-, 4^+)$	6^+	
^{32}S	E (MeV)	8.507	9.065	10.276		
	$J^\pi;T$	0^+	$(2^+, 3^-, 4^+);0$	4^+		
^{40}Ca	E (MeV)	5.2129	5.6296	6.5435	8.0518	10.1371
	$J^\pi;T$	0^+	2^+	4^+	. . .	. . .

basis of a variation after angular-momentum projection. The band has $^{16}O + ^{16}O$ clustering; its properties are discussed in Ref. 4. We note that a shape isomer conjectured earlier[16–18] much below the 8.507 MeV value of Table II does not fit the extrapolated moment of inertia which, in turn, is very close to the result of the nuclear structure calculation. The earlier predictions were based on energy surfaces obtained in liquid-drop plus shell-correction calculations. Apart from the question of applying these methods to light nuclei, it appears[4] that the effect of angular momentum projection before variation is essential in calculating the isomeric band.

In ^{36}Ar we find no level sequence that fits the extrapolated moment of inertia. This may be due to the fact that the currently known experimental spectrum is relatively sparse in ^{36}Ar.

In ^{40}Ca there are two candidates that fit the extrapolated moment of inertia almost equally well: the band in Table II and the sequence 7.7012 MeV 0^+, 8.0912 MeV 2^+, 8.9950 and 10.433 MeV. In the latter case, however, only the first two levels have definite J^π assignments, and the $J(J+1)$ spacing of the higher levels may be accidental due to the high level density. The reservation also applies to the ^{40}Ca band in Table II where only the 0^+, 2^+, and 4^+ levels have known spins. This band, however, coincides with the one predicted in Ref. 16 but is higher in energy than the 3.354 MeV previously conjectured[19] from liquid-drop plus shell-correction results.

We conclude from the results of this study that there are rotational bands in light nuclei that are analogous to the known shape-isomeric rotational bands in actinides in several respects: They have a moment of inertia that a fission-isomeric state would have when shrunk to the size of ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, or ^{40}Ca. The rotational bands are rather long in some cases (at least up to 8^+ in ^{24}Mg, 6^+ in ^{28}Si), and they all follow the $J(J+1)$ rule very accurately (to within 2–5%). For three of the level sequences attributed to shape isomerism in this paper, there is independent evidence that they form a rotational band. The one in ^{28}Si has already been observed[11] through gamma transition measurements; those in ^{24}Mg and ^{32}S are confirmed by nuclear structure calculations.[4,12,13] Further candidates for experimental verification (^{16}O, ^{40}Ca) are given in Table II.

In cases where the intrinsic state of the rotational band is known from previous calculations, it has a cluster structure corresponding to symmetric fragmentation ($^{32}S = ^{16}O + ^{16}O$, Ref. 4) or at least a distinct separation of large parts of the nucleus ($^{24}Mg = ^{16}O + 2\alpha$ or $^{16}O + ^{8}Be$, Refs. 12 and 13; $^{28}Si = ^{12}C + \alpha + ^{12}C$, Ref. 14), and therefore a density distribution with an anomalous wide range (as in Fig. 1). The low excitation energy of these states guarantees high stability against fission and α decay. The calculated overlap between the intrinsic ground and shape-isomeric states[4,14] is only 10^{-7} in ^{28}Si and 10^{-9} in ^{32}S, indicating long gamma-decay half-lives similar to those observed in fission isomers of actinides. Both properties, range and stability, are essential points in the interpretation of anomalous events in relativistic heavy ion collisions. According to the negative result of our search there are no such low-excited long-range metastable states in ^{12}C, ^{20}Ne, and ^{36}Ar. The absence in these nuclei, if confirmed experimentally, could perhaps serve as test cases in the present controversy about the anomalous events in relativistic collisions.

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