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Electric polarizability of the deuteron

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An experimental value of the electric polarizability of the deuteron is extracted from deuteron photoabsoption data, a dispersion relation, and the low-energy theorem for Compton scattering. The experimental number requires the calculation of several small corrections, which are primarily magnetic in origin. Our value is somewhat smaller than, but consistent with, a recently reported experimental determination.

[NUCLEAR STRUCTURE Deuteron; electric polarizability; photodisintegration; sum rules.]

The interesting recent experiment[1] by Rodning, Knutson, Lynch, and Tsang is the first direct measurement of the electric polarizability of the deuteron. In this Communication we emphasize that although accurate numerical values have not previously been obtained, an indirect measurement also exists whose theoretical foundation is as firm as the reported measurement.

Physically, the electric polarizability α_E is the response of a finite system to successive electric impulses, which distort and then reform that system. The analogous magnetic susceptibility β_M exists for magnetic impulses. The reported experiment used the inhomogeneous electric field of a heavy nucleus to provide the electric impulse. Alternatively, one could use a photon, which deforms a system with its electric and magnetic fields, and upon remission allows the system to reform. This is the nuclear Compton effect, whose (nuclear) spin-averaged forward scattering amplitude we denote by $f(\omega^2)$ for an incident photon of energy ω. The imaginary part of this amplitude is related by the optical theorem to the total cross section for photoabsorption: $\sigma_\gamma(\omega)$.

Following the pioneering work of Gell-Mann, Goldberger, and Thirring,[2] one can write a once-subtracted dispersion relation for f, in the form

$$\mathrm{Re}[f(\omega^2)]=f(0)+\frac{\omega^2}{2\pi^2}P\int_0^\infty \frac{\sigma_\gamma(\omega')d\omega'}{\omega'^2-\omega^2} \quad . \qquad (1)$$

The subtraction is necessary for two related reasons: Firstly, the unsubtracted dispersion relation at zero energy has right- and left-hand sides with different signs; secondly, $\sigma_\gamma(\omega)$ does not vanish asymptotically and the integral over that quantity has dubious convergence.

For small ω^2 the slope of f with respect to ω^2 can be easily obtained from Eq. (1) and the known form[3] of f, which is real below threshold, ω_0:

$$f(\omega^2)\cong -\frac{Z^2\alpha}{M_t}+\omega^2(\bar{\alpha}+\bar{\beta})+\cdots \quad , \qquad (2)$$

the two terms being the Thomson and the Rayleigh amplitudes, respectively. This is the simplest and oldest example of a low-energy theorem, a rigorous statement of the behavior of f for long wavelength photons, in terms of the proton and mass numbers Z and A, the total mass $M_t=MA$, the fine structure constant α, the nucleon mass M, and the generalized electric and magnetic polarizabilities $\bar{\alpha}$ and $\bar{\beta}$. We than obtain

$$f'(0)\equiv\bar{\alpha}+\bar{\beta}=\frac{1}{2\pi^2}\int_{\omega_0}^\infty \sigma(\omega')\frac{d\omega'}{\omega'^2}\equiv\bar{\sigma}_{-2}=\frac{\sigma_{-2}}{2\pi^2} \quad , \qquad (3)$$

in terms of the inverse-square energy-weighted photonuclear sum rule. This energy weighting greatly suppresses the contributions of nucleonic excitations and virtual meson production, as we will see later. Other sum rules $\bar{\sigma}_n$ can be constructed analogously.

The only approximation which is required is an evaluation of $\bar{\alpha}$ and $\bar{\beta}$ in terms of nuclear quantities. We write[3]

$$\bar{\alpha}=\frac{2\alpha}{3}\sum_{N\neq 0}\frac{|\langle N|\vec{D}|0\rangle|^2}{E_N-E_0}+\frac{Z\alpha\langle \bar{r}^2\rangle}{3M_t}+\alpha_N+\cdots \tag{4a}$$

and

$$\bar{\beta}=\frac{2\alpha}{3}\sum_{N\neq 0}\frac{|\langle N|\vec{\mu}|0\rangle|^2}{E_N-E_0}+\left[-\frac{Z\alpha\langle \bar{r}^2\rangle}{6M}-\frac{\alpha\langle \vec{D}^2\rangle}{2M_t}\right]+\beta_N+\beta_\pi+\cdots \quad . \tag{4b}$$

Specializing to the deuteron, the three parts of $\bar{\alpha}$ are the electric polarizibility α_E, a recoil correction $\alpha_E'(=\alpha\langle \bar{r}^2\rangle/6M)$, and the sum of intrinsic nucleon polarizabilities α_N. The five parts of $\bar{\beta}$, in order, are the paramagnetic susceptibility β_M, the (ordinary) diamagnetic susceptibility β_d and the center-of-mass correction to it, $\beta_d'(=-\alpha\langle \bar{r}^2\rangle/4M)$, the sum of intrinsic nucleon magnetic susceptibilities β_N, and the contribution of the pionic cloud to the nuclear diamagnetic susceptibility via meson exchange currents β_π. The latter quantity was calculated in Ref. 4 and can be reduced to the following expression:

$$\beta_\pi=-\frac{f^2}{18\mu^3}\int_0^\infty e^{-\mu r}\{[1-7\mu r+\tfrac{2}{3}(\mu r)^2](u^2+w^2)+4\sqrt{2}[\mu r+\tfrac{2}{3}(\mu r)^2](uw-w^2/\sqrt{8})\}\ , \tag{5}$$

where $f^2=0.079$ is the pion-nucleon coupling constant and μ is the pion mass. Additional small corrections due to relativistic effects, other meson exchange currents, etc., are also possible. The quantities $\vec{\mu}$, $\vec{D}$, and $\langle \bar{r}^2\rangle$ are the magnetic and electric dipole operators and the nuclear mean-square radius, respectively. The magnetic dipole operator contains nontrivial contributions ($\sim 10\%$) from meson exchange currents, which affect β_M by a similar amount.

Numerical values of the various quantities in Eq. (4) are listed in Table I, with calculations for the Reid soft core (RSC) potential model and experimental numbers constructed from available photonuclear data.[5] The experimental value of $\bar{\sigma}_{-2}$ is obtained by fitting the experimental data to interpolating functions, which are then integrated to obtain the final number. Several different interpolation schemes led to the same result. The experimental value of α_E is obtained by subtracting the sum of the last five entries in the table from the experimental value of $\bar{\sigma}_{-2}$. Only the paramagnetic susceptibility is non-negligible compared to the experimental error. The two entries for β_M are the values calculated without and with pion-exchange currents. The latter calculation agrees well with experimental transition strengths to the $^1S_0$ deuteron excited state if the RSC $^1S_0$ potential is modified slightly to generate the correct np scattering length (the Reid potential was fitted to pp data). The latter effect is 0.003 and is included in (0.077). All the significant contributions to β_M come from the $^1S_0$ intermediate state in Eq. (4b). It has been assumed in Table I that the neutron's intrinsic polarizabilities in $(\alpha_N+\beta_N)$ are identical to those of the proton, calculated in Ref. 6 using Eq. (3). This quantity is small, as is β_π, for the reason stated below Eq. (3). Our result of 0.61 ± 0.04 is consistent with 0.70 ±0.05 of Ref. 1.

It seems unlikely that there would be much sensitivity of α_E to potential models, if those models reproduce the known binding energy and asymptotic S- and D-state normalizations of the deuteron and the experimental low-energy phase shifts for the p and f states. Two other potential model calculations in Ref. 9 produced 0.615 and 0.628 fm^3 for α_E. It has been known for a generation[7] that the low-energy electric photoabsorption on the deuteron is nearly model independent, and agreement between experiment and theory is good. The effective range expansion[7] for σ, together with Eq. (3), generates 0.638 for α_E (using an effective range of 1.76 fm), and assumes no deuteron D state and plane-wave p states!

We also note that upper and lower bounds can be obtained on α_E and β_M. Replacing E_N-E_0 in Eq. (4) by E_B, the deuteron binding energy, generates an upper bound, and by using the Hölder inequality $(\sigma_{-2}\geq\sigma_{-1}^2/\sigma_0)$, we obtain a lower bound[8]

$$\frac{2\alpha}{3E_B}\langle \bar{r}^2\rangle\geq\alpha_E\geq\frac{8\langle \bar{r}^2\rangle M\alpha}{9(1+\kappa)} \tag{6}$$

TABLE I. Contributions to σ_{-2} for the deuteron in units of fm^3.

	$\bar{\sigma}_{-2}$	α_E	α_E'	$\alpha_N+\beta_N$	β_M	$\beta_d+\beta_d'$	β_π
RSC	0.70	0.624	0.0010	0.0028	0.065(.077)	−0.0024	0.00006
Expt	0.69 ± 0.04	0.61 ± 0.04					

or $1.7 \geqslant \alpha_E \geqslant 0.13$, where we have written

$$\bar{\sigma}_0 = \frac{1}{2\pi^2} \int_0^{\mu} \sigma_\gamma(\omega') d\omega' \cong \alpha(1+\kappa)/4m \ ,$$

where[9] $\kappa \cong 0.8 - 1.0$. These bounds can only eliminate the most dubious values of α_E. A similar treatment of β_M without exchange currents produces

$$\beta_M \leqslant \frac{\alpha}{6M^2 E_B} [\mu_v^2 + 3P_D(\mu_s - \tfrac{1}{2})^2(1 - 3P_D/4)] = 0.11 \ , \quad (7)$$

where μ_s and μ_v are the nucleon isoscalar and isovector magnetic moments ($0.88\mu_N$ and $4.7\mu_N$, respectively), and P_D is the deuteron d state percentage. The complicated second term is the negligible contribution of the ${}^3S_1$-${}^3D_1$ continuum states, whose upper bound is 0.0003. We note that β_M is a large fraction of its upper bound, reflecting the dominance of the ${}^1S_0$ threshold state.

In summary, we have presented a second experimental value of α_E, in agreement with the one recently reported. Better experimental numbers from both techniques would be useful. In the photonuclear case, the most serious limitation is the lack of precise data for $\omega < 5$ MeV. We also note that very low-energy photon scattering ($\ll E_B$) can also, in principle, measure α_E; other methods are clearly more practical.

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