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Strengths of transitions between 0^+ and 1^+ states and their relationship to inelastic electron scattering form factors: Example of ^{24}Mg

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The strengths of transitions in $A=24$ between $J^\pi=1^+$, $T=1$ states and the lowest $J^\pi=0^+$ states of $T=0$ and 2 are calculated from shell-model wave functions constructed in the full eight-particle $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ configuration space. The model Hamiltonian is an empirical interaction which yields a good accounting of the energies of intra-*sd*-shell states throughout the $A=17$—39 region. With these same wave functions the (e,e') form factors for the magnetic dipole excitation of ^{24}Mg to its lowest five states of $J^\pi=1^+$, $T=1$ are calculated in the plane-wave Born approximation. The predicted form factors and the associated predictions for $B(M1)$ and B (Gamow-Teller) are compared to existing experimental values.

[NUCLEAR STRUCTURE Predicted strengths of isospin-changing $M1$, Gamow-Teller, and (p,n) transitions between $J=0^+$ and 1^+ states in ^{24}Mg and form factors for ^{24}Mg(e,e'); shell-model wave functions, complete *sd*-shell basis space, universal *sd*-shell Hamiltonian.]

INTRODUCTION

Inelastic electron scattering measured at 180° yields definitive information about the fundamental $M1$ mode of nuclear excitations.[1,2] The excitation energy of the $M1$, or the related Gamow-Teller, "giant resonance" provides basic constraints upon the form of the effective interaction between nucleons in the nucleus. This has motivated extensive experimental work directed towards mapping this phenomena across the periodic table. While $M1$ strength in heavier nuclei has proved difficult to detect,[3] strong $M1$ transitions have been observed in most even-mass *sd*-shell nuclei.[4–9] Recently, the magnitudes of these excitations have become of increasing interest as constituting possible evidence for the participation of the nucleon-isobar mode in ostensibly nuclear excitations.[10–12] In attempting to isolate the effects of such an "unconventional" mechanism in distinction from the possible effects of several other more conventional processes, all of which can alter the observed total $M1$ excitation strength from the simplest theoretical expectations, it is essential that the conventional processes be examined with the greatest possible precision.

Our aim is to delineate as completely as possible the effects of configuration mixing within the orbitals of a single major shell upon the $M1$ excitation process. To this end we examine the case of ^{24}Mg. This example is advantageous in several respects. The shell-model wave functions for this region of nuclei have been checked to confirm that they reproduce the complete range of spectroscopic features with good accuracy. Since the selection rules for $M1$ excitation confine the transition amplitudes to lie within the *sd*-shell space, the present full-space wave functions can encompass the complete giant resonance strength. The density of states is low enough that the dominant portion of the strength is concentrated into the lowest few 1^+ levels, which facilitates both calculation and comparison to experiment. From the aspect of experimental knowledge, there are measurements of gamma and beta decay strengths involving these states and of their excitation probabilities via the (e,e') and (p,n) (proceeding to the isobaric analog states in ^{24}Al) reactions. Correlations between these data make possible detailed analysis of the structure of the transitions and of the validity of the shell-model wave functions with which we attempt to model them. Finally, the location of ^{24}Mg within the *sd*-shell is such that the system can serve as a paradigm for heavier systems, so that our conclusions may have some implications for the general case of $M1$ excitation. We shall examine the sensitivity of the predicted features of $M1$ excitation to the details of the shell-model wave functions, the relationship between the $M1$ and Gamow-Teller matrix elements for a given transition, and the relationships between the strengths of transitions at zero momentum transfer and at small and intermediate values of finite momentum transfer.

DISCUSSION OF CALCULATIONS

The shell-model wave functions we use to describe the initial and final states of ^{24}Mg have been obtained in new calculations[13] which treat the entire *sd* shell simultaneously with a single formulation of the Hamiltonian. In this formulation the one-body energies are assumed to be independent of A and the two-body matrix elements are scaled for application to each A value by the simple factor $(18/A)^{0.3}$. In order to gain an estimate of typical variations in the predictions for $M1$ phenomena which can result from different choices in the model Hamiltonian, we compare the predictions of the new Hamiltonian with those of the Chung-Wildenthal Hamiltonian,[14] which was obtained in a completely mass-independent treatment of the $A=17$–24 region. In Table I we list the values of the one-body transition matrix elements from the present calculation, which in the shell-model approximation completely characterize each of the transitions $J^\pi=0^+$, $T=0$ to $J^\pi=1^+$, $T=1$.

These one-body transition matrix elements (OBTME) can be combined with the single-particle matrix element (SPME) of the $M1$ and Gamow-Teller operators to yield predictions for values of $B(M1)$ and $B(\mathrm{GT})$ and used as input to electron scattering codes to yield predictions of the form factors for inelastic scattering. The predicted values of transition strengths depend not only upon the OBTME obtained in the shell-model calculation but also upon the choice of the values of the SPME which characterize the physical operator acting within the model context. The conventional choices for the SPME are obtained by normalizing the appropriate combinations of operators in angular momentum and isobaric-spin space so as to match the measured properties of the free neutron and proton. In the case of magnetic excitation processes the relevant nucleonic properties are the neutron and proton magnetic dipole moments. In the case of Gamow-Teller transitions, the relevant experimental property is the half-life of the neutron.

DISCUSSION OF RESULTS AND COMPARISON TO EXPERIMENT

The values of $B(M1)$ for the transitions between the first two $T=1$, $J^\pi=1^+$, states in ^{24}Mg and the $T=0$, $J^\pi=0^+$ ground state are experimentally known, as is the logft value for the analogous beta-decay transition between the metastable $J^\pi=1^+$ first excited state of ^{24}Al to the ^{24}Mg ground state.[15,16]

TABLE I. One-body transition density matrix elements for the transition from the lowest $T=0$ and $T=2$ states of $J^\pi=0^+$ to the lowest four states of J^π, $T=1^+$, 1 in $A=24$.

$$(2\Delta J+1)^{-1/2}(2\Delta T+1)^{-1/2}\langle 1^+,T_f=1,\#_f|||(a_j^\dagger\times a_{j'})_{\Delta J=1,\ \Delta T=1}|||0^+,T_i\rangle$$

	jj'	$\#_f=1$	2	3	4
$T_i=0$	$0d_{5/2}0d_{5/2}$	−0.24035	−0.16280	−0.11784	0.03306
	$0d_{5/2}0d_{3/2}$	−0.18750	−0.08589	0.06006	−0.08009
	$1s_{1/2}1s_{1/2}$	−0.05106	−0.01987	0.05096	−0.14779
	$1s_{1/2}0d_{3/2}$	−0.04746	0.15993	0.12446	0.06879
	$0d_{3/2}0d_{5/2}$	−0.01057	−0.22470	0.32792	−0.09588
	$0d_{3/2}1s_{1/2}$	−0.04787	−0.01227	0.06961	−0.03532
	$0d_{3/2}0d_{3/2}$	−0.02131	0.02039	−0.07015	−0.07538
$T_i=2$	$0d_{5/2}0d_{5/2}$	−0.36988	−0.10690	−0.05176	−0.29643
	$0d_{5/2}0d_{3/2}$	−0.03407	0.02957	−0.08874	−0.05728
	$1s_{1/2}1s_{1/2}$	−0.12887	−0.05493	0.10155	−0.18092
	$1s_{1/2}0d_{3/2}$	0.05132	0.00450	−0.06836	−0.07487
	$0d_{3/2}0d_{5/2}$	0.23248	−0.03709	0.14462	0.10628
	$0d_{3/2}1s_{1/2}$	0.04586	−0.08233	−0.02129	−0.07960
	$0d_{3/2}0d_{3/2}$	−0.00956	−0.00877	−0.01604	0.01509

TABLE II. Comparison between theoretical and experimental values of the transition strengths to 1^+, $T=1$ states of $A=24$.

State	$J\# = 1^+$, $T=1$; #1	$J\# = 1^+$, $T=1$ #2	$J\# = 1^+$, $T=1$; #'s 3,4
	Excitation energy in ^{24}Mg (MeV)		
Experimental[a]	9.966	10.712	13.30, 13.60[b]
Calculated (present)[c]	9.991	10.635	12.753, 13.142
Calculated (CW)[d]	10.218	11.114	13.301, 13.592
	$(0.591)\times B(M1)$, 0^+, $T=0$ to 1^+, $T=1$ (μ_N) (^{24}Mg)		
Experimental[e]	1.8±0.5[f]	3.3±0.4	
Calculated (present)[c]	0.59	1.94	0.27, 0.08
Calculated (CW)[d]	0.34	2.07	0.18, 0.14
	B(GT), 0^+, $T=0$ to 1^+, $T=1$ (^{24}Al$\leftrightarrow^{24}$Mg)		
Experimental[a]	0.038±0.011	0.82[g]	(0.31[g])
Calculated (present)[c]	0.090	1.51	0.82, 0.06
Calculated (CW)[d]	0.037	1.44	0.72, 0.24
	$(0.888)\times B(M1)$, 0^+, $T=2$ to 1^+, $T=1$ (μ_N) (^{24}Mg)		
Experimental[a]	1.00±0.24	0.36±0.23	
Calculated (present)[c]	1.14	0.54	0.45, 1.40
Calculated (CW)[d]	0.68	0.75	0.01, 1.14
	B(GT), 0^+, $T=2$ to 1^+, $T=1$ (^{24}Ne$\rightarrow^{24}$Na)		
Experimental[a]	0.261±0.007	0.243±0.013	
Calculated (present)[c]	0.28	0.38	0.67, 0.63
Calculated (CW)[d]	0.14	0.43	0.19, 0.53

[a]Reference 16.
[b]Energy values inferred from analog states in ^{24}Na.
[c]Reference 13.
[d]Reference 14.
[e]References 15 and 16.
[f]This value includes the strength of the 9.83 MeV 1^+, $T=0$ state, which we assume arises from isospin mixing with the 9.966 MeV state.
[g]Values inferred from (p,n) cross sections of Ref. 18.

The single beta-decay datum can be augmented by the results of the ^{24}Mg(p,n) reaction. It has been shown that the cross sections of the (p,n) reaction leading to $J=1^+$ states, measured at 0° and at intermediate bombarding energies, are closely proportional to the strengths of corresponding Gamow-Teller beta decay transitions.[17] From this proportionality relationship, the results of the ^{24}Mg(p,n) reaction[18] can be converted to yield equivalent GT strengths for higher lying 1^+, $T=1$ states in ^{24}Mg. In addition to these transitions involving the ground state of ^{24}Mg, the $M1$ and Gamow-Teller strengths from the 0^+, $T=2$ state of $A=24$ to the same 1^+, $T=1$ states are also known.[16] While these latter data do not have the same direct relationship to the inelastic scattering form factors as do the decays of the 1^+ states to the 0^+, $T=0$ ground state, they do offer a further critique of the shell-model wave functions which is very germane to the issue at hand.

The experimental values of these transition strengths are presented and compared to the shell model predictions for these quantities in Table II. In order to make an absolute comparison between the $M1$ and GT strengths of a given transition we have multiplied the $B(M1)$ values by the factor

$$R=\left[\frac{\sqrt{2}(g_A/g_v)\begin{pmatrix} T_f & 1 & T_i \\ -T_{zf} & \Delta T_z & T_{zi}\end{pmatrix}}{\left[\frac{3}{4\pi}\right]^{1/2}\frac{g_{sp}-g_{sn}}{2}\begin{pmatrix} T_f & 1 & T_i \\ -T_z & 0 & T_z\end{pmatrix}}\right]^2$$

so that within the confines of the shell model alone and in the limit $g_l=0$, $B(\mathrm{GT})=B(M1)$. Measured differences between the quantities $RB(M1)$ and B(GT) for a given transition thus can reflect both the orbital contribution to $M1$ transitions and differences in the mesonic-exchange contributions to the weak and electromagnetic processes.[19] The most significant general feature of the experimental results and of the corresponding theoretical predictions is that the transition strengths between the ^{24}Mg ground state and the lowest 1^+, $T=1$ states, which are the strongest such transitions either observed or calculated, are much smaller than the simple single-particle transition estimates. From a jj coupled, seniority-zero $(d_{5/2})^8$ model for the ground state of ^{24}Mg only two transitions to $J^\pi=1^+$, $T=1$ states exist. One proceeds by "spin-flip," for which only the $d_{5/2}$-$d_{3/2}$ OBTME would be nonzero, having a value of 0.8165 and a corresponding $B(M1)$ of 13.52 μ_N, and the other by "orbital recoupling," for which only the $d_{5/2}$-$d_{5/2}$ path has a nonzero amplitude, having an OBTME value of 0.3564 and a corresponding $B(M1)$ of 5.73 μ_N. Hence the observed values and the predictions of the configuration-mixed shell model are both quenched by a factor of roughly 5 from the simplest estimate. Moreover, the total strength predicted by the configuration-mixing model is only 6.6 μ_N, compared to the single-particle limit of 19.2 μ_N. Hence, the reduction in the excitation strengths of the lowest two states below the single-particle estimates is to be understood as due primarily to an overall quenching induced by correlations in the ^{24}Mg ground state rather than to a fragmentation of this strength over the many other, higher-lying 1^+ states in the model spectrum.

Comparison of $M1$ and Gamow-Teller strengths for the transitions between the ground state and the lowest three 1^+, $T=1$ states can yield information on the relative importance of the spin and orbital contributions to the $M1$ excitation process.[20,18] Such a comparison suggests that the transition to the first 1^+, state with its $M1$ strength much larger than its GT strength, is dominated by the orbital part of the $M1$ operator. Similarly, the transition to the second 1^+ state, with its comparable $M1$ and GT strengths, would seem to be dominated by the spin part of the operator. The detailed model predictions for these transitions are in good accord with these empirically-based suggestions. The predicted transition strength of the third 1^+ state is smaller in its $M1$ guise than in Gamow-Teller, which suggests an important orbital contribution, but one which cancels part of the spin contribution.

On the basis of the comparisons in Table II, no significant evidence exists that any other mechanism beyond the configuration-mixing shell model need be invoked to explain the experimental $M1$ data. On the other hand, the stronger GT transitions are on the average quenched almost 50% from the theoretical estimates. This difference between $M1$ and GT strengths is consistent with that deduced from the "single-particle" $A=17$, $J^\pi=5/2^+$, $T=1/2$ isovector magnetic moment and beta decay.[21,22] It has been shown that the mesonic-exchange and pair diagrams give rise to similar differences for $A=3$ (Ref. 19), but we are not aware of similar quantitative calculations of this specific effect in heavier nuclei. Beyond this general effect of GT quenching, the differences between theory and experiment in Table II are large only for some matrix elements which are less than about 1% of the simple single-particle estimates because of cancellation between various OBTME. Agreement within a factor of 2 for such cases is as good as should be expected.

As mentioned, the theoretical results incorporate the properties of free neutrons and protons via the single-particle matrix elements. The Chung-Wildenthal wave functions have been used to obtain alternatives to these "free nucleon" values of the SPME. These alternative SPME optimize the average agreement over the entire shell between shell-model predictions for magnetic moments and Gamow-Teller beta decay and experiment.[21,22] Use of these empirical SPME with the OBTME of Table I would tend to produce smaller transition strengths, with the quenching greater for GT than for $M1$, as noted above for the ^{24}Mg results.

We have not presented results obtained with such "empirical" SPME since the proper procedure for this would involve redetermining these "empirical" values with the present wave functions. These new values are not yet available and it remains to establish the degree of consistency between them and the analogous values determined with the Chung-

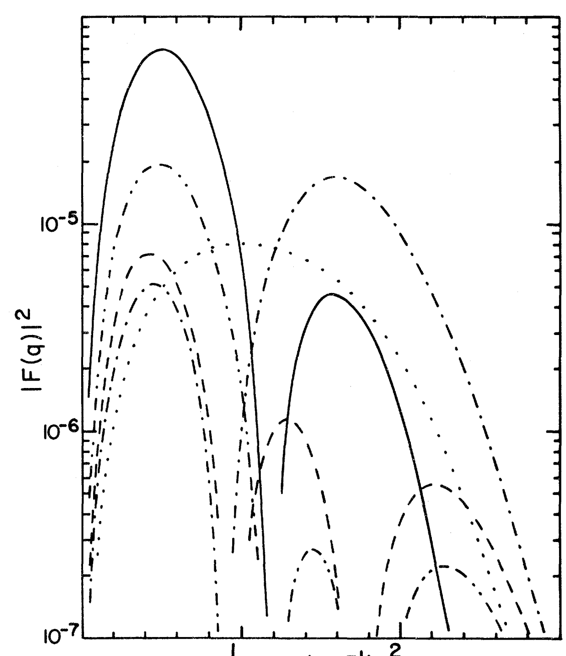

FIG. 1. Theoretical transverse form factors for the lowest five 0^+, $T=0\rightarrow1^+$, $T=1$ transitions in ^{24}Mg. Free-nucleon g factors were used. The curves shown correspond to the theoretical excitation energies (in MeV): 9.991 (—··—), 10.635 (—), 12.753 (— —), 13.142 (· · ·), and 14.033 (—·—).

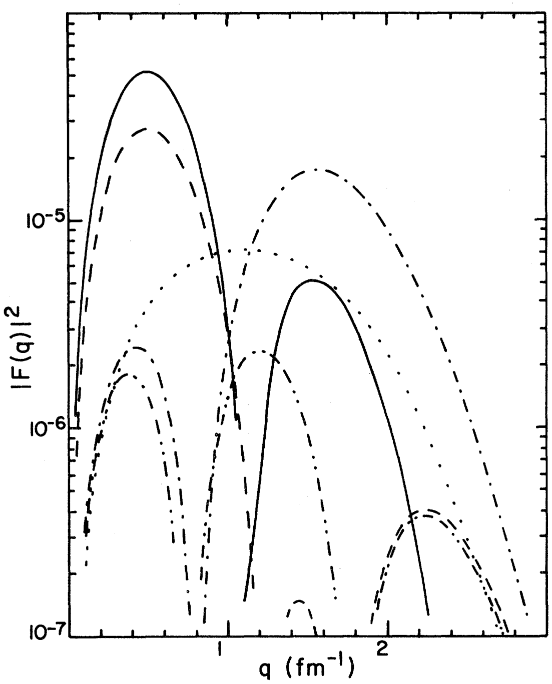

FIG. 2. Theoretical transverse form factors for the same transitions shown in Fig. 1. The only difference between the calculations shown here and those of Fig. 1 is that here we use orbital g factors $g_{lp} = g_{ln} = 0$.

Wildenthal wave functions. We would, at least, expect that their use not result in a deterioration of the agreement between experiment and theory shown in Table II.

It is possible that the momentum-transfer dependence of the strength of these spin-type transitions might provide a more discriminating key to unraveling the contributions to the excitation process which are not explicitly included in the shell-model calculation than is provided by the strengths at the $q = 0$ limit. We have used the one-body transition matrix elements of Table I as input to plane-wave Born approximation calculations[23] in order to obtain predictions for the inelastic electron-scattering form factors of these 1^+ states. These predictions are presented in Fig. 1. From Fig. 1 it is clear that feasible combinations of intra-shell one-body transition contributions can produce form factors with a wide variety of shapes. Hence, we would conclude that caution should be exercised in interpreting deviations of observed form factors from simple single-particle shapes as clear-cut evidence for unconventional effects. In Fig. 2 we show the form factors for these same states which result from setting the orbital part of the $M1$ operator to zero. This illustrates the effects of the relative importance of the

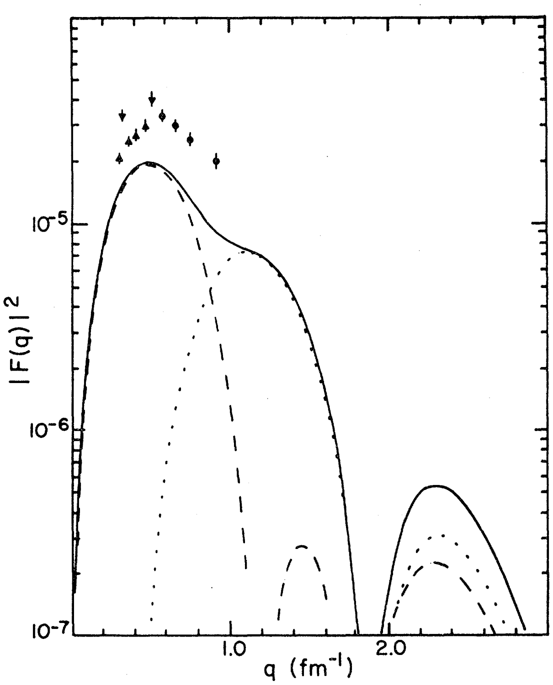

FIG. 3. Transverse form factor (solid line) for the transition to the $1_1^+ - 2_1^+$, $T = 1$ doublet in ^{24}Mg. The individual contributions from the 1_1^+ ($E_{th} = 9.991$ MeV) (— —) and 2_1^+ ($E_{th} = 10.131$ MeV) (· · ·) states are also shown. The experimental data are from Ref. 6.

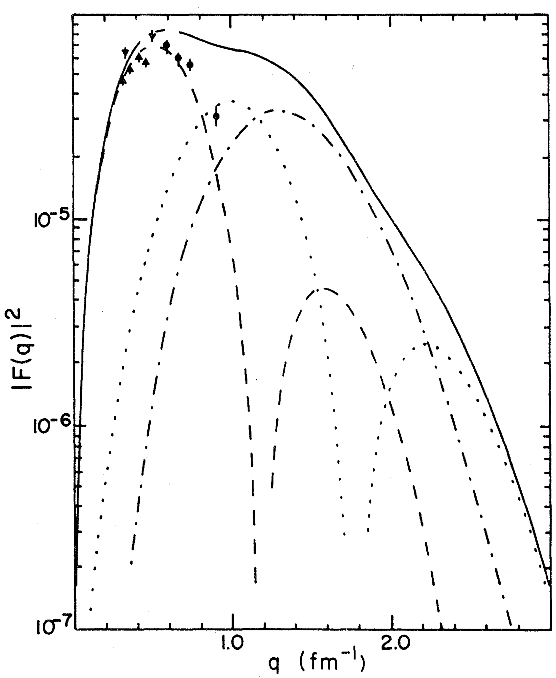

FIG. 4. Transverse form factor (solid line) for the transition to the $1_2^+ - 2_2^+ - 3_2^+$, $T = 1$ triplet in ^{24}Mg. The individual contributions from the 1_2^+ ($E_{th} = 10.635$ MeV) (— —), 2_2^+ ($E_{th} = 10.676$ MeV) (· · ·), and 3_1^+($E_{th} = 10.917$ MeV)(—·—·—·) are also shown. The experimental data are from Ref. 6.

spin and orbital components in these transitions that we previously mentioned.

In comparing such predicted form factors to experimental data, it is necessary in the practical case to take account of limitations in experimental resolution. Resolutions of ~ 100 keV, in conjunction with typical level densities at the excitation energy of the giant $M1$ resonance in light nuclei, can result in several states in addition to the desired 1^+ state being included in the observed cross sections. The form factors for these other states can be significant at higher values of momentum transfer, thus yielding an additional requirement for care in interpreting the experimental results. In Figs. 3 and 4 we compare the results of electron scattering experiments at rather low values of momentum transfer[6] to predictions of the present calculations. In these predicted form factors we have included the contributions of all states which should fall within the acceptance of the experimental resolution. It is evident that at larger momentum transfer the contributions of $M3$ and transverse $E2$ scattering are comparable to or larger than the $M1$ contributions.

SUMMARY

We have studied examples of "strong" $\Delta T=1$, $0^+ \rightarrow 1^+$ transitions as they are found in the $A=24$ system by comparing a variety of the observed properties of the 1^+ states to the predictions of a "complete" classical shell model calculation. The measured $q=0$ properties of the $\Delta J=1$, $\Delta T=1$ transitions involving these states are in qualitative terms well reproduced by the model wave functions. By "qualitative" we mean the magnitudes of transition strength relative to simple estimates, the relative strengths from state to state, and the relative strengths for $M1$ and Gamow Teller decays. Quantitative discrepancies between experiment and theory seem explainable in terms of errors in some of the experimental values[24] and in the need for global, state-independent renormalizations of the $M1$ and Gamow-Teller operators to compensate for the limitations of the classical shell model. Our conclusion is that we probably can reproduce in quantitative detail the $q=0$ decay properties of the lowest two 1^+, $T=1$ states in $A=24$. From this standpoint, we ask how far this grasp of $q=0$ details will allow us to predict what happens in these transitions at $q \neq 0$.

We note first that the $M1$ form factors are very sensitive to the structure of the specific shell model wave functions and to the relative contributions of the orbital and spin parts of the $M1$ operator. We conclude that in view of this sensitivity, caution should be exercised in attributing "anomalies" in $M1$ from factors to sources more exotic than shell model configuration mixing. We note, in addition, that at larger values of momentum transfer, transitions of higher multipolarity compete in strength with $M1$ transitions. Hence, within practical limits of experimental energy resolution, caution must be exercised in attributing form-factor strength at large q to the $M1$ transition that dominated that energy bite at lower q.

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