Radiative electron-capture decay of ^{49}V

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Inner bremsstrahlung accompanying electron-capture decay of ^{49}V has been measured with a (4.5×5.1)-cm NaI(Tl) scintillation spectrometer in the energy region 180–580 keV. The counts spectrum was unfolded following the procedure of Lidén and Starfelt. Results were compared with the $(1s+2s)$-capture relativistic calculations of Martin and Glauber. At energies higher than 340 keV, the shape of the spectrum agrees with the predictions of the theory. At energies less than 340 keV, experimental values are lower than the theoretical values.

[RADIOACTIVITY ^{49}V measured IB $(1s+2s)$; NaI(T1) detector; spectrum shape.]

I. INTRODUCTION

The inner-bremsstrahlung (IB) spectrum accompanying orbital electron capture (EC) by a nucleus is weak continuous electromagnetic radiation. IB emission in EC is a second-order process in perturbation theory that induces both beta and electromagnetic transitions. These transitions connect the initial and final states of the system through a set of virtual intermediate states. Møller[1] and Morrison and Schiff[2] independently developed a nonrelativistic theory of IB spectra in allowed transitions, neglecting the influence of the nuclear Coulomb field on the capture process. The energy spectrum of the electromagnetic radiation was calculated to be continuous up to some maximum energy, namely, the end point. This theory is restricted to 1s-electron capture and predicts an energy spectrum of the form $k(k_0-k)^2$, where k is the energy of the emitted photon and k_0 is the end point. Early experimental IB (EC) data obtained at higher energy were consistent with the Morrison-Schiff theory, whereas measurements that included the low-energy region showed an unexpected steep rise in the intensity.[3] Relativistic calculations for allowed transitions were made by Martin and Glauber.[4] This theory successfully explained the observed excess of low-energy photons as due to the capture of p electrons. It provides approximate expressions for the calculation of partial spectra corresponding to capture of electrons from different shells. The energy spectra of IB are mainly determined by s radiation ($l=0$ capture) for energies greater than αz and by p radiation ($l=1$ capture) at low energies. The shape of the s-radiation spectrum can be described to a good approximation by the formula of Morrison and Schiff

$$N(k)=C(k)k(k_0-k)^2 . \qquad (1)$$

This relation permits the possible application of a Jauch plot for the determination of k_0. Changes in the shape due to Coulomb effects may, however, limit the accuracy of this method.

IB spectra have been measured mostly for electron-capture isotopes decaying through ground-to-ground transitions, where k_0 is large. IB has also been investigated in transitions where the daughter nucleus may be left in an excited state by partial EC. In these cases, the gamma ray that follows EC has an energy much less than k_0 corresponding to the ground-to-ground transition.[5–7] Coincidence techniques can be employed in the study of IB from such EC isotopes.[8]

The agreement between theory and experiment is not completely satisfactory.[9] The shapes of total spectra and the partial spectra have been measured for a number of allowed transitions and have been found to be in agreement with theory.[10] However, the intensities of these spectra measured using IB-gamma coincidences show deviations from the theoretical predictions.[10,11] Only Kadar *et al.*[12] report agreement with theory.

The IB accompanying ground-to-ground allowed EC decay of ^{49}V ($\Delta J=0$, $\Delta\pi=$no, $\log ft=6.2$) has been measured only once previously by Hayward and Hoppes[13] with a well-type NaI(T1) gamma-ray spectrometer. The source was prepared inside a celluloid tube and kept inside the detector well. The results of these authors were compared with the nonrelativistic theory of Glauber and Martin. Hayward and Hoppes[13] followed the differential method of Owen and Primakoff[14] for the resolution correction. By normalizing the results at 200 keV with the total spectrum of Glauber and Martin,[15] the authors[13] found agreement with the nonrelativistic theory within the experimental error except, perhaps, at the very lowest energy where the corrections were quite high. Since Hayward and Hoppes[13] used a thick celluloid tube as a source container, absorption of IB in the container could not be neglected. Furthermore, an accurate estimate of the efficiency of a well-type crystal is difficult to make. Because of the fact that IB from ^{49}V has been investigated only once before, it was thought worthwhile to reinvestigate the same with a different type of source following a different procedure for unscrambling the measured spectrum, and to compare the experimental results with the relativistic theory of Martin and Glauber.

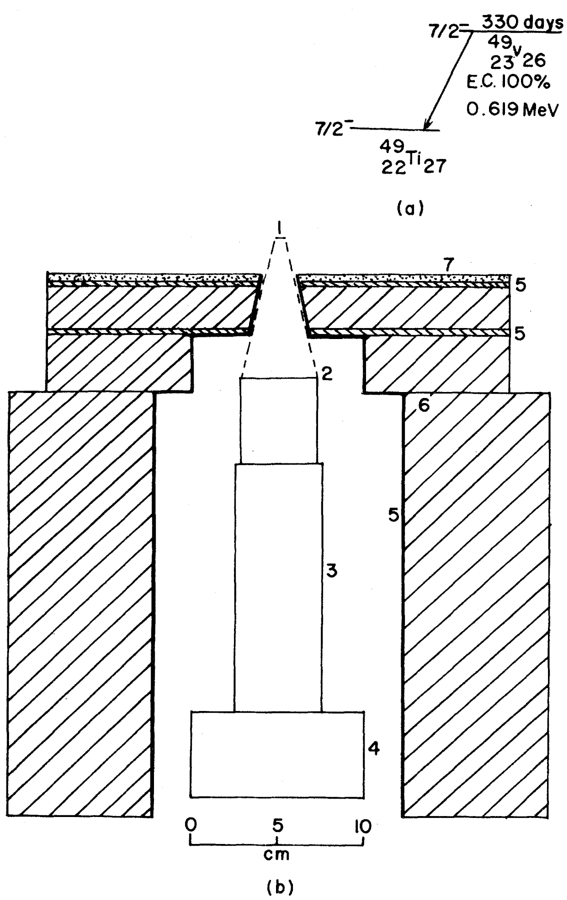

FIG. 1. (a) Decay scheme of ^{49}V. (b) Experimental setup; (1) source, (2) (4.5×5.1)-cm NaI(Tl) crystal, (3) photomultiplier, (4) cathode follower and preamplifier, (5) aluminum lining, (6) lead, and (7) Perspex sheet.

II. EXPERIMENT

The decay scheme[16] of ^{49}V and the experimental setup are shown in Fig. 1. The IB spectrum accompanying ground-to-ground EC in this isotope was measured in the energy region 180−580 keV with a (4.5×5.1)-cm NaI(Tl) scintillation counter coupled to a multichannel analyzer with 256 channels. The source was obtained in the form of carrier-free vanadyle chloride solution from Radiochemical Centre, Amersham, United Kingdom. A known quantity of the solution was evaporated drop by drop on a thin Mylar film ($\sim$1.7 mg/cm^2) mounted on a Perspex ring of 2.4-cm inner diameter and 1-mm wall thickness. Sufficient care was taken to get a uniform spread by adding drops only after a regular drying interval. The spread of the source was limited to a circular area of 0.5-cm diameter. Two sources of low activity ($\approx$10 μCi) were used in the present study.

Counts were accumulated once with the source in position and again without it. Counting time was varied from 5 to 20 h. The background level was minimized by housing the NaI(Tl) crystal and photomultiplier assembly in a lead shield of 8.4-cm wall thickness. The linearity of the spectrometer was checked with the following gamma-ray lines: 84 keV (^{170}Tm), 123 keV (^{57}Co), 145 keV (^{141}Ce), 280 keV (^{203}Hg), 412 keV (^{198}Au), 662 keV (^{137}Cs), and 835 keV (^{54}Mn). The stability of the counting system was ensured by using a stabilized power supply and maintaining the laboratory temperature constant at 23±1 °C. Stability was checked before and after each individual run with the 662-keV gamma ray line of ^{137}Cs. Since IB is of very low intensity, the background was recorded for the same time before and after each measurement. This is done because a measurable amount of background can originate in radioactivity carried by air, in the form of trace amounts of radioactive gases such as ^{222}Rn and ^{220}Rn which are short-lived ones in the uranium and thorium chains present in the construction materials of the laboratory.[17] From the data accumulated in several runs, the average of ten consistent readings was considered for final evaluation of the spectrum. Figure 2 shows the raw IB spectrum with the corresponding background spectrum recorded for the same time. Although the spectrum was recorded for the entire energy range, the data

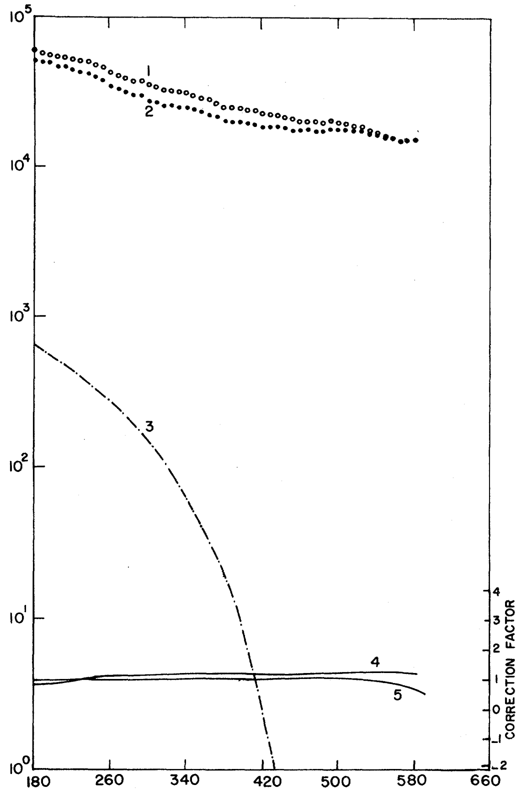

FIG. 2. (1) Experimental pulse-height distribution, (2) background, (3) Compton electron distribution, (4) correction factor for energy resolution, and (5) correction factor for detection efficiency.

for the final analysis were limited to the energy region of 180–580 keV in order to be compared with the relativistic theory restricted to s capture only. In this isotope the contribution to total IB due to p capture is negligible[10] beyond 180 keV, and the measured IB is mostly due to 1s and 2s capture.

III. DATA ANALYSIS

The measured pulse-height distribution is due to the folding of the incident spectrum with the detector response function. Hence, in order to get the original spectrum, the raw spectrum has to be unscrambled. In the present analysis the method due to Liden and Starfelt[18] is followed. After subtracting the background from the measured spectrum, corrections for finite energy resolution were applied using the relation

$$N(E)=\int_0^\infty \frac{N'(E_\gamma)}{(2\pi K E_\gamma)^{1/2}} e\left[\frac{-(E_\gamma-E)^2}{2KE_\gamma}\right] dE_\gamma\ , \quad (2)$$

where $N(E)$ is the background-corrected spectrum, $N'(E_\gamma)$ is the distribution after correcting for energy resolution and we have

$$K=W^2(E_\gamma)/2E_\gamma \ln 2\ ,$$

where $2W$ is the full width at half maximum. As a first approximation, the background-corrected distribution was substituted for $N'(E_\gamma)$ and the integration was carried out numerically. The resultant distribution obtained after integration was again substituted and the iteration process was repeated until convergence was reached. This method was found to be more appropriate than using the simple differential formula of Morton,[19] which is only valid at high energies. The resolution-corrected spectrum was then subjected to correction for the Compton-electron distribution and gamma-detection efficiency. The detector efficiency was calculated for the experimental geometry following Wolicki *et al.*[20] but using recent values for the gamma absorption coefficients compiled by Hubbell.[21] The correction due to back scattering was found to be negligible. Figure 2 also shows the Compton-electron distribution and the correction factors for geometric detection efficiency and energy resolution.

IV. COMPARISON WITH THEORY

According to the theory of Martin and Glauber for (1s + 2s) IB, we have

$$W_{\rm IB}^{\rm th}(k)dk=\frac{\alpha}{\pi}k\,dk[P_K(1-k/k_0{}^{1s})^2R_{1s}(Z,k)$$
$$+P_{L_1}(1-k/k_0{}^{2s})^2R_{2s}(Z,k)]\ , \quad (3)$$

where $W_{\rm IB}^{\rm th}(k)dk$ is the IB spectrum per electron capture decay between k and $k+dk$: α is the fine-structure constant; k is the photon energy in units of the electron rest energy; P_K and P_{L_1} are relative capture probabilities in 1s and 2s shells respectively; $R_{ns}(Z,k)$ are the relativistic correction factors; and k_0^{ns} are the end-point energies of the partial ns-IB spectra. The quantity k_0^{ns} is the difference between the transition energy and the corresponding binding energy of the captured electron. The correction factors R_{ns} were calculated from the theory of Martin and Glauber.[3] The most recent value for the transition energy[14] was used in the calculation of the IB spectrum. Experimental values[22] for the ratios of the relative capture probabilities $P_{L_1}/P_K=0.088$ and $P_{L_2}/P_K=0.0003$ were used for finding P_K and P_{L_1}.

For comparing the measured $(1s+2s)$ IB spectrum with theory, Eq. (2) can be written in terms of the Coulomb-free appoach of Morrison and Schiff by introducing an overall shape factor $R_s(Z,k)$ for the s-IB spectrum in the following form

$$W_{\rm IB}^{\rm th}(k)dk = R_s(Z,k)W_{\rm IB}^{\rm CF}(k,k_0{}^{1s})dk \ . \quad (4)$$

Here,

$$W_{\rm IB}^{\rm CF}(k,k_0{}^{1s})dk = \frac{\alpha}{\pi}k(1-k/k_0{}^{1s})^2 dk \quad (5)$$

and

$$R_s(Z,k) = P_K R_{1s} + P_{L_1} R_{2s} h(k) \ , \quad (6)$$

where

$$h(k) \approx 1 + k_{Kx}/(k_0{}^{1s}-k)^2 \ , \quad (7)$$

where k_{Kx} is the difference between the binding energy of the $1s$ and $2s$ electrons. The overall shape factor was modified by including a screening correction for calculating the theoretical spectrum.

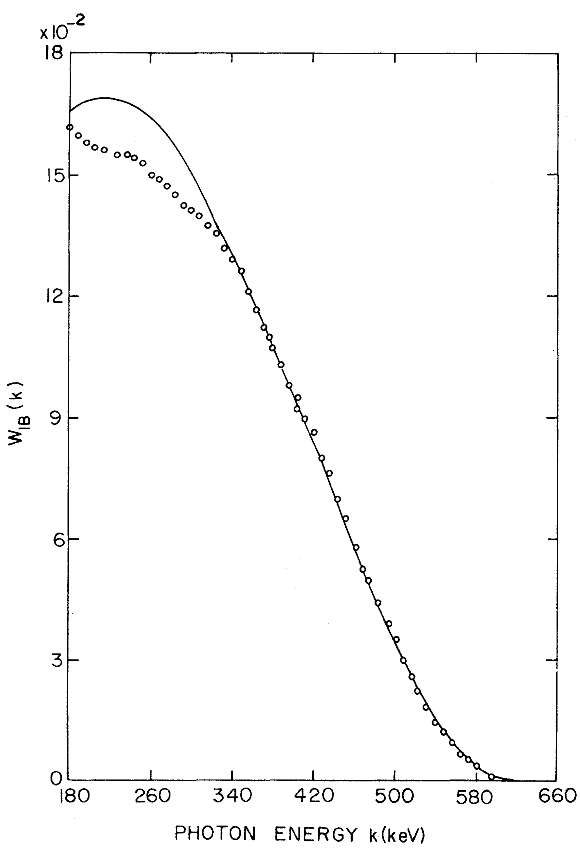

FIG. 3. IB photon spectrum $W_{\rm IB}(k)$ of ^{49}V per ground state EC decay. The open circles (○ ○ ○) show the experimental spectrum. The full line represents the theoretical spectrum calculated from Glauber-Martin theory.

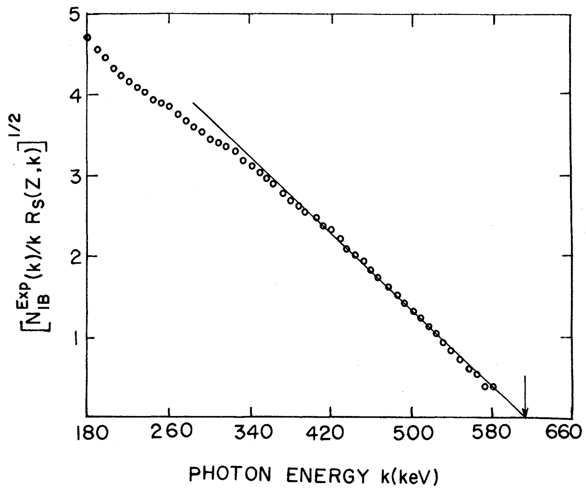

FIG. 4. Jauch plot $N_{\rm IB}(k)/kR_s(Z,k)^{1/2}$ vs k of the IB spectrum of ^{49}V, yielding an end point energy 613 ± 7 keV.

V. RESULTS AND DISCUSSIONS

Figure 3 shows the comparison of the measured IB spectrum per electron capture decay with the theoretical spectrum normalized to an energy interval of 1 keV. Figure 4 shows the Jauch plot of the IB spectrum. It can be seen that the Jauch plot has linearity in the higher-energy region showing the spectral shape essentially to be of the form given by the theory of Morrison and Schiff. It is for this reason that the experimental distribution was normalized to the theoretical spectrum at 380 keV.

Large counting times were used in order to keep errors as low as 2% to 5% in the counting statistics between the lower and higher limits in the energy range of our investigation. The error involved in the estimation of detection efficiency is about 2%, whereas the combined error introduced in resolution and Compton distribution corrections would be less than 4%. Therefore, the overall error in the final

TABLE I. Measured IB counting rates $N_{\rm IB}^{\rm exp}(k)$ per ground state EC decay of ^{49}V compared with those calculated from the theoretical spectrum.

Energy (keV)	$N_{\rm IB}^{\rm exp}(k)\times10^4$	$N_{\rm IB}^{\rm th}(k)\times10^4$
180–580	38.28	39.99
260–580	26.02	26.59
340–580	14.83	14.66
420–580	6.18	6.10
500–580	1.33	1.38

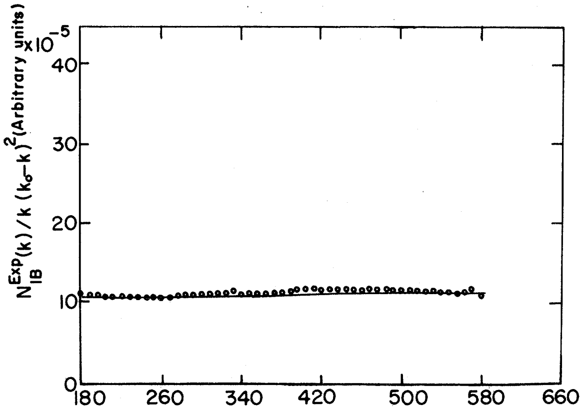

FIG. 5. The overall shape factor of the ^{49}V IB spectrum in the energy region 180–580 keV. The open circles (○○○) show the experimental points. The full line represents the theoretical prediction.

data ranges from 5% at 180 keV to 14% at 560 keV.

Although the agreement between experiment and theory is good in the high-energy region, there is deviation in the lower-energy region. Similar trends were also observed in earlier measurements[7,9] on ^{7}Be where agreement between theory and measurement was expected to be quite close. Earlier results[13] on ^{49}V, however, showed agreement with theory. It is hard to account for the discrepancy between the present measurements and the results of Hayward and Hoppes. It would be of interest to see the comparison of the experimental results, with the theory of Martin and Glaubers, worked out for all orders of αz including a more realistic representation for the nuclear charge.

The numerical values of the integral rates

$$N_{IB}(E_1)=\int_{E_1}^{580} W_{IB}(k)dk$$

are given in Table I and compared with the corresponding theoretical values. Figure 5 shows the overall shape factor for s-IB as a function of photon energy. It can be seen from this figure that the energy dependence of the shape factor cannot be conclusively established from the present data.

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