PHYSICAL REVIEW C VOLUME 27, NUMBER 3 MARCH 1983

Autoionization accompanying emission of internal bremsstrahlung in K capture

K. Pisk and M. Krčmar
Rudjer Bošković Institute, Zagreb, Yugoslavia

B. A. Logan
Department of Physics, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5
(Received 18 October 1982)

We have calculated the probability of a process in which internal bremsstrahlung associated with K electron capture in an allowed transition is accompanied by the simultaneous ejection of the uncaptured K electron. Consideration has been given to the case where the ejected electron is produced by the sudden change in the nuclear Coulomb field in the capture process, and to ejection produced by interactions between the two K electrons. The relative importance of the respective contributions is discussed.

[RADIOACTIVITY Calculated simultaneous IB and autoionization in EC; ^{37}Ar, ^{55}Fe.]

I. INTRODUCTION

Both internal bremsstrahlung (IB) and autoionization are phenomena which have been studied extensively in electron capture decays. A comprehensive review of electron capture decays which also discusses IB and autoionization processes is available.[1] Recent reviews of autoionization in electron capture have also been made.[2,3] It is possible for IB and autoionization to occur simultaneously, and while it is expected that this higher order process will have a low probability, autoionization accompanying IB has recently been observed in β decay.[4]

We have calculated the probability of an internal bremsstrahlung emission (IBE) process, which involves emission of IB associated with K electron capture being accompanied by the simultaneous ejection of the uncaptured K electron. The IB can be emitted from either the electron involved in the capture process or from the electron ejected in the autoionization process. Although there are discrepancies between different calculations of the autoionization process, there is general agreement that the probability of autoionization is strongly reduced at higher ejected electron energies. If we limit ourselves to higher energy photons it is reasonable to neglect contributions to the IB from the electrons ejected in the autoionization process.

It is expected that two mechanisms will contribute to the IBE process. The first is due to the sudden change in the nuclear Coulomb field resulting from the K capture. The second is due to the interaction between the K electrons during the IB process in K capture. The autoionization due to the electron interaction is often described in terms of a direct collision, but it can also be regarded as a process of double internal bremsstrahlung in K capture, where one of the emitted photons is absorbed by the ejected electron. Both theoretical and experimental investigations of double internal bremsstrahlung in electron capture have been made.[5–8] Our calculations are based on a nonrelativistic description of the electrons and on the Morrison-Schiff[9] approach to the IB process in K capture. We have employed the perturbation method introduced by Intemann and Pollock[10] to treat the electron-electron interaction.

II. CALCULATION OF THE MATRIX ELEMENT

The lowest order Feynman diagrams for the IBE process are shown in Fig. 1. The same order diagrams representing the emission of radiation from the ejected electron lines are omitted. Figure 1(a) represents the contribution due to the sudden change in the nuclear Coulomb field during the K-capture process in which IB is emitted. Figures 1(b) and (c) represent the contribution from the interaction between the two electrons during the course of the IB process in K capture.

The matrix elements corresponding to the diagrams of Fig. 1 are derived by using a nonrelativistic description for the electrons and the Morrison-Schiff approach to the IB process. In effect, we use Schrödinger-type wave functions for the initial and final electron states; we assume an instantaneous Coulomb interaction between the electrons, and a

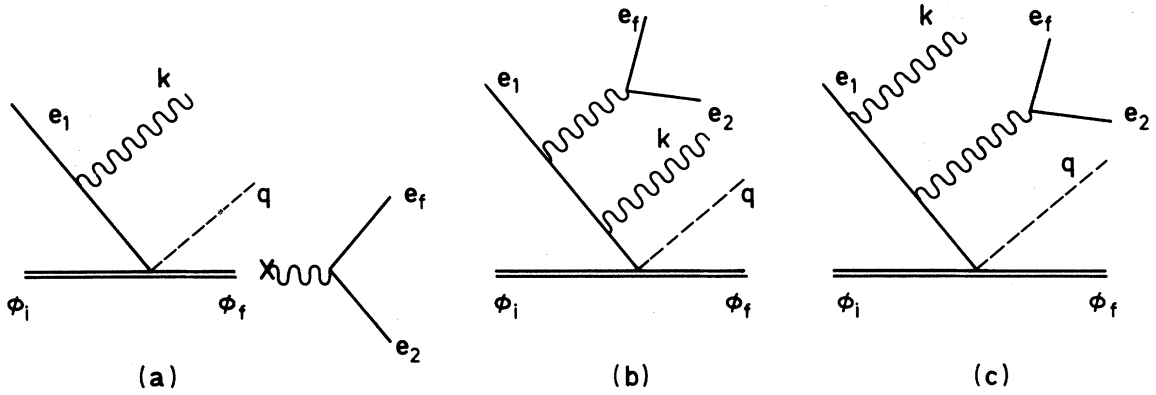

FIG. 1. Feynman diagrams for the IBE process. The same order diagrams representing the emission of radiation from the ejected electron lines are omitted.

Martin-Glauber scalar Green's function[11,12] for the propagator of those electrons which strongly experience the nuclear Coulomb field. The Morrison-Schiff approach assumes the field-free Dirac propagator for electrons after the photon emission takes place and only zero-momentum contributions to the electron wave function. These approximations are reasonable for relatively small electron energies and relatively large photon energies. When taken together these approximations are consistent with omission of the diagrams describing the photon emission from the ejected electron.

For allowed transitions, when we can take the lepton current at the origin and with the above considerations, the matrix elements M_a, M_b, and M_c, corresponding to Figs. 1(a), 1(b), and 1(c), respectively, can be written in the standard V-λA weak interaction theory as follows:

$$M_a = 2\pi e C_v \delta(E_{\max}+\epsilon_0-k-q-\epsilon)(1-P_{12})\chi_f^\dagger \chi_2 N_\mu \times \int d\vec{r}_1 \bar{\Psi}_\nu(0)\Lambda_\mu S(0,\vec{r}_1)\hat{a}^\dagger(\vec{r}_1)\phi_0(\vec{r}_1)\chi_1 \int d\vec{r}_2 \phi_f^{*(Z-1)}(\vec{r}_2)\phi_0(\vec{r}_2)\,, \tag{1}$$

$$M_b = 2\pi e\alpha C_v \delta(E_{\max}+\epsilon_0-k-q-\epsilon)(1-P_{12})\chi_f^\dagger \chi_2 N_\mu \times \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 \bar{\Psi}_\nu(0)\Lambda_\mu S(0,\vec{r}_1)\hat{a}^\dagger(\vec{r}_1)G(\epsilon_b;0,\vec{r}_3)\phi_0(\vec{r}_3)\chi_1 \frac{1}{|\vec{r}_3-\vec{r}_2|}\phi_f^*(\vec{r}_2)\phi_0(\vec{r}_2)\,, \tag{2}$$

$$M_c = 2\pi e\alpha C_v \delta(E_{\max}+\epsilon_0-k-q-\epsilon)(1-P_{12})\chi_f^\dagger \chi_2 N_\mu \times \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 \bar{\Psi}_\nu(0)\Lambda_\mu G(\epsilon_c;0,\vec{r}_3)S(\vec{r}_3,\vec{r}_1)\hat{a}^\dagger(\vec{r}_1)\phi_0(\vec{r}_1)\chi_1 \frac{1}{|\vec{r}_3-\vec{r}_2|}\phi_f^*(\vec{r}_2)\phi_0(\vec{r}_2)\,. \tag{3}$$

We have employed the units $m=c=\hbar=1$ (m is the electron mass), $e^2/4\pi=\alpha=\frac{1}{137}$, and the Dirac matrices are $\vec{\gamma}=-i\beta\vec{\alpha}$, $\gamma_4=\beta$, $\gamma_5=\gamma_1\gamma_1\gamma_3\gamma_4$, and $\bar{\phi}=\phi^\dagger\gamma_4$, $\Lambda_\mu=\gamma_\mu(1+\lambda\gamma_5)$, C_v is the weak interaction coupling constant, and $\lambda=|C_A/C_V|$.

The scalar product of the four vectors $a_\mu=(\vec{a},ia_0)$ and $b_\mu=(\vec{b},ib_0)$ is $a\cdot b=a_\mu b_\mu=\vec{a}\cdot\vec{b}-a_0b_0$ and $\hat{a}=a_\mu\gamma_\mu$.

In Eqs. (1)—(3) the initial K electrons space-part wave functions are ϕ_0, and ϵ_0 is the total K electron energy. The ejected electron space-part wave function is ϕ_f, and ϵ is the total ejected electron energy. $\phi_f^{(Z-1)}$ is the space-part wave function of the ejected electron which has a total energy ϵ and is in a Coulomb field of $(Z-1)$ units of charge. P_{12} is an electron exchange operator which ensures the antisymmetrization of the two K electrons in the initial state and applies to the constant electron spinor functions $\chi_{1,2}$. S represents the field-free Dirac propagator, and G is a Martin-Glauber scalar Green's function as defined in the Intemann-Pollock paper.[10] The neutrino wave function Ψ_ν is characterized by a momentum $\vec{q}$ and energy $q=|\vec{q}|$, and a photon wave $\vec{a}$ is characterized by a polarization vector $\vec{e}$, a momentum $\vec{k}$, and an energy $k=|\vec{k}|$. $E_{\max}$ is the energy available in the K capture process and N is a nonrelativistic nuclear matrix element. From energy conserva-

tion we also have the following:

$$\epsilon_b = 2\epsilon_0 - \epsilon \ , \qquad \epsilon_c = 2\epsilon_0 - \epsilon - k \ . \tag{4}$$

Using the method introduced by Intemann and Pollock,[10] we can express the electron wave function $\phi^{(Z-1)}$ through a perturbation series of a $\phi^{(Z)}$ functions. We obtain for M_a

$$M_a = 2\pi e\alpha C_v \delta(E_{\max} + \epsilon_0 - k - q - \epsilon) \frac{A}{\epsilon - \epsilon_0} (1 - P_{12}) \chi_f^\dagger \chi_2 N_\mu \int d\vec{r}\, \bar{\Psi}_\nu(0) \Lambda_\mu S(0, \vec{r}) \hat{a}^\dagger(\vec{r}) \phi_0(\vec{r}) \chi_1 \ , \tag{5}$$

where A is an integral defined as

$$A = \int d\vec{r} \frac{1}{r} \phi_f^*(\vec{r}) \phi_0(\vec{r}) \ . \tag{6}$$

A further simplification of M_c can be achieved by retaining only the first term in the multiple expansion of $1/|\vec{r}_3 - \vec{r}_2|$ in Eq. (3). This is reasonable, as the effective ranges of the $\vec{r}_3$ and $\vec{r}_2$ integrations in Eq. (3) are on the electron Compton wavelength and atomic scale, respectively. After this simplifcation M_c can be written

$$M_c = 2\pi e\alpha C_v \delta(E_{\max} + \epsilon_0 - k - q - \epsilon) A (1 - P_{12}) \chi_f^\dagger \chi_2 N_\mu \int d\vec{r}\, \bar{\Psi}_\nu(0) \Lambda_\mu S'(0, \vec{r}) \hat{a}^\dagger(\vec{r}) \phi_0(\vec{r}) \chi_1 \ , \tag{7}$$

where the spinor function S' is defined and calculated in the Appendix.

The total matrix element M for the IBE process is now expressed through Eqs. (2), (5), and (7) as

$$M = \left[\frac{2}{k}\right]^{1/2} \pi e\alpha C_v \delta(E_{\max} + \epsilon_0 - k - q - \epsilon)(1 - P_{12}) \chi_f^\dagger \chi_2 N_\mu \bar{v}(\vec{q}) \Lambda_\mu$$
$$\times \left\{ \phi_0 \left[\frac{1}{\epsilon - \epsilon_0} + f(\vec{k}^2) \right] A \frac{i\hat{\tau}_1 - 1}{\tau_1^2 + 1} + B \frac{i\hat{\tau}_2 - 1}{\tau_2^2 + 1} \right\} \vec{e} \cdot \vec{\gamma} \chi_1 \ . \tag{8}$$

Here v is the neutrino spinor function and four-vectors τ are

$$\tau_1 = (-\vec{k}, i(1 - k)) \ , \qquad \tau_2 = (-\vec{k}, i(1 - k - \epsilon + \epsilon_0)) \ . \tag{9}$$

The quantity B in Eq. (8) is defined as a double integral

$$B = \int d\vec{r}_1 d\vec{r}_2 G(\epsilon_b; 0, \vec{r}_1) \phi_0(\vec{r}_1) \frac{1}{|\vec{r}_1 - \vec{r}_2|} \phi_f^*(\vec{r}_2) \phi_0(\vec{r}_2) \ . \tag{10}$$

The integrals A and B and the function f are given in the Appendix.

The differential probability for the IBE process $dW_{\rm IBE}$ is obtained, for unpolarized photons and electrons, and unoriented nuclei, by squaring the matrix element M and multiplying by the density of final states. The standard procedure of averaging over the initial spin states and summing over final spin states is performed and the neutrino's degrees of freedom are also summed over. The result, normalized to the K electron capture rate W_K, is as follows:

$$\frac{dW_{\rm IBE}}{W_K dE\, dk} = \frac{2^5 \alpha^3 a^4}{\pi} \frac{(E_{\max} - k - E - a^2/2)^2}{E_{\max}^2} \frac{k(E+1)}{e^{2\pi a/p} - 1}$$
$$\times \left\{ \left[h(E, \vec{k}^2) I_a + \frac{k}{\kappa} I_b \right]^2 + \left[h(E, \vec{k}^2) I_a + \frac{k + E + a^2/2}{\kappa} I_b \right]^2 \right\} , \tag{11}$$

where p and E are the electron nonrelativistic momentum and energy, respectively, and $a = \alpha Z$. In Eq. (11) we have introduced the functions h and κ

$$h(E, \vec{k}^2) = 1 + (E + a^2/2) f(\vec{k}^2) \ , \tag{12}$$

$$\kappa = k(1 - E - a^2/2) + E + a^2/2 - (E + a^2/2)^2/2 \ . \tag{13}$$

The functions I_a and I_b are given in the Appendix.

III. DISCUSSION OF THE RESULTS

Equation (11) indicates that there is no angular correlation between the emitted photons and the electrons in the IBE process. For the contribution from the sudden change in the Coulomb field this is obvious, as the angular degrees of freedom of the photons and electrons are separated. In the case of the electron-electron interaction when one of the two photons in double internal bremsstrahlung is absorbed by the electron, the lack of an angular correlation is because our approximations only allow s-intermediate state contributions to M_b and M_c. However, an experimental investigation of the angular correlation of photons emitted in double internal bremsstrahlung with ^{37}Ar showed the correlation was weak.[7]

In Fig. 2 the ejected electron spectra, normalized to K capture events, for the IBE process for the allowed decays of ^{37}Ar and ^{55}Fe are shown. An energy range of $0.2mc^2$ to $E_{\max}-a^2/2$ has been summed over for the emitted photons. The spectra are similar to those obtained in the autoionization process, without IBE, and show a strong energy dependence.

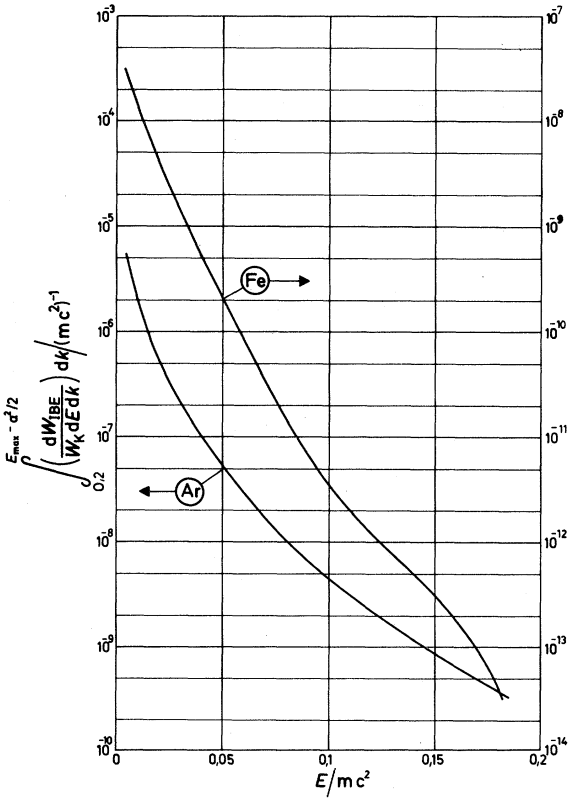

FIG. 2. Ejected electron spectra for the IBE process for the allowed decays of ^{37}Ar and ^{55}Fe normalized to K capture events.

In Fig. 3 we present h, as defined by Eq. (12) for various photon and electron energies. This function represents the interference between M_a and M_c, a unit value of h corresponding to the contributions being limited to M_a. At higher photon energies the shake-off mechanism is dominant.

In Fig. 4 the ratio I_b/I_a is shown. In the limit of $E/k\rightarrow 0$ this ratio measures M_b/M_a. Our result is in agreement with the conclusion of Intemann[13] for autoionization without IBE, in which he showed that the intermediate states had to include contributions from the continuum. Paquette,[14] in the case of IB in K capture, has also shown the importance of the continuum contributions to the intermediate states. Our results show that when the electron-electron interaction occurs before IBE it makes a significant contribution to the IBE process. Overall, as in the case of autoionization without IBE, our results indicate a destructive interference between M_a, and M_b and M_c, can be deduced.

IV. CONCLUSIONS

Our results give an estimate of the probability of IBE in allowed transitions which may serve as a guide for experimental investigations. The accuracy of our calculations is limited by such factors as neglecting the relativistic description of the electrons and by our simplified analysis of the IB process.

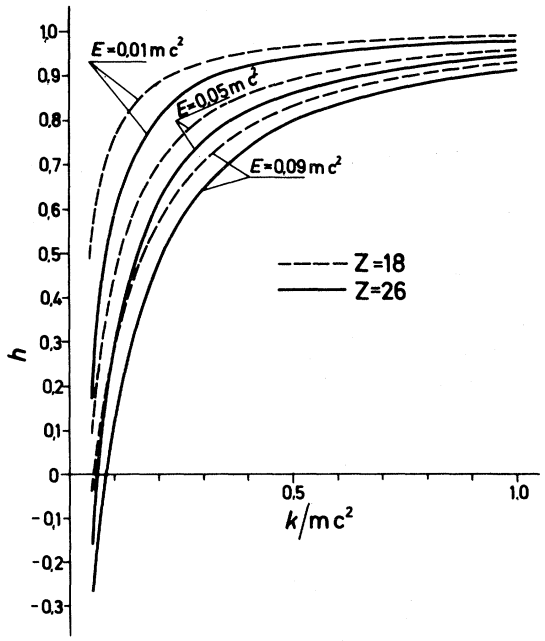

FIG. 3. Dependence of the function h on the photon energy for given electron energies and atomic numbers.

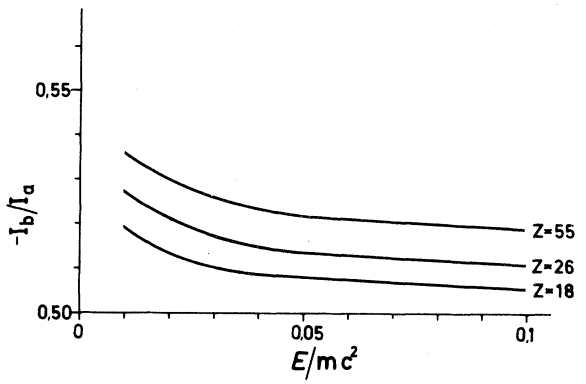

FIG. 4. Dependence of the ratio $-I_b/I_a$ on the electron energy for given atomic numbers.

Our calculations are reasonable for relatively large photon energies and lower electron energies. If lower photon energies are to be considered, contributions from the IB associated with the electrons ejected in autoionization will have to be considered and this will require a more detailed analysis.

ACKNOWLEDGMENTS

One of us (B.A.L.) would like to thank the Natural Sciences and Engineering Council of Canada for financial support. One of us (K.P.) is grateful to the University of Ottawa, where part of this work was performed, for kind hospitality.

APPENDIX

The electron wave functions ϕ_0,ϕ_f used in our calculations are the same (except for a different normalization for ϕ_f with an adequate compensation in the phase-space factor) as defined in the paper of Intemann and Pollock.[10] Also, we have used similar techniques to calculate integrals through the residue theorem in the complex plane, where a hypergeometric function is represented.

The integral A, Eq. (6), is calculated as

$$A=4\sqrt{\pi}a^{3/2}\Gamma\left[1-i\frac{a}{p}\right]\exp\left[-\frac{\pi}{2}\frac{a}{p}\right]I_a\ , \tag{A1}$$

$$I_a=\frac{\exp\left[2\frac{a}{p}\text{arctg}\frac{a}{p}\right]}{p^2+a^2}\ , \tag{A2}$$

where Γ is a gamma function, $a=\alpha Z$, and p is the nonrelativistic electron momentum.

The integral B, Eq. (10), is calculated by using a multipole series expansion of $1/|\vec{r}_1-\vec{r}_2|$. The result is

$$B=8a^3\exp\left[-\frac{\pi}{2}\frac{a}{p}\right]\Gamma\left[1-i\frac{a}{p}\right]I_b\ , \tag{A3}$$

where I_b is defined through the integral representation

$$I_b=-4\mu_b\int_0^\infty ds\left[\frac{1+s}{s}\right]^{\eta_b}\left\{\frac{1}{z^3}\frac{\exp\left[2\frac{a}{p}\arctan\frac{a}{p}\right]}{p^2+a^2}-\frac{1}{z}\frac{\exp\left[2\frac{a}{p}\arctan\frac{z+a}{p}\right]}{[p^2+(z+a)^2]^2}-\frac{1}{z^3}\frac{\exp\left[2\frac{a}{p}\arctan\frac{z+a}{p}\right]}{p^2+(z+a)^2}\right\}. \tag{A4}$$

In Eq. (A4) we introduced the quantities

$$\mu_b=[2(1-\epsilon_b)]^{1/2}\ ,$$

$$\eta_b=a/\mu_b\ , \tag{A5}$$

$$z=\mu_b(1+2s)+a\ ,$$

where ϵ_b is given by Eq. (4).

The spinor function S' from Eq. (7) is defined as

$$S'(0,\vec{r})=\int d\vec{r}_1 G(\epsilon_c;0,\vec{r}_1)S(\vec{r}_1,\vec{r})\ . \tag{A6}$$

By using the explicit forms of G and S functions, the calculation gives

$$S'(0,\vec{r})=\frac{1}{(2\pi)^3}\int d\vec{\rho}\frac{-i\hat{\rho}+1}{\rho^2+1}f(\vec{\rho}^{\,2})e^{-i\vec{\rho}\cdot\vec{r}}\ . \tag{A7}$$

Here the scalar function f is given as

$$f(\vec{\rho}^{\,2})=-8{\mu_c}^2\int_0^\infty ds\left[\frac{1+s}{s}\right]^{\eta_c}\times\frac{1+2s}{[{\mu_c}^2(1+2s)^2+\vec{\rho}^{\,2}]^2}\ , \tag{A8}$$

where μ_c and η_c are defined by Eq. (A5) for the energy ϵ_c, Eq. (4).

[1]W. Bambynek, H. Behrens, M. H. Chen, B. Crasemann, M. L. Fitzpatrick, K. W. D. Ledingham, H. Genz, M. Mutterer, and R. L. Intemann, Rev. Mod. Phys. 49, 77 (1977).

[2]M. S. Freedman, Annu. Rev. Nucl. Sci. 24, 209 (1974).

[3]R. J. Walen and C. Briançon in *Atomic Inner-Shell Processes,* edited by B. Crasemann (Academic, New York, 1975), Vol. 1, p. 233.

[4]A. Zide and H. Lancman, Phys. Rev. Lett. 44, 861 (1980).

[5]A. Ljubičić, R. T. Jones, and B. A. Logan, Nucl. Phys. A236, 158 (1974).

[6]K. Pisk, A. Ljubičić, and B. A. Logan, Nucl. Phys. A267, 77 (1976).

[7]B. A. Logan, A. Ljubičić, K. Pisk, and Z. Roller-Ivanović, Phys. Rev. C 16, 2078 (1977).

[8]Z. Roller, K. Pisk, A. Ljubičić, and B. A. Logan, Nucl. Phys. A390, 43 (1982).

[9]P. Morrison and L. I. Schiff, Phys. Rev. 58, 24 (1940).

[10]R. L. Intemann and F. Pollock, Phys. Rev. 157, 41 (1967).

[11]R. J. Glauber and P. C. Martin, Phys. Rev. 104, 158 (1956).

[12]P. C. Martin and R. J. Glauber, Phys. Rev. 109, 1307 (1958).

[13]R. L. Intemann, Nucl. Phys. A219, 20 (1974).

[14]G. Paquette, Can. J. Phys. 40, 1765 (1962).