Heavy neutrinos and the beta spectra of ^{35}S, ^{18}F, and ^{19}Ne

F. P. Calaprice
Department of Physics, Princeton University, Princeton, New Jersey 08540

D. J. Millener
Brookhaven National Laboratory, Upton, New York 11973
(Received 4 October 1982)

We evaluate the weak magnetism and other higher order matrix elements for the beta decays of ^{18}F, ^{19}Ne, and ^{35}S. The matrix elements were obtained from *sd* shell-model wave functions derived from the one- and two-body effective interactions of Chung and Wildenthal. The shapes of the beta spectra are calculated assuming zero-mass neutrinos and allowing for the higher order matrix elements. The calculations suggest that precise measurements of these spectra would be useful to search for heavy neutrinos with intensities of the order of $>10^{-3}$ over the neutrino mass range from 20 keV to 2 MeV. While other observations may rule out such parameters for mixing of massive neutrinos, the measurements of spectra would provide the most direct and least uncertain limits. For the allowed decays of ^{18}F and ^{19}Ne the weak magnetism form factor appears to be the dominant higher order effect. The analog $M1$ matrix elements are known experimentally in both cases and thus the weak magnetism terms are determined through the conserved vector current theory. One might alternatively regard measurements of these spectra as tests of the conserved vector current predictions.

[RADIOACTIVITY ^{18}F, ^{19}Ne, ^{35}S; calculated nuclear matrix elements, shapes of beta spectra. Relationship to searches for heavy neutrinos.]

I. INTRODUCTION

At the present time the status of neutrino mass is not clear. The experiment of Lubimov *et al.*[1] on tritium beta decay indicates that the mass of the electron antineutrino is around 30 eV and the reactor experiment of Reines *et al.*[2] suggests neutrino oscillations which also imply a finite neutrino mass. Other experiments,[3] however, are completely consistent with a vanishingly small neutrino mass. In particular, the reactor experiment of Boehm *et al.*,[4] which is similar to the Reines experiment, shows no evidence for neutrino oscillations. Though unsettled experimentally, there seems, nevertheless, to be a natural place for neutrinos with mass in certain grand unification theories[5] and in the explanation of cosmological missing mass effects.[6]

It was recently suggested by Shrock,[7] and previously by Nakagawa *et al.*,[8] that some of the mass eigenstates of the neutrino might have masses of MeV, or larger. Certainly this would be consistent with the present mass limits on ν_e, ν_μ, and ν_τ. These weak states of the neutrino are not, however, the mass eigenstates and thus it is possible that the common electron neutrino ν_e, which like the other weak states is a superposition of the mass states, might possess a small amplitude of a state with large mass. These heavy neutrinos would produce "kinks" in the beta spectrum at energies where they become energetically allowed. From a survey of experimental spectra, Shrock[7] has placed limits on the possible masses and amplitudes for heavy neutrinos. A similar study of spectra and ft values of the pure Fermi decays was made by Calaprice.[9] These studies show that the allowable intensities ($\sin^2\theta$) of a heavy neutrino in a two state system are $\lesssim 10^{-2}$ for masses in the range from ~ 100 keV to ~ 10 MeV. Simpson[10] has obtained tight bounds from the tritium spectrum ranging from $\sin^2\theta \lesssim 10^{-1}$ to $\lesssim 5\times 10^{-3}$ for masses between 350 eV and 10 keV. Between 10 keV and ~ 100 keV there is a gap which deserves further investigation.

In the following work we discuss a few selected nuclear beta transitions which could be investigated experimentally to tighten the permissible amplitudes and extend the mass range of heavy neutrinos. We will show that the theoretical shapes of the beta spectra of ^{35}S ($T_0=167$ keV, $\log ft=5.0$), ^{18}F ($T_0=633$ keV, $\log ft=3.6$), and ^{19}Ne ($T_0=2216$ keV, $\log ft=3.2$) (where $T_0=W_0-511$ is the maximum electron kinetic energy) are sufficiently certain to permit one to detect heavy neutrino intensities of 10^{-3}. The ^{35}S and ^{18}F decays would be especially useful in filling in the low mass gap.

The limits imposed on heavy neutrinos from such studies of beta spectra are direct and involve few assumptions. One should note that more stringent but less direct constraints are obtained in other neutrino processes. In one of these, Toussaint and Wilczek[11] point out that the ^8B neutrinos emanating from the sun could have mass components up to the endpoint energy of 14 MeV. These massive neutrinos would decay on their way to the earth through the process $\nu' \rightarrow e^+e^-\nu$ giving rise to a stream of positrons with kinetic energies of a few MeV. The positron flux from the sun is sufficiently small to establish upper limits on the mixing angle $\sin^2\theta$ of 10^{-4} to 10^{-6} for masses between 1.1 and 14 MeV. There is no sensitivity to masses below $2m_e$.

Other constraints on heavy neutrinos have been set in studies of neutrinoless double beta decay which is an allowed process if the intermediate neutrino is a massive Majorana neutrino. Neutrinoless double beta decay has not been observed and the lower limit for the half-life in ^{76}Ge [$T_{1/2}(\beta\beta 0\nu) > 10^{21.7}$ y] (Ref. 12) has been used by Haxton *et al.*[13] to establish an upper limit for the mass of $m_\nu < 15$ eV, assuming it is a Majorana neutrino. Simpson[14] has extended this argument to allow mixing of a heavy neutrino, with mass m_2 and a light neutrino to obtain the condition $m_2\sin^2\theta < 40$ eV. This very tight constraint corresponds to $\sin^2\theta < 4\times 10^{-5}$ for $m_2=1$ MeV and $\sin^2\theta < 10^{-3}$ for $m_2=40$ keV.

Finally, astrophysical and cosmological observations have been used to constrain the properties of neutrinos. In a summary of these constraints Turner[15] has shown that m_ν must be less than 200 eV or greater than 10–100 MeV for neutrinos that decay radiatively with a lifetime proportional to m_ν^{-5}.

The constraints based on solar neutrinos, double beta decay, and cosmology are stringent but involve assumptions and calculations which are perhaps somewhat uncertain. It may therefore still be of interest to investigate beta spectra, especially if comparable limits could be achieved. The ^{35}S, ^{18}F, and ^{19}Ne decays should provide mass limits from ~ 20 to ~ 2000 keV at intensities of $\lesssim 10^{-3}$. For the ^{18}F and ^{19}Ne superallowed decays the largest distortion in the beta spectrum is due to the weak magnetism form factor. This quantity is determined by the experimental isovector magnetic dipole matrix element by virtue of the conserved vector current (CVC) theory. One might alternatively regard careful measurements of these spectra as tests of the CVC theory. The kinks in the beta spectra due to heavy neutrinos have a unique shape and are distinguishable from the weak magnetism effect. For three neutrino mass eigenstates the beta spectrum would consist of an incoherent superposition of three beta spectra

$$d\lambda=\frac{2}{\pi^2}g_v^2[F(Z,W)g(W,W_0)pW(W_0-W)^2S(W)]\sum_{i=1}^{3}B_i\left[1-\left[\frac{m_{\nu_i}}{W_0-W}\right]^2\right]^{1/2}dW\,, \tag{1}$$

with B_i the branching ratio for the emission of the ith neutrino with mass m_{ν_i}. F is the Fermi function, g is a radiative correction factor, and S is the shape factor.

In Sec. II we discuss the calculation of the shape factor and compare the expressions given by Behrens *et al.*[18,19] and by Holstein.[20] Then in Sec. III we discuss the results obtained for the shape factors of the ^{35}S, ^{18}F, and ^{19}Ne decays and state our conclusions.

II. CALCULATION OF THE SHAPE FACTORS

The form of the shape factor $S(W)$ depends on nuclear matrix elements and on small Coulomb terms which vary with the definition of the Fermi function, which we take to be $F(Z,W)=F_0L_0$, as specified by Behrens and Jänecke.[16] We write the shape factor for allowed transitions in the form

$$S(W)=C(W)R(W)\,. \tag{2}$$

$R(W)$ takes into account[17] the nuclear recoil with

$$R(W)=1-\frac{2W_0}{3M}+\frac{10W}{3M}-\frac{2}{3MW}$$

or (3)

$$R(W)=1+\frac{2W}{M}$$

TABLE I. One body density matrix elements defined in Eq. (5). To obtain the OBDME of Chung and Wildenthal (Refs. 22 and 23) our OBDME must be multiplied by $\hat{J}_f\hat{T}_f(\hat{\Delta J}\hat{\Delta T})^{-1}$ and an adjustment made for different definitions of the single-particle wave functions. Our single-particle wave functions are defined with $l\times s\rightarrow j$, radial wave functions are positive near the origin, and an i^l factor is included with the spherical harmonic.

Decaying nucleus	^{18}F^{a}	^{19}Ne β		^{35}S^{b}		
$T_iJ_i\rightarrow T_fJ_f$	$01\rightarrow 10$	$\frac{1}{2}\ \frac{1}{2}\rightarrow\frac{1}{2}\ \frac{1}{2}$		$\frac{3}{2}\ \frac{3}{2}\rightarrow\frac{1}{2}\ \frac{3}{2}$		
ΔJ	1	0	1	0	1	2
$(2j_1,2j_2)$						
(11)	3.141(−1)*^{c}	3.153(−1)	5.772(−1)*	4.941(−3)*	3.044(−3)*	
(31)	2.761(−2)		2.187(−3)		2.179(−1)*	1.527(−1)*
(13)	5.711(−2)*		2.187(−3)*		2.176(−1)*	8.388(−2)*
(33)	1.780(−2)	4.142(−2)	2.003(−2)	3.831(−2)	4.143(−1)*	1.185(0)*
(51)						6.002(−2)
(15)						3.770(−2)
(53)	1.599(−1)*		1.812(−1)*		7.850(−2)*	1.516(−2)
(35)	4.000(−1)		1.812(−1)		9.777(−2)*	1.190(−1)*
(55)	5.230(−1)*	2.842(−1)	3.458(−1)*	2.843(−2)*	1.288(−2)*	1.798(−2)

^{a}Calculated using the "particle" Hamiltonian of Chung and Wildenthal (Ref. 21).
^{b}Calculated using the "hole" Hamiltonian of Chung and Wildenthal (Ref. 21).
^{c}The exponent is given in parentheses. An asterisk indicates that the sign is negative.

for axial and vector decays, respectively, M being the nuclear mass and W the total electron energy.

$$C(W)=1+aW+b/W+cW^2\ . \qquad (4)$$

To evaluate the coefficients a, b, and c, which depend on ratios of nuclear matrix elements, we follow Behrens *et al.*[18,19] who provide a consistent treatment of the influence of the Coulomb interaction between the ejected β particles and the nucleus. We also compare these results with those obtained using the formalism of Holstein.[20]

The nuclear matrix elements were computed using one-body density matrix elements (OBDME) ob-

TABLE II. Important form factor coefficients. The form factor coefficients listed are sufficient to construct the quantities C_1, C_2, and D_0 defined in the text and in Appendix B of Ref. 19. To a good approximation A_0 and C_0 are the Fermi and Gamow-Teller matrix elements, respectively. The values of nuclear matrix elements may be extracted by comparison with Table 6 of Ref. 18. Note that $\lambda>0$, $Z>0$, etc. so that for positron decay sign changes from the formal substitutions $\lambda\rightarrow-\lambda$, $fp\rightarrow-fp$, $Z\rightarrow-Z$ have been incorporated into the matrix elements. $\lambda=g_A$, $f_M=(g_M-1)/2M_N$, $f_p=g_p/2M_N$, where g_A, g_M, and g_p are given in Table III.

FFC	Decay: ^{19}Ne	^{18}F	^{35}S
$^{A}F^{0}_{101}$	λ1.6828(0)*	1.3492(0)*	2.6813(−1)*
$^{A}F_{101}{}^{1}$	λ1.3732(0)* + f_p/R 9.6902(−2)^{a}	1.1155(0)*, 7.8727(−2)	2.0278(−1)*, 1.3747(−2)
$^{A}F_{101}{}^{1}(1,1,1,1)$	λ1.6105(0)* + f_p/R 7.6870(−2)	1.3165(0)*, 5.9915(−2)	2.4637(−1)*, 1.7641(−2)
$^{A}F_{101}{}^{1}(1,2,2,1)$	λ1.7567(0)* + f_p/R 9.7601(−2)	1.4328(0)*, 7.7582(−2)	2.6535(−1)*, 1.8713(−2)
$^{A}F^{0}_{121}$	λ1.6257(−2)+ f_p/R 3.4260(−1)^{a}	4.1128(−2)*, 2.7834(−1)	2.6890(−1), 4.8601(−2)
$^{A}F^{0}_{121}(1,1,1,1)$	λ1.9593(−2)+ f_p/R 3.3773(−1)	4.6428(−2)*, 2.6803(−1)	3.2656(−1), 6.5780(−2)
$^{A}F^{0}_{110}$	λ0.0 $+f_p/R(W_0R)$8.3920(−2) $+f_p/R(\alpha Z)$7.5550(−2)^{a}	3.1332(−2)*, 6.8179(−2), 5.9536(−2)*	2.4750(−2)*, 1.1905(−2), 1.5742(−2)
$^{V}F^{0}_{111}$	1.6357(−1) + f_M/R4.1220(0)^{b}	1.4601(−1), 3.3047(0)	3.5204(−2), 6.5678(−1)*
$^{V}F_{000}{}^{1}$	8.1599(−1) + f_M/R 2.8029(−1)		
$^{V}F_{000}{}^{1}(1,1,1,1)$	9.5840(−1) + f_M/R 2.4702(−1)		
$^{V}F_{000}{}^{1}(1,2,2,1)$	1.0449(0) + f_M/R 3.0057(−1)		

^{a}Note that in the combination $-5\,{}^{A}F_{101}{}^{1}+\sqrt{2}\,{}^{A}F^{0}_{121}$ terms in f_p cancel out; f_p has a significant effect only in D_0, mainly through the $f_p(\alpha Z)$ term in ${}^{A}F^{0}_{110}$.
^{b}We do not list very small terms in $f_M(W_0R)$ and $f_M(\alpha Z)$.

TABLE III. Matrix elements in the Holstein-Treiman formalism.

Quantity[a,b]	^{18}F	^{19}Ne	^{35}S
$M_F=\langle f\|\|\tau^{\pm}\|\|i\rangle_{HT}=M^0_{000}$		1.0	
$M_{GT}=\langle f\|\|\tau^{\pm}\underline{\sigma}\|\|i\rangle_{HT}=M^0_{101}$	-1.3491	-1.6828	0.2681
$M_r{}^2=\langle f\|\|\tau^{\pm}r^2\|\|i\rangle_{HT}=R^2M_{000}{}^1$		10.869	
$M_{\sigma r}{}^2=\langle f\|\|\tau^{\pm}\sigma r^2\|\|i\rangle_{HT}=R^2M_{101}{}^1$	-14.471	-18.290	3.4075
$M_L=\langle f\|\|\tau^{\pm}\underline{l}\|\|i\rangle_{HT}=\sqrt{2/3}M_NRM^0_{111}-M^0_{101}$	-0.6939	-0.6364	-0.8288
$M_{\sigma L}=\langle f\|\|\underline{\tau\sigma}\times\underline{l}\|\|i\rangle_{HT}=-2/\sqrt{3}M_NRM^0_{110}$	-0.6200	0.0	0.5574
$M_{1y}=\sqrt{16\pi/5}\langle f\|\|\tau^{\pm}r^2(\sigma,Y^2)^1\|\|i\rangle_{HT}=-2/\sqrt{5}R^2M^0_{121}$	0.4772	-0.1937	4.0415
$a_1=M_F$		1.0	
$c_1=g_AM_{GT}$	-1.6864	-2.1035	0.3352
$a_2=\frac{1}{6}M_r{}^2$		1.8114	
$c_2=\frac{1}{6}G_A[M_{\sigma r}{}^2+1/\sqrt{10}M_{1y}]$	-2.9834	-3.8112	0.9762
$b_{WM}=A[g_MM_{GT}+g_VM_L]$	-126.77	-162.56	15.156
$d=Ag_AM_{\sigma L}$	-13.951	0.0	24.388
$h=2/\sqrt{10}M^2g_AM_{1y}/(\hbar c)^2-A^2g_PM_{GT}$	$-97\,173$	$-135\,045$	$-73\,016$

[a]The nuclear matrix elements M^N_{KLS} may be extracted from Table II.
[b]We use $g_A=1.25$, $g_V=1$, $g_M=4.706$, $g_P=-222.3$, $M_N=938.93$ MeV, $M=A\times930.6$ MeV.

TABLE IV. Constants and results.

Constants	^{18}F	^{19}Ne	^{35}S
W_0 (MeV)	1.145	2.727	0.679
R fm[a]	3.6018	3.6496	4.0993
b_{os} fm[b]	1.7506	1.7622	1.9055
W_0R[c]	0.02089	0.05044	0.01410
αZ[c]	0.05838	0.06568	0.12406
f_M/R[c]	0.10807	0.10665	0.09495
f_p/R[c]	-6.4862	-6.4013	-5.6991
Results			
$\log ft$(th)[d]	3.34	3.06	4.74
$\log ft$(exp)	3.58	3.24	5.02
a_B	$-4.643(-3)$	$-3.121(-3)$	$-3.629(-4)$
a_H	$-4.532(-3)$	$-2.990(-3)$	$-8.151(-4)$
b_B	$7.584(-4)$	$6.440(-4)$	$-5.382(-4)$
b_H	$7.253(-4)$	$6.390(-4)$	$-4.376(-4)$
c_B	$-1.929(-4)$	$-1.867(-4)$	$-3.312(-4)$
c_H	$-2.019(-4)$	$-1.921(-4)$	$-3.324(-4)$

[a]$R=r_0A^{1/3}$, $r_0=1.614-0.1067\ln A+0.00545(\ln A)^2+6.112/(A-1.76)^2$ following Ref. 17.
[b]$b_{os}{}^2=41.467(\hbar w)^{-1}$, $\hbar w=45A^{-1/3}-25A^{-2/3}$ MeV.
[c]Dimensionless.
[d]$ft=6177/(M_F{}^2+M_{GT}{}^2)$. The fitted Gamow-Teller single-particle matrix elements of Brown, Chung, and Wildenthal (Ref. 23) yield $\log ft$ values of 3.56, 3.22, and 5.02 for ^{18}F, ^{19}Ne, and ^{35}S, respectively.

tained from *sd* shell wave functions deriving from the effective Hamiltonians (two-body matrix elements and single particle energies) of Chung and Wildenthal.[21] The effective Hamiltonians of Chung and Wildenthal have proved very successful in describing $M1$ properties[22] and Gamow-Teller β decay[23] in *sd*-shell nuclei. The one-body density matrix elements relevant to the transitions in ^{18}F, ^{19}Ne, and ^{35}S are listed in Table I. They are defined as the following reduced matrix elements (Brink and Satchler's[24] definition).

$$\langle J_fT_f\|(a^\dagger_{j_1}\tilde{a}_{j_2})^{\Delta J\Delta T}\|J_iT_i\rangle\ . \tag{5}$$

The OBDME are then combined with single-particle matrix elements (SPME), evaluated according to the formulae in Table 7 of Ref. 18, to yield the nuclear matrix elements which occur in the expressions for the form-factor coefficients, defined in Table 6 of Ref. 18. Simple expressions for those SPME which do not involve the division of the radial integral into two parts are given in Appendix F of Ref. 19. Numerical values for nuclear matrix elements which play an important role in defining the shape factor $C(W)$ are given in Table II. These matrix elements were calculated using harmonic oscillator wave functions; the values of b_{os} and R, the equivalent charge radius, are specified in Table IV. The corresponding matrix elements which are necessary to evaluate the shape factor in the Holstein formalism are given in Table III. Our final results for the quantities *a*, *b*, and *c*, which define the shape factor of Eq. (4), are given in Table IV. To obtain *a*, *b*, and

c we evaluated the linear combinations[19] of form factor coefficients $A_0, A_1, \ldots, H_1$ and used Eqs. (2.2)–(2.4) of Ref. 19. However, to a very good approximation, a, b, and c are given by simpler expressions which we now list and compare with the corresponding expressions of Holstein.[25,26] For an allowed Gamow-Teller transition (upper signs refer to electron emission; natural units) where C_0 and c_1 are the Gamow-Teller matrix elements in the two formalisms:

$$a_B \approx 2RC_1/C_0 ,$$
$$C_1 = -\tfrac{2}{3}(\tfrac{2}{3})^{1/2}\,{}^{V}F^0_{111} + \tfrac{2}{27}(W_0R)[-5\,{}^{A}F_{101}{}^1 + \sqrt{2}\,{}^{A}F^0_{121}]$$
$$\pm\tfrac{1}{27}(\alpha Z)[{}^{A}F_{101}{}^1(1,1,1,1) - 2\sqrt{2}\,{}^{A}F^0_{121}(1,1,1,1) + 9\,{}^{A}F_{101}{}^1(1,2,2,1)] , \tag{6}$$

or

$$a_H = \pm\frac{4b_{WM}}{3AM_Nc_1} + \frac{40c_2W_0}{9c_1(\hbar c)^2} \mp \frac{20c_2\alpha Z}{3Rc_1(\hbar c)} ,$$
$$b_B \approx -2\gamma_1 RD_0/C_0 \approx -2RD_0/C_0 , \tag{7}$$
$$D_0 = -\tfrac{1}{3}\{(\tfrac{1}{3})^{1/2}\,{}^{A}F^0_{110} + (\tfrac{2}{3})^{1/2}\,{}^{V}F^0_{111}\} + \frac{W_0R}{27}\{-{}^{A}F_{101}{}^1 + 2\sqrt{2}\,{}^{A}F^0_{121}\} \pm \tfrac{1}{6}\alpha Z\,{}^{A}F_{101}{}^1(1,2,1,1) ,$$

or

$$b_H = -\frac{4W_0c_2}{9c_1(\hbar c)^2} - \frac{1}{3AM_Nc_1}\{d \pm 2b_{WM}\} + \frac{W_0h}{6(AM_N)^2c_1} ,$$
$$C_B \approx 2R^2C_2/C_0 ,$$
$$C_2 = -\tfrac{2}{27}[-5\,{}^{A}F_{101}{}^1 + \sqrt{2}\,{}^{A}F^0_{121}] , \tag{8}$$

or

$$C_H = -\frac{40c_2}{9c_1(\hbar c)^2} .$$

On comparing the expressions a_H and a_B we note that, apart from contributions to ${}^{V}F^0_{111}$ which depend on $f_M W_0 R$ and $f_M \alpha Z$ and are typically one percent effects, the first terms are identical, as are the second terms. The αZ terms, however, are not identical. Since terms involving f_p cancel exactly in C_1 we can replace $|{}^{A}F^N_{KLS}|$ by $\lambda\,|{}^{A}M^N_{KLS}|$. Then using rectangular single-particle wave functions and the correct form[18,19] of the integrands for $r \le R$ in the nuclear matrix elements we have

$$\frac{M_{101}{}^1(1,1,1,1)}{M_{101}{}^1} = \frac{M^0_{121}(1,1,1,1)}{M^0_{121}} = \frac{9}{7} ;$$
$$\frac{M_{101}{}^1(1,2,2,1)}{M_{101}{}^1} = \frac{19}{14} , \tag{9}$$

and finally for the third term in a_B

$$\frac{\alpha Z}{10}[5F_{101}{}^1 - \tfrac{20}{21}\sqrt{2}F^0_{121}] . \tag{10}$$

Thus the αZ term in a_H is larger than that in a_B by a factor $\approx \frac{20}{18}$; this factor is very close to that found in calculations with more realistic single-particle wave functions. For $A=18$ the resultant difference between a_H and a_B is small. However, in $A=35$, where there is a strong cancellation between the weak magnetism and αZ terms, the difference is over a factor of 2 (as a function of Z a_H and a_B go through zero at $Z \approx 12.3$ and 14.5, respectively). Differences in b_H and b_B are in part owing to Coulomb corrections to induced terms which are explicitly excluded in the Holstein treatment, e.g., such a term which occurs in ${}^{A}F^0_{110}$ gives rise to a contribution to b_B of the form

$$\pm(2/5M_N)(fp/R)(\alpha Z/\lambda) .$$

We note that $f_p \neq 0$ affects only b; setting $f_p = 0$ in our calculation changes b by 6–14 % in the three cases.

III. RESULTS AND DISCUSSIONS

The calculated values of the coefficients a, b, and c which define the shape factor are given in Table IV; a_B, b_B, and c_B contain contributions[19] from small terms not appearing in Eqs. (6)–(8). The

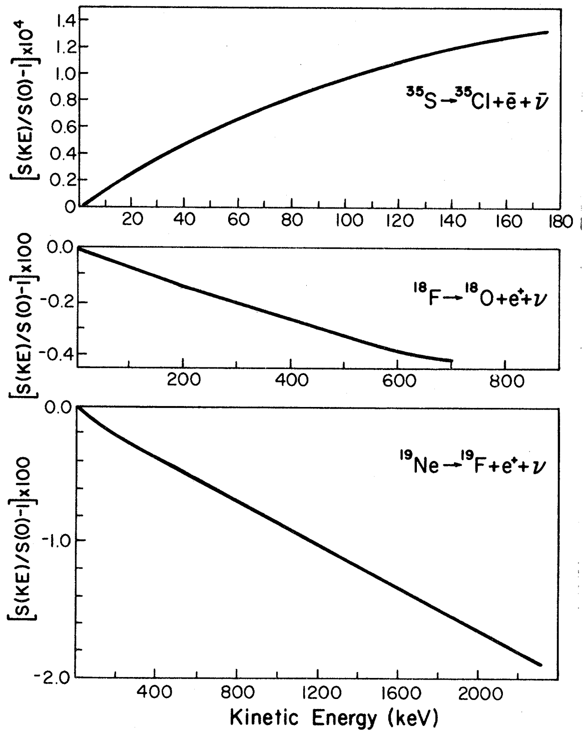

FIG. 1. The theoretical spectrum shape factors for ^{35}S, ^{18}F, and ^{19}Ne decays. Note the expanded vertical scale for the ^{35}S shape factor.

shape factors for the three transitions are displayed graphically in Fig. 1. For convenience of comparison with experiment we plot the deviation of the normalized shape factor from unity as a function of the electron kinetic energy.

In the case of ^{18}F and ^{19}Ne the weak magnetism term dominates and gives rise to linear terms with coefficients of -0.46% and -0.31% MeV^{-1}, respectively. Note, however, that all three energy dependent terms contribute constructively to the slope of the shape factor so that the slopes near the high energy end of the spectrum are $\sim -0.57\%$ and -0.42% MeV^{-1}, respectively.

In ^{35}S decay the weak magnetism matrix element $b_{\rm WM}$ is computed to be small, the result of a small spin matrix element canceling against the orbital matrix element (see Table III); we have $b_{\rm WM}/(Ac_1)\approx 1.6$ instead of the value of 4.7 for a spin dominated transition. The normal cancellation between the weak magnetism term and the αZ term leads to an almost vanishing linear coefficient (a). The $1/W$ term thus dominates and is gradually overcome by the linear and quadratic terms as the electron kinetic energy increases (the shape factor would turn over at ~ 780 keV). The calculated variation of the shape factor is extremely small, the shape factor being allowed to within $\sim 0.02\%$.

As one measure of the reliability of the results we can compare the theoretical Gamow-Teller matrix element with the one extracted from the experimental ft value. For ^{18}F and ^{19}Ne decay the ratio of observed to calculated matrix elements is ~ 0.76. The quenching of the Gamow-Teller matrix element is a well known phenomenon and an explanation requires the consideration[27] of several physical effects. We could use the empirical single-particle matrix elements of Brown, Chung, and Wildenthal,[23] which give good agreement with the experimental $\log ft$ values (see footnote d to Table IV), but they are not available for all of the operators needed for our purposes. We therefore use bare single-particle matrix elements with the precaution that we should not expect an accuracy of better than a factor of 2. The energy dependence of the shape factor actually depends on ratios of matrix elements such as c_2/c_1, $b_{\rm WM}/c_1$, etc., and it is possible that the computed ratios are more reliable than the individual values, especially in cases where the operators are similar, as in $M_{\sigma r}{}^2/M_{\rm GT}$. However, in ^{18}F and ^{19}Ne decay the dominant effects come from the ratio $b_{\rm WM}/c_1$ and the CVC theory may be used to relate $b_{\rm WM}$ to electromagnetic matrix elements. In particular, for $A=18$

$$B(M1;0^+,1\rightarrow 1^+,0)=3\frac{3}{4\pi}2\left[\frac{b_{\rm WM}}{2A}\right]^2\mu_N{}^2 =1/5\left[\frac{b_{\rm WM}}{A}\right]^2 \text{W.u.}\,, \tag{11}$$

and for $A=19$

$$\mu(^{19}\text{Ne})-\mu(^{19}\text{F})=1/\sqrt{3}\left[\frac{b_{\rm WM}}{A}\right]\mu_N\,. \tag{12}$$

The $B(M1)$ calculated from Eq. (11) is 9.9 W.u., in excellent agreement with the measured[28] value of 10.4 ± 1.5 W.u. Similarly, the calculated value for the difference in magnetic moments, $-4.94\ \mu_N$, is close to the measured value of $-4.52\ \mu_N$. These agreements are consistent with the fact that most large $M1$ matrix elements in light nuclei can be reproduced quite well using good shell model wave functions and bare nucleon g factors (e.g., Ref. 22); core polarization and mesonic effects, while possibly large individually, appear to cancel.[29] Thus it is very likely that the values of $b_{\rm WM}/c_1$ for ^{18}F and ^{19}Ne decay should be $\sim 30\%$ larger than those we have calculated. For ^{35}S decay the variation in the shape factor is so small that such refinements to b_{WM}/c_1 would be academic.

The beta spectrum of ^{35}S has been the subject of many experimental investigations.[30] These measurements, though not all consistent, and the measurements on ^{18}F and ^{19}Ne (Refs. 30 and 31) are typically sensitive enough to rule out 10% intensities for heavy neutrinos in the mass range for 50—2000 keV. More precise measurements at the level of 0.1% would be very desirable to restrict neutrino mixing-mass parameters and to observe the interesting weak magnetism effect.

[1]V. A. Lubimov, E. G. Novikov, V. Z. Nozik, E. F. Tretyakov, and V. S. Kosik, Phys. Lett. 94B, 266 (1980).

[2]F. Reines, H. W. Sobel, and E. Pasierb, Phys. Rev. Lett. 45, 1307 (1980).

[3]For a recent review of neutrino oscillation experiments, see C. Baltay, in *Neutrino 81, Proceedings of the 1981 International Conference on Neutrino Physics and Astrophysics, Hawaii,* edited by R. J. Cence, E. Ma, and A. Roberts (University of Hawaii Press, Honolulu, 1981), Vol. II, p. 295.

[4]F. Boehm, J. F. Cavaignac, F. V. Feilitzsch, A. A. Hahn, H. E. Henrikson, D. H. Koang, H. Kwon, R. L. Mössbauer, B. Vignon, and J. L. Vuilleumier, Phys. Lett. 97B, 310 (1980).

[5]L. Wofenstein, see Ref. 3, p. 329.

[6]R. Brent Tully, see Ref. 3, Vol. I, p. 73.

[7]R. E. Shrock, Phys. Lett. 96B, 159 (1980).

[8]M. Nakagawa, H. Okonogi, S. Sakata, and A. Toyoda, Prog. Theor. Phys. 30, 727 (1963).

[9]F. P. Calaprice, Princeton University Internal Report, 1981 (unpublished).

[10]J. J. Simpson, Phys. Rev. D 24, 2971 (1981).

[11]D. Toussaint and F. Wilczek, Nature 289, 777 (1981).

[12]E. Fiorini, A. Pullia, G. Bertolini, F. Cappellani, and G. Restelli, Nuovo Cimento 13A, 747 (1973).

[13]W. C. Haxton, G. J. Stephenson, and D. Strottman, Phys. Rev. Lett. 47, 153 (1981); see also Phys. Rev. D 25, 2360 (1982).

[14]J. J. Simpson, Phys. Lett. 102B, 35 (1981).

[15]M. S. Turner, see Ref. 3, Vol. I, p. 95.

[16]H. Behrens and J. Jänecke, in, *Landolt-Börnstein: Numerical Data and Functional Relationships,* edited by K.-H. Hellwege (Springer, Berlin, 1969), New Series, Group I, Vol. 4.

[17]D. H. Wilkinson, *Les Houches 1977, Session XXX: Nuclear Physics with Heavy Ions and Mesons,* edited by R. Balian, M. Rho, and G. Ripka (North-Holland, Amsterdam, 1978), pp. 935—944.

[18]H. Behrens and W. Bühring, Nucl. Phys. A162, 111 (1971).

[19]H. Behrens, H. Genz, M. Conze, H. Feldmeier, W. Stock, and A. Richter, Ann. Phys. (N.Y.) 115, 276 (1978).

[20]B. Holstein, Rev. Mod. Phys. 46, 789 (1974).

[21]W. Chung, Ph.D. thesis, Michigan State University, 1976 (unpublished).

[22]B. H. Wildenthal and W. Chung, *Mesons in Nuclei,* edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979), Chap. 18; *The (p,n) Reaction and the Nucleon-Nucleon Force,* edited by C. D. Goodman, S. M. Austin, S. D. Bloom, J. Rapaport, and G. R. Satchler (Plenum, New York, 1980), p. 89.

[23]B. A. Brown, W. Chung, and B. H. Wildenthal, Phys. Rev. Lett. 40, 1631 (1978); and (to be published).

[24]D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon, Oxford, 1968).

[25]F. P. Calaprice, W. Chung, and B. H. Wildenthal, Phys. Rev. C 15, 2178 (1977).

[26]B. Holstein (private communication). Note that the αZ terms in Eqs. (6)—(8) differ from those quoted in Ref. 25.

[27]I. S. Towner and F. C. Khanna, Phys. Rev. Lett. 42, 51 (1979); E. Oset and M. Rho, *ibid.*, 42, 47 (1979).

[28]J. Keinonen, H.-B. Mak, P. Skensved, J. R. Leslie, and W. McLatchie, Phys. Rev. C 22, 351 (1980); J. Keinonen, H.-B. Mak, T. K. Alexander, G. C. Ball, W. G. Davies, J. S. Forster, and I. V. Mitchell, *ibid.* 23, 2073 (1981).

[29]A. Arima and H. Hyuga, *Mesons in Nuclei,* edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979), Chap. 17; K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. A226, 282 (1974).

[30]*Table of Isotopes,* 7th ed., edited by C. M. Lederer and V. S. Shirley (Wiley, New York, 1978).

[31]I. Hofmann, Acta. Phys. Austriaca 18, 309 (1964).