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Extension of Fuda's πN P wave separable model

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Fuda's method to construct a πN separable model for the P wave interaction is extended to further include the inelasticity effect for all partial waves and the sign changing property of the P_{11} phase shift.

[NUCLEAR REACTIONS πN P wave separable model, inverse scattering problem with crossing symmetry; t matrix contains nucleon poles, right and left cuts, and inelasticity effect; P_{11} changes the sign.]

I. INTRODUCTION

Recently, interest in deepening our understanding of the πN P-wave interaction was revived by several groups. The main reason for the revived interest is the gradual recognition of the crucial importance of this partial wave in pion absorption phenomena[1] in many-body and three-body calculations. Associated with this absorption phenomenon is the often neglected sign changing and resonance producing P_{11} partial wave.

Ernst and Johnson[2] reexamined one of the basic theories for the πN P-wave interaction, the static Chew-Low model calculation,[3] and found that the model can describe only the P_{33} channel adequately, and hence proposed a separable[4] model with a special treatment for the P_{11} partial wave.

An improvement of the Chew-Low model by including the nucleon recoil effect was carried out by Miller,[5] who has also shown a justification for the existence of the πN potential in the model. The Chew-Low model itself without the static assumption was examined in more detail by Wei and Banerjee,[6] who have included rigorous crossing kinematics. They concluded[7] that without an additional Z graph[8] the general Chew-Low model cannot produce the resonance in the P_{33} channel and that the inelasticity must be included in the P_{11} channel to account for the sign changing behavior.

Mizutani *et al.*[1,9] examined in greater detail the properties of the P_{11} partial wave, and to explain its complex behavior, they proposed a two-potential approach in which the nucleon pole term plays an important role.

On the other hand, a much simpler parametrization of the t matrix by the conventional separable model still continues.[10,11] Although only the P_{33} channel was shown[7] to be separable in the general Chew-Low model, in view of the incomplete description of the P-wave phase shifts, there remain various possibilities to exploit the simplicity embodied in the separable model, the latest of which was introduced by Fuda.[12] Starting from the Feshbach-Villars Hamiltonian formulation,[13] Fuda[14] develops a time dependent scattering theory which provides a framework for constructing a separable potential model with built-in structures for accommodating the field theoretically required properties[3] such as right and left hand cuts, and the direct and crossed nucleon poles, plus the nucleon recoil effect.[5–7] Since many of the analytical properties of the scattering matrix in the complex energy plane are independent of the details of dynamical models, a separable model with built-in structure for the analytical properties mentioned above will provide both greater confidence in our future calculations and greater simplicity. The analysis of the analytical properties of the scattering amplitude and its applications to the πN interaction were further extended by Reiner[15] using dispersion relations.

In this paper we improve upon Fuda's work[12] by including the inelasticity effect for all partial waves, and the sign changing character in the P_{11} wave.[1,4,7,11] The resulting separable model will have achieved vast improvements over all separable models proposed thus far[16] and differs from more fundamental models only in the separability assumption and details of the functional form of the form factors. We regret to report, however, that we were unable to provide any numerical values for the form factors.

In Sec. II a new dispersion relation including the inelasticity effect is written, and an analytical solution for the form factor will be given. Section III discusses a choice of the phase convention in order to give unique meaning to the form factors obtained.

Special care is taken for the P_{11} wave in Sec. IV, so that the form factor can accommodate the sign changing character of the phase shift. Some discussions on numerical solutions will be given in the last section.

II. DISPERSION RELATIONS

Using similar notations as in Ref. 12, we can write the denominator function of a separable T matrix when inelastic channels are open:

$$\begin{aligned} d_\alpha(z) &= \gamma_\alpha(z)^{-1} - \int_0^\infty dp\, p^2 \\ &\quad \times \left[\frac{G_\alpha^{(+)}(p)^2}{x-z} + \frac{G_\alpha^{(-)}(p)^2}{x+z} \right] \\ &= \gamma_\alpha(z)^{-1} - \frac{1}{\pi} \int_\mu^\infty dx\, p(x)^3 \\ &\quad \times \left[\frac{v_\alpha(x)^2}{x-z} + \frac{u_\alpha(x)^2}{x+z} \right], \\ x &= \sqrt{\mu^2+p^2} + \frac{p^2}{2m}, \end{aligned} \tag{1}$$

where $\gamma_\alpha(z)$ is the energy dependent coupling constant to ensure the elastic unitarity,[17] and $G_\alpha^{(+)}$ and v_α are the original and reduced form factors for the positive energy state. This equation shows that the $\gamma_\alpha(z)$, as well as $d_\alpha(z)$, must have a left hand cut in addition to the usual right hand cut. This consequence can be easily understood if the reader goes back to the definition of γ in Ref. 18 and uses the Feshbach-Villars Hamiltonian formalism[13] in Feshbach's projection operator method,[19] instead of the standard Hamiltonian formalism. (Replace the usual Green's function for the direct channel by the one which includes also the crossing channel.)

Corresponding to the use of the energy dependent coupling constant, the on-shell scattering matrix element is parametrized according to[4,11]

$$\begin{aligned} T_\alpha(\omega) &= -\frac{m+\xi}{\pi k m \xi} \hat{\eta}_\alpha(\omega) \sin\hat{\delta}_\alpha(\omega) e^{i\hat{\delta}_\alpha(\omega)} \\ &= -\frac{m+\xi}{\pi m \xi} \frac{k^2 v_\alpha(\omega)^2}{d_\alpha(\omega+i\epsilon)}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \omega &= (\mu^2+k^2)^{1/2} + \frac{k^2}{2m} = \xi + \frac{k^2}{2m}, \\ \tan\hat{\delta}_\alpha &= \tan\delta_\alpha \left[1 + \frac{1-\eta_\alpha}{2\eta_\alpha \sin^2\delta_\alpha} \right], \\ \hat{\eta}_\alpha &= \eta_\alpha \frac{\sin 2\delta_\alpha}{\sin 2\hat{\delta}_\alpha}, \end{aligned} \tag{3}$$

with δ and η being the real phase shift and absorption parameter, and μ and m being the pion and nucleon masses. Both Eqs. (1) and (2) have explicitly included the proper kinematical factors[4] for the form factor which are suggested by the low energy behavior of the T matrix. Equation (2) suggests that we can introduce a phase factor along the right hand cut as either

$$\begin{aligned} d_\alpha(\omega+i\epsilon) &= |d_\alpha(\omega+i\epsilon)| e^{-i\theta_\alpha}, \\ |d_\alpha(\omega+i\epsilon)| &= \frac{k^3 v_\alpha{}^2}{\hat{\eta}_\alpha |\sin\hat{\delta}_\alpha|}, \end{aligned} \tag{4a}$$

or

$$d_\alpha(\omega+i\epsilon) = \frac{k^3 v_\alpha{}^2}{\hat{\eta}_\alpha \sin\hat{\delta}_\alpha} e^{i\hat{\delta}_\alpha}. \tag{4b}$$

Although Eq. (4b) is a special case of Eq. (4a), the difference will have an important consequence in our future development.

In order to relate γ to v and u, we introduce the static crossing symmetry[2,3,12] at *all* energies. This assumption is introduced to avoid the complexity[6,7] in evaluating the general crossing matrix elements. Thus the difference between the static and general crossing matrices is absorbed in the form factors v and u. Fuda[12] used crossing symmetry for the scattering amplitude in the form

$$\frac{G_\alpha^{(+)}(k)^2}{d_\alpha(-\omega-i\epsilon)} = \sum_{\beta=1}^4 A_{\alpha\beta} \frac{G_\beta^{(+)}(k)^2}{d_\beta(\omega+i\epsilon)}, \tag{5}$$

where $A_{\alpha\beta}$ is a crossing matrix element.[2,3,14] On the other hand, the usual crossing relation[2,3] is

$$\frac{1}{d_\alpha(-\omega-i\epsilon)} = \sum_{\beta=1}^4 A_{\alpha\beta} \frac{1}{d_\beta(\omega+i\epsilon)}. \tag{6}$$

Therefore if we introduce a phase factor along the left hand cut by[12]

$$d_\alpha(-\omega-i\epsilon) = |d_\alpha(-\omega-i\epsilon)| e^{-i\varphi_\alpha}, \tag{7}$$

then for the phase convention of Eq. (4a) and for the general crossing relation of Eq. (6),

$$\varphi_\alpha = \tan^{-1}\left[\frac{Y_\alpha}{X_\alpha} \right] \pm l\pi, \tag{8}$$

(l an integer) and also

$$\mathrm{Im}\, d_\alpha(-\omega-i\epsilon) = -k^3 \frac{Y_\alpha}{X_\alpha{}^2 + Y_\alpha{}^2} \equiv k^3 w_\alpha(\omega), \tag{9}$$

where

$$X_\alpha \equiv \sum_{\beta=1}^4 A_{\alpha\beta} |\sin\hat{\delta}_\beta| \hat{\eta}_\beta v_\beta{}^{-2} \cos\theta_\beta, \tag{10a}$$

$$Y_\alpha \equiv \sum_{\beta=1}^{4} A_{\alpha\beta} \,|\sin\hat{\delta}_\beta|\, \hat{\eta}_\beta v_\beta{}^{-2} \sin\theta_\beta \;. \tag{10b}$$

Correspondingly, for the choice of Eq. (4b), we have similar expressions with the replacement of $|\sin\hat{\delta}_\beta|\cos\theta_\beta$ and $|\sin\hat{\delta}_\beta|\sin\theta_\beta$ of Eq. (10) by $\sin\hat{\delta}_\beta\cos\hat{\delta}_\beta$ and $\sin^2\hat{\delta}_\beta$, respectively. Fuda's crossing symmetry yields the results which are obtained by omitting $v_\beta{}^{-2}$ in Eq. (10).

By comparing the imaginary parts of Eqs. (1) and (4), with the help of Eq. (9), we can express the imaginary parts of the coupling constant in terms of form factors as

$$\mathrm{Im}\gamma_\alpha(\omega+i\epsilon)^{-1} = k^3 v_\alpha{}^2(1-\nu_\alpha{}^{-1}) \;, \tag{11a}$$

$$\mathrm{Im}\gamma_\alpha(-\omega-i\epsilon)^{-1} = k^3(u_\alpha{}^2+w_\alpha) \;, \tag{11b}$$

where

$$\nu_\alpha{}^{-1} = \frac{\sin\theta_\alpha}{\hat{\eta}_\alpha\,|\sin\hat{\delta}_\alpha|}, \quad \text{for Eq.(4a)}$$

$$= \frac{1}{\hat{\eta}_\alpha}, \quad \text{for Eq.(4b)} \;. \tag{11c}$$

Since in the high energy limit $d_\alpha(z)$ must approach unity, this imposes another constraint[18] on $\gamma(z)$ as

$$\gamma_\alpha(z) \overset{|z|\to\infty}{\to} 1 \;. \tag{12}$$

Thus by assuming no other singularities for $\gamma(z)$ on the first sheet of the complex Z plane aside from those given by Eq. (11), we can write a dispersion relation

$$\gamma_\alpha(z)^{-1}-1 = \frac{1}{2\pi i}\oint_{c_1}\frac{dz'}{z'-z}\{\gamma_\alpha(z')^{-1}-1\} = \frac{1}{\pi}\int_\mu^\infty dx\left\{\frac{\mathrm{Im}\gamma_\alpha(x)^{-1}}{x-z}+\frac{\mathrm{Im}\gamma_\alpha(-x)^{-1}}{x+z}\right\}$$
$$= \frac{1}{\pi}\int_\mu^\infty dx\, p^3\left[\frac{1-\nu_\alpha{}^{-1}}{x-z}v_\alpha{}^2+\frac{u_\alpha{}^2+w_\alpha}{x+z}\right], \tag{13}$$

where the contour c_1 consists of an infinite circle joined by parallel lines along the cuts for $|z|>\mu$. In the second step, the following assumption was made for $\gamma(z)$ across the cuts

$$\gamma_\alpha(z)^* = \gamma_\alpha(z^*) \;. \tag{14}$$

Hence by substituting Eq. (13) into Eq. (1), we arrive at

$$d_\alpha(z) = 1-\frac{1}{\pi}\int_\mu^\infty dx\, p^3\left\{\frac{v_\alpha{}^2}{\nu_\alpha(x-z)}-\frac{w_\alpha}{x+z}\right\}. \tag{15}$$

Since we want $d_\alpha(z)$ to have a nucleon pole at $z=0$,[12] putting the condition

$$d_\alpha(0)=0 \tag{16}$$

back into Eq. (15), we obtain

$$d_\alpha(z) = -\frac{z}{\pi}\int_\mu^\infty \frac{dx}{x}p^3\left\{\frac{v_\alpha{}^2}{\nu_\alpha(x-z)}+\frac{w_\alpha}{x+z}\right\}. \tag{17}$$

Thus by calling the residue of $d_\alpha(z)^{-1}$ at $z=0$ λ_α, we finally arrive at

$$d_\alpha(z) = z\lambda_\alpha{}^{-1}\left[1-\frac{z\lambda_\alpha}{\pi}\int_\mu^\infty\frac{dx}{x^2}p^3\left\{\frac{v_\alpha{}^2}{\nu_\alpha(x-z)}-\frac{w_\alpha}{x+z}\right\}\right], \tag{18}$$

where λ_α is going to be given by[2,3,12]

$$\lambda_\alpha = \frac{2}{3}\left[\frac{f}{\mu}\right]^2 \begin{cases} -4 \\ -1 \\ -1 \\ 2 \end{cases} \text{for } \alpha = \begin{cases} 1 & (1,1) \\ 2 & (1,3) \\ 3 & (3,1) \\ 4 & (3,3) \end{cases}, \tag{19}$$

with $f^2=0.086$.[12]

In order to determine v^2 and w, a dispersion relation for $\ln d(z)$ may be employed, and Fuda[12] has introduced an ingenious function $D(z)$ to cancel out the zero of $d(z)$. According to this method, the solution of Eq. (18) is given by

$$d_\alpha(z) = \frac{z}{z-\mu}\exp\left\{-\frac{1}{\pi}\int_\mu^\infty dx\left[\frac{\theta_\alpha}{x-z}+\frac{\varphi_\alpha}{x+z}\right]\right\}, \quad \alpha=1,2,3 \tag{20a}$$

and

$$d_4(z) = \frac{z}{z+\mu}\exp\left\{-\frac{1}{\pi}\int_\mu^\infty dx\left[\frac{\theta_4}{x-z}+\frac{\varphi_4}{x+z}\right]\right\}. \tag{20b}$$

Now we eliminate illusory poles at $z=\pm\mu$ via

$$\int_\mu^\infty \frac{dx}{x(x-z)}=\frac{1}{z}\ln\frac{\mu}{z-\mu}\ ,$$
$$\int_\mu^\infty \frac{dx}{x(x+z)}=\frac{1}{z}\ln\frac{z+\mu}{\mu}\ , \tag{21}$$

and introduce the coupling constant λ_α, but the order in which this is done is subtlely important. In the case of negative λ, we first take the residue of $d(z)$ and then apply Eq. (21). This order ensures that λ is negative through an intermediate step. However, for positive λ, this order is not important and both orders produce the same result. In the end we obtain

$$d_\alpha(z)=\mp z\lambda_\alpha{}^{-1}\exp\left\{-\frac{z}{\pi}\int_\mu^\infty\frac{dx}{x}\left[\frac{\theta'_\alpha}{x-z}-\frac{\varphi'_\alpha}{x+z}\right]\right\}\quad \text{for}\begin{cases}\lambda<0\\ \lambda>0\end{cases} \tag{22a}$$

with

$$\theta'_\alpha=\begin{cases}\theta_\alpha-\pi\\ \theta_\alpha\end{cases},\ \varphi'_\alpha=\begin{cases}\varphi_\alpha\\ \varphi_\alpha-\pi\end{cases}\ \text{for}\begin{cases}\lambda<0\\ \lambda>0\end{cases}. \tag{22b}$$

The functional form of v^2 can be obtained by comparing the positive energy imaginary parts of Eqs. (18) and (22a) as

$$v_\alpha(\omega)^2=\frac{\omega\hat{\eta}_\alpha}{k^3\lambda_\alpha}\,|\sin\hat{\delta}_\alpha|\times\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\theta'_\alpha}{x-\omega}-\frac{\varphi'_\alpha}{x+\omega}\right]\right\}, \tag{23a}$$

where P means taking the principal value integral. Similarly from the negative imaginary parts,

$$w_\alpha(\omega)=-\frac{\omega}{k^3\lambda_\alpha}\sin\varphi_\alpha\times\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\varphi'_\alpha}{x-\omega}-\frac{\theta'_\alpha}{x+\omega}\right]\right\}. \tag{24a}$$

Correspondingly, had we started from the definition of Eq. (4b), we would obtain

$$v_\alpha(\omega)^2=\frac{\omega\hat{\eta}_\alpha}{k^3\lambda_\alpha}\sin\hat{\delta}_\alpha\times\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\hat{\delta}'_\alpha}{x-\omega}-\frac{\varphi'_\alpha}{x+\omega}\right]\right\} \tag{23b}$$

and

$$w_\alpha(\omega)=-\frac{\omega}{k^3\lambda_\alpha}\sin\varphi_\alpha\times\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\varphi'_\alpha}{x-\omega}-\frac{\hat{\delta}'_\alpha}{x+\omega}\right]\right\}. \tag{24b}$$

Clearly Eq. (23a) is not acceptable for $\lambda<0$ interactions. Fuda's crossing symmetry, Eq. (5), and the usual one, Eq. (6), are the same in the Chew-Low model, as the form factor is state independent, and one should have one v^2 instead of four v's and w's. Therefore Fuda's choice seems to take the Chew-Low model as the lowest order approximation and relegate whatever is left behind in this approximation into eight independent form factors. Naturally this approximation works well only if Eq. (5) contains a good deal of physics. We can also take the Chew-Low model as the first approximation, but in higher order iteration processes, to calculate φ, the general crossing symmetry, Eq. (6) may be used.

III. DETERMINATION OF PHASES

The above solutions we obtained are not complete until the meaning of phases is unambiguously given.[20] In the present problem there are two phases to be determined, θ and φ. In the case of Eq. (4b), θ is predetermined to be $\hat{\delta}$.

The first criterion for the phases is obtained by a modified Levinson theorem,[12,20,21] namely,

$$\frac{1}{2\pi i}\oint_{c_1} d\ln d_\alpha(z)=1\ , \tag{25}$$

as there is only one nucleon pole within the contour c_1. This, with the help of Eqs. (4), (7), and (23a), leads to

$$\theta_\alpha(\mu)+\varphi_\alpha(\mu)-\theta_\alpha(\infty)-\varphi_\alpha(\infty)=\pi\ . \tag{26}$$

The second condition comes from the requirement that $d(z)$ must approach unity as $|z| \to \infty$. Because of Eq. (21), if both θ and φ approach certain constants in the high energy limit, then

$$\exp\left\{-\frac{z}{\pi}\int_{\mu}^{\infty}\frac{dx}{x}\left[\frac{\theta'}{x-z}-\frac{\varphi'}{x+z}\right]\right\} \overset{|z|\to\infty}{\longrightarrow} \exp\left\{\frac{\theta'(\infty)}{\pi}\ln\frac{|z-\mu|}{\mu}+\frac{\varphi'(\infty)}{\pi}\ln\frac{z+\mu}{\mu}\right\}. \quad (27)$$

Thus in order for $d(z)$ to remain finite, we must have

$$\theta(\infty)+\varphi(\infty)=0 \quad (28)$$

which, combined with Eq. (26), yields

$$\theta(\mu)+(\mu)=\pi\ . \quad (29)$$

Thirdly, from Eqs. (2) and (4a), θ must be equal to $\hat{\delta}$ or different from it only by an odd multiple of π, and finally the choice of θ and φ must be consistent with Eqs. (8) and (24). Similar conclusions could be drawn for the case of Eq. (4b).

IV. P_{11} PARTIAL WAVE

The P_{11} wave has been known to change the sign of its phase shift at about $\omega_0=256$ MeV, and hence Eq. (23b) leads to a conflict. Sign changing phenomena of the phase shift or of the t matrix passing through zero was extensively studied in Ref. 18, along with the occurrence of poles in the t matrix. Their study shows that zeros of the t matrix are the result of $\gamma(z)$ passing through zeros or $\gamma(z)^{-1}$ having poles, and that the poles of the t matrix result from zeros of $d(z)$. Although the zero of the t matrix for the P_{11} channel lies outside the contour c_1, and hence Eq. (13) is still valid, we want to modify that equation so that the singularity of $\gamma(z)$ may more clearly manifest. Thus $\gamma(z)$ for the P_{11} channel can be written as

$$\gamma_a(z)=(z-\omega_0)\bar{\gamma}_1(z)\ . \quad (30)$$

Then because of Eq. (12), $\bar{\gamma}_1(z)^{-1}$ must approach $z+a$ as $|z|\to\infty$, where a is a complex constant. Hence by assuming that the singularities of $\gamma_1(z)$ are to be given by Eqs. (11) and (30), we have the following dispersion relation, instead of Eq. (13):

$$\bar{\gamma}_1(z)^{-1}=z+a+\frac{1}{\pi}\int_{\mu}^{\infty}dx\,p^3\left\{\frac{x-\omega_0}{x-z}(1-\nu_1^{-1})v_1^2-\frac{x+\omega_0}{x+z}(u_1^2+w_1)\right\}. \quad (31)$$

By using similar techniques as in Sec. II, i.e., by eliminating a, introducing λ, and so on, we finally arrive at

$$d_1(z)=z\lambda_1^{-1}\left[1-\frac{z}{z-\omega_0}-\frac{z\lambda_1}{\pi(z-\omega_0)}\int_{\mu}^{\infty}\frac{dx}{x^2}p^3\left[\frac{x-\omega_0}{x-z}\frac{v_1^2}{\nu_1}+\frac{x+\omega_0}{x+z}w_1\right]\right]. \quad (32)$$

Again a dispersion relation involving $\ln d_1$ must be written to obtain v_1^2 and w_1. This time, however, we must impose the sign changing capability on Fuda's D_1 function.[12]

In Refs. 18 and 20 the authors have also indicated a way to write a dispersion relation when the t matrix has zeros and poles. Unfortunately their method of mapping the complex energy plane onto the complex momentum plane will introduce an additional cut along the imaginary momentum axis when it is applied to the present case. Therefore we take a different path and introduce a new function $D_1(z)$ by

$$D_1(z)=\frac{\sqrt{z-\mu}+i\sqrt{\mu}}{\sqrt{z-\mu}-i\sqrt{\mu}}\times\frac{z-\omega_0}{(\sqrt{z-\mu}+i\sqrt{2\mu})^2}d_1(z)\ . \quad (33)$$

The first factor on the right hand side was introduced by Fuda[12] to cancel the zero of $d_1(z)$. The second factor was introduced so that Canchy's integral theorem may be applied for $\ln D_1(z)$, which incorporates the phase differences across the cuts, and that $d_1(z)$ may have a capability to cope with the sign changing property of phase shift in the end. Now the second factor has only the square root cut for $z>\mu$ as an argument of logarithmic function and nowhere does it becomes real negative within the contour c_1. Since the other factor has no singularity within c_1, which was discussed in detail in Ref. 12, we can immediately write, for the phase convention of Eq. (4a),

$$\begin{aligned}\ln D_1(z)&=\frac{1}{2\pi i}\oint_{c_1}\frac{dz'}{z'-z}\ln D_1(z')\\&=-\frac{1}{\pi}\int_{\mu}^{\infty}dx\left[\frac{\theta_1}{x-z}+\frac{\varphi_1}{x+z}\right]\\&\quad+\ln\frac{(\sqrt{z-\mu}+i\sqrt{\mu})^2}{z-\mu}+F(z)\ .\end{aligned} \quad (34)$$

Here

$$F(z)=\frac{1}{2\pi i}\oint_{c_1}\frac{dz'}{z'-z}\ln\left\{\frac{z'-\omega_0}{(\sqrt{z'-\mu}+i\sqrt{2\mu})^2}\right\}$$
$$=\frac{1}{2\pi i}\int_\mu^\infty\frac{dx}{x-z}\ln\left\{\frac{(-\sqrt{x-\mu}+i\sqrt{2\mu})^2}{(\sqrt{x-\mu}+i\sqrt{2\mu})^2}\right\}, \tag{35}$$

where we have made use of the property of the integrand that it has only a square root cut for $z>\mu$ and that $(z'-\omega_0)$ has no discontinuity. Therefore by considering another contour c_2 which consists of an infinite circle joined by parallel lines along the cut for $z>\mu$ *only,* we see that Canchy's theorem tells us

$$F(z)=\ln\left[\frac{-\sqrt{z-\mu}+i\sqrt{2\mu}}{\sqrt{z-\mu}+i\sqrt{2\mu}}\right]. \tag{36}$$

Hence by comparing Eq. (33) with the combined results of Eqs. (34)—(36), we arrive at

$$d_1(z)=-\frac{z}{z-\mu}\frac{z+\mu}{z-\omega_0}\times\exp\left\{-\frac{1}{\pi}\int_\mu^\infty dx\left[\frac{\theta_1}{x-z}+\frac{\varphi_1}{x+z}\right]\right\}. \tag{37}$$

By first calling the residue of d_1^{-1} at $z=0$ λ_1 and then including the factors $\mu/(z-\mu)$ and $(z+\mu)/\mu$ via Eq. (21) into the exponential functions, the final expression for $d_1(z)$ becomes

$$d_1(z)=z\lambda_1^{-1}\frac{\omega_0}{z-\omega_0}\times\exp\left\{-\frac{z}{\pi}\int_\mu^\infty\frac{dx}{x}\left[\frac{\theta_1-\pi}{x-z}-\frac{\varphi_1+\pi}{x+z}\right]\right\}. \tag{38}$$

The form factors are again obtained by comparing the imaginary parts of Eqs. (32) and (38) as

$$v_1(\omega)^2=-\frac{\omega\hat{\eta}_1}{k^3\lambda_1}\frac{\omega_0}{\omega-\omega_0}\,|\sin\hat{\delta}_1|\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\theta_1-\pi}{x-\omega}-\frac{\varphi_1+\pi}{x+\omega}\right]\right\}, \tag{39a}$$

and

$$w_1(\omega)=\frac{\omega}{k^3\lambda_1}\frac{\omega_0}{\omega+\omega_0}\sin\varphi_1\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\varphi_1+\pi}{x-\omega}-\frac{\theta_1-\pi}{x+\omega}\right]\right\}. \tag{40a}$$

Clearly Eq. (39a) is not acceptable. However, if we start with the phase convention of Eq. (4b), then we obtain

$$v_1(\omega)^2=-\frac{\omega\hat{\eta}_1}{k^3\lambda_1}\frac{\omega_0}{\omega-\omega_0}\sin\hat{\delta}_1\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\hat{\delta}_1-\pi}{x-\omega}-\frac{\varphi_1+\pi}{x+\omega}\right]\right\} \tag{39b}$$

and

$$w_1(\omega)=\frac{\omega}{k^3\lambda_1}\frac{\omega_0}{\omega+\omega_0}\sin\varphi_1\exp\left\{-\frac{\omega}{\pi}\mathrm{P}\int_\mu^\infty\frac{dx}{x}\left[\frac{\varphi_1+\pi}{x-\omega}-\frac{\hat{\delta}_1-\pi}{x+\omega}\right]\right\}. \tag{40b}$$

The form of Eq. (39b) can accommodate a sign changing phase shift without leading to a contradiction. Thus for $\lambda<0$ interactions we must use Eq. (4b), contrary to $\lambda>0$ cases for which both Eqs. (4a) and (4b) may be used. The phase φ_1 is determined by Eq. (8) subject to the conditions

$$\hat{\delta}_1(\infty)+\varphi_1(\infty)=0\,, \tag{41a}$$

$$\hat{\delta}_1(\mu)+\varphi_1(\mu)=\pi\,. \tag{41b}$$

V. DISCUSSION

Although there is one analytical expression for the right hand side form factor v^2 in either Eq. (23) or (39), there are two analytical solutions for the left hand side form factor w in Eqs. (9) and (24) or (40). The two solutions for w should agree with each other if the static crossing relation [Eq. (6)] and analyticity assumptions embody the correct interaction mechanism. Otherwise the degree of resemblance between the two solutions represents the extent of validity of Eq. (6) and analyticity. Since in practice the static crossing symmetry cannot be valid everywhere, neither solution should be able to reproduce the input phase shift $\hat{\delta}$ exactly.

In order to obtain solutions for v^2 and w, first solutions for v^2 and φ may be obtained from Eqs. (8), (23b), and (39b), since these do not involve w. w can be calculated after v^2 and φ are determined. However, v^2 and φ constitute coupled nonlinear integral equations and hence solutions cannot be readily obtained. Thus one may assume that the Chew-Low result[3] or Fuda's crossing symmetry[12] is a good first approximation for evaluating φ which does not involve v^2, and then start an iteration procedure by calculating v^2 and φ in turn. The solutions so obtained must be self-consistent, i.e., must reproduce the input $\hat{\delta}$.

Aside from the analytical solutions, one may try obtaining a numerical solution. Since a completely numerical solution is impossible to obtain, one may determine only the function w numerically, v^2 and φ being determined by the same method as in the preceding paragraph. Or one may go one step further, forgetting about the analytical solution and assuming a functional form of v. The w can be set through Eq. (9), and Eq. (18) or (32) may be solved.

We have tried a number of possible solutions by selecting suitable values for θ and φ, but were unable to reproduce the input $\hat{\delta}$ which was calculated from the data of Almehed and Lovelace.[22] We have also tried several numerical solutions by assuming a functional form of w, but were unsuccessful. Moreover, we have sought after a solution, to no avail, by assuming a functional form of v and using Eqs. (18) and (32). The sources of failure to obtain an analytical solution may be ascribed to the use of static crossing symmetry at all energies, the poor method of solving nonlinear equations for v and φ, and the presence of hidden singularities. On the other hand, the parameter fitting of numerical solutions has failed because of the mixing process of all partial waves which was required by the crossing symmetry. In principle, one has to determine all four sets of parameters at the same time.

We have improved in this paper Fuda's pioneering work[12,14] to construct a πN separable model which satisfies most of the field theoretically required analytical properties for the scattering amplitude. Although formal solutions for the form factors were obtained, because of the underlying crossing symmetry, an acceptable numerical solution was not found for the moment. Since it is so hard to fully implement even the static crossing symmetry and obtain a reasonable solution, it may be necessary to modify the content of crossing symmetry in future calculations. Thus far, none of the models for πN interaction in actual calculations has implemented the crossing symmetry.

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