Reduced transition probabilities of vibrational states in ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb

R.M. Ronningen,* R. S. Grantham, J. H. Hamilton, R. B. Piercey, A. V. Ramayya, B. van Nooijen,† H. Kawakami,‡ W. Lourens,† and R. S. Lee
Department of Physics, Vanderbilt University, Nashville, Tennessee 37235

W. K. Dagenhart
Fusion Energy Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

L. L. Riedinger
Department of Physics, University of Tennessee, Knoxville, Tennessee 37916
(Received 25 January 1982)

The vibrational states of ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb have been studied via Coulomb excitation. Thin high purity targets were prepared in an isotope separator. The Coulomb excitation of these nuclei was studied by the scattering of 12–16 MeV α particles, which were detected in an Enge split-pole spectrograph. The reduced transition probabilities $B(E\lambda;0^+_{\text{g.s.}}\rightarrow I^\pi K)$ were obtained for $I^\pi K=2^+2$, 2^+0, and $I^\pi=3^-$ states in each nucleus. Limits on $B(E\lambda)$ values and values extracted from published (d,d') cross sections are also presented. Trends in $B(E2)$ values and energy levels for the Dy, Er, and Yb nuclei show that the $K^\pi=2^+$ levels are reasonably constant in energy for only Dy and Er. In all three nuclei $B(E2)$ values decrease with increasing neutron number. Except for the Dy isotopes, the lightest isotope has the largest $B(E2)$ value for the lowest lying $I^\pi K=2^+0$ state. $I^\pi=3^-$ states level energies and $B(E3)$ values are found to be in fair agreement with microscopic calculations. Values of $B(E2)$ for $K^\pi=2^+$ states are in fair agreement with microscopic calculations when these exist.

[NUCLEAR REACTIONS ^{156,158}Dy(α,α'), ^{162,164}Er(α,α'), ^{168}Yb(α,α'), $E=12-16$ MeV; measured σ, deduced $B(E\lambda)$. Enriched targets.]

I. INTRODUCTION

The development of techniques for studies of high-spin states in the yrast and side bands in the rare earth region has emphasized the importance of measuring absolute transition elements to members of these bands in order to learn of their collective natures, at least at the beginnings of these bands. For example, the use of heavy ions in Coulomb-excitation measurements of high-spin collective nature of states relies on bandhead $B(E2)$ and $B(E3)$ values from light-ion measurements to facilitate the interpretation of the γ-ray yields for the high-spin states. We therefore have engaged in a program of measurement of absolute $B(E2)$ and $B(E3)$ values to $I^\pi=2^+$ and 3^- states of energies up to ≈ 2 MeV in the most neutron-deficient stable dysprosium, erbium, and ytterbium nuclei by α-particle Coulomb excitation. Our results from these measurements on $E2$ and $E4$ matrix elements for the 2^+ and 4^+ members of the ground bands in these nuclei have already appeared.[1]

Of interest in these types of measurements is the trend of $B(E2)$ or $B(E3)$ values to systematically occurring vibrational states in an isotopic sequence. It was noted in early Coulomb excitation[2] and inelastic deuteron scattering studies[3–7] of deformed rare-earth nuclei, that both $E2$ and $E3$ excitations of vibrational-like $I=2^+$ and 3^- states were, in general, stronger for increasingly more neutron deficient isotopes of a given nucleus. Our Coulomb excitation studies of deformed Gd and Hf nuclei[8] substantiates this for these nuclei. The trends for the stable Dy, Er, and Yb isotopes are presented here.

While many theories of collective motion are successful in explaining level spacings in vibrational or quasiparticle bands, precise reduced $E2$ or $E3$ transition probabilities provide very stringent tests of these theories. Comparisons of our measured values to microscopic calculations are made.

II. EXPERIMENTAL PROCEDURE

Alpha particles were produced in the EN tandem Van de Graaff accelerator at Oak Ridge National Laboratory (ORNL). Experiments using laboratory angles of 150° and 90° were performed on each target, except for ^{156}Dy, where only 150° was used. When the scattered particles were to be detected at 90° we used higher beam energies than for the 150° studies so that about the same distances of closest approach would be achieved at each angle. The beam energies were (for 150° and 90°, respectively) 12 and 15 MeV for ^{158}Dy, 13 and 15.5 MeV for ^{162,164}Er, 13 and 16 MeV for ^{168}Yb, and 13 MeV (150° only) for ^{156}Dy. Our choices of beam energies were governed by Coulomb-nuclear interference studies[9,10] of the excitations of the first 2^+ and 4^+ states of some neighboring nuclei. We chose beam energies by requiring the target-projectile surface separation to be ~ 7 fm at closest approach, assuming spherical nuclei with $R_0 = 1.2$ fm,[9] a separation judged to be safe in view of these studies. Under the influence of the nuclear force some α particles may be detected from trajectories which would carry them into more forward angles than if only the Coulomb force is present. This process should have a negligible effect in our 150° data because our beam energies are based on "safe" energies from existing data at the same angle.[9,10] For 90° and the higher beam energies we cannot rule out this process. But, the $B(E2)$ values for the first 2^+ states,[1] and for the vibrational states as we note below, obtained from both the 150° and 90° data agree. The first 2^+ state data is most sensitive because of the larger excitation probabilities and the smaller statistical uncertainties. We conclude that the influence of the nuclear force in our experiment is small. As in our Gd and Hf studies[8] we assumed that the onset of nuclear excitation of the vibrational states occurs at the same distance of closest approach of target and projectile as for the first 2^+ and 4^+ states.

The detection system consisted of a position-sensitive gas-flow proportional counter mounted in the focal plane of an Enge spit-pole magnetic spectrograph. Efficiency and linearity calibrations of the system were made by measuring the yields and positions of the two most intense α groups in the decay of ^{244}Cm as functions of the magnetic field strength.

Because we were also interested in obtaining precise values of the $E2$ and $E4$ reduced matrix elements to the first 2^+ and 4^+ states of these nuclei, we obtained very high purity ($\gtrsim 99\%$) thin ($\sim 20-30\ \mu$g/cm^2 material on $\sim 80\ \mu$g/cm^2 carbon foil backings) targets. These were prepared at the ORNL 180° isotope separator[11] using natural material as feed, with the high purities obtained after one pass.

III. DATA ANALYSIS

Figures 1 and 2 show α-particle spectra in the region of 1 MeV excitations from our Dy studies. Typical spectra for ground band states were presented in Ref. 1. Experimental ratios of inelastic-to-elastic scattering differential cross sections were found from ratios of the areas of peaks of inelastic scattering to the elastic peak. These ratios were compared with those calculated with the aid of the semiclassical Coulomb-excitation code by Winther and de Boer,[12] modified[13] to include $E1$, $E3$, and $E4$ excitations. The matrix elements $\langle 0^+_{\text{g.s.}} || M(E\lambda) || I^\pi = \lambda^\pi \rangle$ were free parameters with all other nonzero matrix elements deduced from adiabatic rotor-vibrator (collective) model predictions.[14] The states included were $0^+_{\text{g.s.}} - 6^+_{\text{g.s.}}$, and excited states with $I^\pi = 3^-$ and $I^\pi K = 0^+0$, 2^+0, 2^+2, and 4^+2. To test the validity of the use of collective model matrix elements to describe the excitations of the vibrational states, calculations were done for ^{168}Yb using matrix elements deduced from experimental $B(E2)$ ratios from γ-ray studies. The calculated cross sections differed by less than one percent from the ones resulting from the use of collective model matrix elements. Since multistep excitations of 3^- states were estimated to be less than 2%, as deduced by coupling the $I^\pi = 2^-$ and 3^- states in ^{158}Dy to the $0^+ - 6^+$ ground band states by

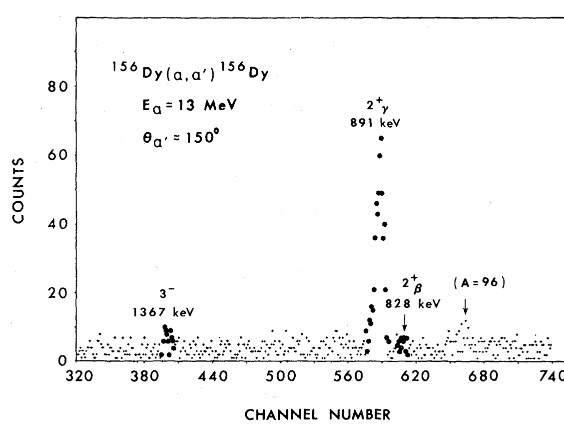

FIG. 1. Alpha particle spectrum for low-lying vibrational states in ^{156}Dy. Although the 828 keV state is labeled "β," it is not collective.

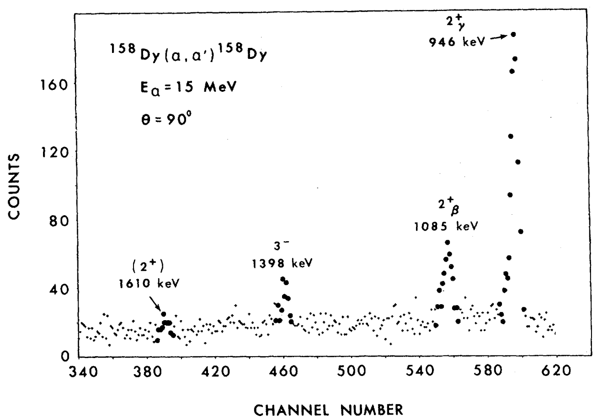

FIG. 2. Alpha particle spectrum for low-lying vibrational states in ^{158}Dy. The lowest lying $I^\pi K=2^+0$ state at 1085 keV is much more strongly excited than the 828 keV state in ^{156}Dy, as seen by comparing this spectrum with the one in Fig. 1.

all nonzero $E2$ and $E3$ collective model matrix elements, our $B(E3)$ values are from calculations with only direct excitations of the 3^- states. The value of $\langle 2^+||M(E2)||2^+\rangle$ is known (e.g., Refs. 15 and 16) to be important in these calculations; it is proportional to the static quadrupole moment of that 2^+ state. We have used the values of this matrix element as predicted by the collective model. That is, the ground band and excited bands are assumed to have quadrupole moments of equal magnitude, with that of a $K=0$ band differing in only the sign from that for a $K=2$ band. For the nuclei we have studied this should be a very good approximation in view of the study of ^{166}Er by McGowan *et al.*[16] and the study of ^{180}Hf by Baker *et al.*[17] However, for transitional nuclei the rotational model assumption may not be a good one, as recent studies[16,18] of some W isotopes show.

Because there are differences between quantum mechanical and semiclassical calculations of cross sections, which may be non-negligible for vibrational states,[15] five level (0, 2, 4, 2′, 4′) or (0, 2, 4, 0′, 2′) calculations were performed using both kinds of calculations for a test case of ^{168}Yb ($E=13$ MeV, $\theta=150°$, and $E=16$ MeV, $\theta=90°$). When $\theta=150°$, the quantum mechanical calculation gave a larger ($\sim 3.7\%$) cross section for the $I^\pi K=2^+2$ state and $\sim 2.8\%$ smaller ones for the $I^\pi K=2^+0$ states. When $\theta=90°$, these differences were $\sim 3\%$ and $\sim 3.6\%$, respectively. Our $B(E2)$ values deduced from semiclassical calculations were adjusted by using these results.

Finally, the actual scattering potential is not a pure Coulomb ($V \propto 1/r$) one. It contains small contributions from the atomic screening of the target nuclear charge, the dipole polarizability of the target nucleus, the vacuum polarization, and from relativistic terms. First order estimates of these effects were calculated by using the prescriptions of Alder and Winther.[19] For our experimental conditions the atomic screening and vacuum polarization effects were the largest ones by at least one order of magnitude. These were very nearly equal in magnitude, but opposite in sign, and would produce a net correction of less than 1% to the calculated cross sections. These corrections were thus not applied.

IV. RESULTS AND DISCUSSION

A. Experimental results

Our experimental results for absolute $B(E2)$ and $B(E3)$ values are summarized in Table I. The uncertainties in our $B(E\lambda)$ values are from the statistical errors. These values are the weighted averages of the results of the experiments for which $\theta=150°$ and 90°, except for ^{156}Dy, where $\theta=150°$ only. Agreement between the results at the two angles was very good except for the $I^\pi K=2^+0$ state in ^{158}Dy, where the two results differed by two standard deviations. For the level energies and spin-parity assignments (with exceptions as discussed below) we rely on those adopted by the *Nuclear Data Sheets*[20–24] compilers. In Table I we also make a comparison to $B(E\lambda)$ values from (d,d') reaction studies.[5–7] However, those values as published were obtained by normalizing the (d,d') cross sections to earlier Coulomb excitation results (e.g., Ref. 2) whose precision is currently surpassed. Instead, $B(E\lambda)$ values from this work and other recent Coulomb excitation studies[15,25–33] were compiled for each stable even-A Dy, Er, and Yb nucleus, when these values existed. If there were several measurements, these were error-weight-averaged. The $B(E\lambda)$ values for the (d,d') studies, as given in Table I, were then deduced as was done originally, except that a weight averaging of the normalization constants was also performed. A comparison between Coulomb excitation and (d,d') values is beneficial as it allows a reasonably accurate determination of $B(E\lambda)$ values for states which have small (d,d') cross sections, or might not be observed in a Coulomb excitation experiment.

1. ^{156}Dy

This nucleus has several structural peculiarities, some common to other $N=90$ nuclei, such as

TABLE I. Experimental absolute $B(E\lambda)$ strengths for ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb from (α,α') Coulomb excitation studies. A comparison is made with $B(E\lambda)$ values deduced from (d,d') cross sections as discussed in the text. Some states are given which have proposed 2^+ or 3^- spin-parity assignments but are not observed (N) in one of the reactions.

Nucleus	E (keV)	$I^\pi K$	$B(E\lambda;0^+_{\mathrm{g.s}}\rightarrow I^\pi K)$[a] (e^2b^λ)	$B(E\lambda)$ from (d,d') studies[b] (e^2b^λ)	$\frac{B(E\lambda)^{\mathrm{c}}}{B(E\lambda)_{\mathrm{sp}}}$
^{156}Dy	138	$2^+_{\mathrm{g.b.}}$	3.72(3)		149(2)
	828	2^+0	0.008(5)	0.004	0.3(2)
	891	2^+2	0.180(11)	0.217	7.2(5)
	1367	$3^-?$	0.22(7)	0.26	22(7)
	1408	(3^-)	N	0.009	0.9
^{158}Dy	99	$2^+_{\mathrm{g.b.}}$	4.67(4)		184(2)
	946	2^+2	0.149(8)	0.158	5.9(4)
	1085	2^+0	0.053(8)	0.039	2.1(4)
	1398	$3^-?$	0.23(5)	0.22	22(5)
	1610	(2^+)	<0.023	0.014	0.6
	1710	(2^+)	N	0.004	0.2
^{162}Er	102	$2^+_{\mathrm{g.b.}}$	5.01(3)		191(2)
	901	2^+2	0.164(8)	0.170	6.3(4)
	1171	2^+0	0.042(7)	0.043	1.6(3)
	1357	$3^-?$	0.19(4)	0.17	17(4)
	1430	(2^+0)	0.018(8)	0.010	0.7(5)
	1500[d]	(2^+)			
			0.011(11)	N	0.4(4)
	1523[d]	$(1^+,2^+)$			
	1623	$3^-?$	<0.072	0.03	3
^{164}Er	91	$2^+_{\mathrm{g.b.}}$	5.48(4)		206(2)
	860	2^+2	0.148(6)	0.170	5.6(3)
	1315	2^+0	0.006(3)	N	0.23(12)
	1433	$3^-?$	0.15(3)	0.11	13(3)
	1484	2^+	0.030(9)	0.28	1.1(4)
	1568	$3^-?$	0.091(34)	0.05	8(3)
^{168}Yb	88	$2^+_{\mathrm{g.b.}}$	5.77(4)		210(2)
	984	2^+2	0.127(6)	0.128	4.6(3)
	1234	2^+0	0.050(5)	0.05	1.8(2)
	1277	(2^+0)	N	N	
	1480	$3^-?$	0.22(4)	0.20	19(4)
	1600	(3^-)	0.09(2)	0.10	8(2)

[a]This work. The uncertainties (in parentheses) are one-standard-deviation values and represent variations in the last digits of the best values. For example, 0.143(11) may be written as 0.143 ± 0.011. The $B(E2)$ values to the 2^+ members of the ground band ($2^+_{\mathrm{g.b.}}$) are from Ref. 1.
[b]References 5–7.
[c]$B(E\lambda)_{\mathrm{sp}}=(2\lambda+1)/4\pi[3/(\lambda+3)]^2(0.12A^{1/3})^{2\lambda}\ e^2b^\lambda$ for $I_i=0$ and $I_f=\lambda$.
[d]These levels are not resolved in our study. The summed strength is given.

^{154}Gd. For example, both the ground and $K^\pi=0^+$ excited bands backbend,[34] as happens in ^{154}Gd. A "superband" has been proposed[35] to help explain the "twin-backbending" features, and searches for the low spin members have been made.[36,37] But, unlike the $I=2$ member of the excited $K^\pi=0^+$ band in ^{154}Gd, which has an absolute $B(E2)$ strength of ~1.5 spu (Ref. 8), the corresponding state in ^{156}Dy, at 828 keV, is only weakly excited in our (α,α') study as well as in the (d,d') reaction.[5]

In our study, the $I^{\pi}K=2^{+}2$ state at 891 keV is strongly excited, as is the $I^{\pi}K=3^{-}1$ state at 1368 keV. There is no evidence for the state at 1408 keV which is now proposed[37] to have $I^{\pi}=3^{-}$.

2. ^{158}Dy

The ^{158}Ho decay studies[38] and the (d,d') reaction studies[5] have located many of the levels in this nucleus. Strongly excited in our study are the $I^{\pi}K=2^{+}2$, $2^{+}0$, and 3^{-}(?) states at 946, 1085, and 1398 keV, respectively. Evidence for a weak excitation is seen near 1610 keV, where a state with $I^{\pi}=(1,2^{+})$ has been proposed.[38] If we are observing the excitation of this state, we would support an $I^{\pi}=2^{+}$ assignment. There is no evidence for the $I^{\pi}=(2^{+})$ state at 1710 keV, which was weakly excited in the (d,d') reaction.

3. ^{162}Er

Most of the levels in this nucleus have been placed either by the (d,d') reaction study[6] or from the decay of $^{162}\mathrm{Tm}^{m}$ (Ref. 39). The $K^{\pi}=2^{+}$ band, whose bandhead is at 901 keV, is now known[40,41] to (12^{+}). One of the two possible negative parity bands, whose bandhead has $I^{\pi}K=3^{-}1$ at 1352 keV, is now possibly known[40] to $I=13$. We observed strong excitations of $I^{\pi}K=2^{+}2$, $2^{+}0$, and $3^{-}1$ states at 901, 1171, and 1352 keV, respectively. A state at 1430 keV assigned[39] $I^{\pi}K=2^{+}(0)$ was weakly excited. Two other states, candidates for having $I^{\pi}=2^{+}$, lie at 1500 and 1523 keV. A peak is seen near these energies. Since the 23 keV separation is near our resolving capability, we can only estimate a combined strength. The $B(E3)$ value for a state with $I^{\pi}=3^{-}$ at 1623 keV can only be estimated by its (d,d') cross section.

4. ^{164}Er

This backbending nucleus is the subject of intense experimental[42–44] and theoretical[45,46] effort to understand its high-spin nature. The γ vibrational band is known[43,44] to at least $I=21$; its bandhead at 860 keV is strongly excited in our study. However, the $I^{\pi}=2^{+}0$ state at 1315 keV is excited only weakly, this being consistent with a small (d,d') cross section.[6] Instead we find the 2^{+} state at 1484 keV to have about five times more $E2$ strength. The 3^{-} state at 1433 keV was strongly excited, but there is no evidence for excitation of another 3^{-} state at 1568 keV, which was only weakly excited in the (d,d') studies.[6]

5. ^{168}Yb

States adopted[24] as having $I^{\pi}=2^{+}$ are at 984 keV $(K=2)$, and at 1234 keV $(K=0)$. The 984 and 1234 keV states were strongly excited in our studies. A level at 1277 keV is observed in the (p,t) reaction[47] but not in (d,d') studies.[7] It has possibly[51] $I^{\pi}K=2^{+}0$ with the $I^{\pi}=0^{+}$ state being at 1197 keV (Ref. 48). It is not excited in our study. We also see the strong excitation of the state at 1480 keV, which has been given the assignments of $I^{\pi}=3^{-}$ (Ref. 7), $I^{\pi}K=4^{+}0$ (Ref. 47), and $I=(2,3,4)$ (Ref. 24). While we cannot absolutely rule out the possibility of $I^{\pi}=2^{+}$, we obtain consistent $B(E3)$ values at the two different scattering angles, and thus we prefer the $I^{\pi}=3^{-}$ assignment. Also, it would be very unusual to find that ^{168}Yb does not have a collective $I^{\pi}=3^{-}$ state. A state at 1600 keV is weakly excited in our study. It is possibly the state seen in the (p,t) reaction at 1600 keV, in the (d,d') reaction at 1595 keV, and at 1598 keV in the ^{168}Lu decay,[49] with assignments of $I^{\pi}=(2^{+})$, (3^{-}), and (4^{-}) from these studies, respectively. Graetzer *et al.*[48] observe a state at 1543 keV with $I^{\pi}=0^{+}$, and speculate that these two levels may form the lowest part of a $K^{\pi}=0^{+}$ band. But because of its strong excitation in the (d,d') reaction we propose that it has $I^{\pi}=3^{-}$. An unidentified peak is seen at 1940 keV. Riedinger *et al.*[33] performed multiple Coulomb excitation studies of the even-Yb isotopes. In ^{168}Yb they observe the $I^{\pi}K=2^{+}2$ state at 984 keV, and their measured value of $B(E2;0^{+}_{\mathrm{g.s.}}\rightarrow 2^{+}2)=0.132(12)e^{2}\mathrm{b}^{2}$ is in good agreement with our value. They also observe the strong excitation of the $I^{\pi}K=0^{+}0$ state at 1154 keV, which is the bandhead of a $K=0$ band of which the $I^{\pi}=2^{+}$ state at 1234 keV is a member.

B. Collective strength and energy level trends for $I^{\pi}=2^{+}$ states

The trends of the $B(E2)$ strengths (in Weisskopf single particle units) and energy levels are presented, as functions of A, in Fig. 3 for known or proposed $I^{\pi}=2^{+}$ states in the stable, even-A isotopes of Dy, Er, and Yb. Where there exists no precise Coulomb excitation measurement of $B(E2)$, the renormalized value from the (d,d') studies is used.

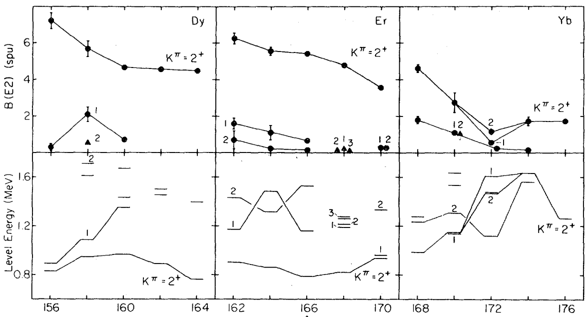

FIG. 3. $B(E2)$ and level energy trends for $I^\pi=2^+$ states in the stable, even -A dysprosium, erbium, and ytterbium nuclei. The trends for $K=2$ states are labeled as such; the other states are suspected to have $I^\pi=2^+$ and are known to, or most likely, have $K=0$. The I^π assignments and compiled from Refs. 15 and 20–33. Circles represent Coulomb excitation measurements (Refs. 15 and 25–33) and triangles represent estimates from (d,d') cross sections (Refs. 5–7) as discussed in the text. The numerical labels relate level energy to $B(E2)$ strength. Connecting lines show how $B(E2)$ strengths are distributed and are not an attempt to classify states as to microscopic structure (it may only be possible to do this for the most collective states).

As in the deformed Gd and Hf nuclei,[8] the lightest Dy, Er, and Yb isotopes have the largest $B(E2)$ value to the $I^\pi K=2^+2$ state. Every stable even-even Dy, Er, and Yb isotope has a collective 2^+2 state. Except for the Yb isotopes, the level energies of these states remain constant, within 200 keV with increasing N.

In our Gd and Hf study[8] we noted that the $B(E2)$ strength to the $I^\pi K=2^+0$ states was largest in the lightest isotope, with decreasing or disappearing strengths accompanied by level energy increases in the heavier isotopes. In our present study we find that the Dy nuclei present a counter example, at least in $B(E2)$ strength. The $B(E2)$ strength to the $I^\pi K=2^+0$ in ^{156}Dy is smaller than to that state in ^{158}Dy; but with its lower level energy it would be expected that this $B(E2)$ value would be larger than that in ^{158}Dy. It is possible that the $K^\pi=0^+$ band in ^{156}Dy (the bandhead is at 675 keV) is not collective, that is, has little β-vibrational character. Or, it could be collective, but mixes very strongly with the ground band to reduce the measured $B(E2)$ value. El Masri *et al.*[35] have studied ^{156}Dy using the ^{159}Tb $(p,4n\gamma)$ reaction. Using a two-band band mixing procedure they find that no unique value of the band mixing parameter, Z_β, can describe the transition rate ratios between the excited $K=0$ band and the ground band; the extracted mixing parameter values decrease with increasing spin. The inclusion of the mixing with the $K=2$ band does not improve the situation. In a simple band mixing picture[50] the intrinsic $B(E2)$ value multiplied by $(1-6Z_\beta)^2$ yields the measured value. If we take the largest value of Z_β from these data, we find the intrinsic $B(E2)$ value to the $I^\pi K=2^+0$ state is about 1.1 spu, still somewhat smaller than the observed value in ^{158}Dy. A better test of a $K^\pi=0^+$ band's collectivity is from the value of $B(E2;0^+0\to 2^+_{\text{g.s.}})$ which can be obtained by a multiple Coulomb excitation experiment.

Kolata and Oothoudt[51] also question a β-vibrational interpretation of this $K^\pi=0^+$ band. The 0^+ member is populated in their (p,t) study with a strength of $\sim 24\%$ compared to the ground state, this being almost twice the observed strength for the "analogous" 0^+ state in ^{154}Gd (Ref. 52). [In the neighboring ^{158}Dy the 0^+ level at 991 keV is populated in a higher energy (p,t) study,[37] with a strength 9% of the ground state.] Also they suggest that the $I^\pi=2^+$ state at 1520 keV is the bandhead of a $K=2$, two-phonon excitation. The (p,t) reaction populates this state with an intensity equal to that of the $2^+_{\text{g.s.}}$, unlike any other case. We can only estimate its collectivity from the (d,d') study (0.3 spu).

These data call for an explanation at a microscopic level. Indeed, Peker and Hamilton[53] have shown that Coriolis coupling effects due to the $i_{13/2}$ neutron configurations in this region are responsible for nonadiabatic effects which at least alter the energy spacings of the $K^\pi=0^+$ and 2^+ band members. A formidable task for a microscopic theory of ^{156}Dy is thus not only to describe such strong transfer reaction strengths and a weak collective nature, but also to locate the β-vibrational strength.

We have noted in our Gd and Hf studies[8] that the lowest $K^\pi=0^+$ excited band may not always have the largest $B(E2)$ value to its $I=2$ member. Of the two $K^\pi=0^+$ bands in ^{178}Hf, the $I^\pi K=2^+0$ state at 1496 keV has a larger $B(E2)$ value to it than to the one at 1277 keV. We believe that this happens in ^{164}Er and ^{170}Yb. In ^{164}Er, the state at 1484 keV has a $B(E2)$ value about five times the value for the state at 1315 keV. However, the lower one is known to have $K^\pi=0^+$, whereas de Boer *et al.*[54] suggest that the 1484 keV state is the member of a band having possibly $K^\pi=1^+$ with a bandhead at 1417 keV. Preliminary multiple Coulomb excitation data[55] from ^{16}O on ^{164}Er show that the state at 1417 keV is rather strongly populated. This reaction favors two-step $E2$ excitations through the $2^+_{\mathrm{g.s.}}$ state so that we suspect the 1417 keV state has $I^{\pi}K=0^{+}0$. Maher, Kolata, and Miller[37] have studied ^{164}Er using the (p,t) reaction. A state at 1420 keV is excited with one-half the strength (at 25°) of the 0^+ state at 1248 keV. But they make no spin-parity assignments. The nucleus ^{170}Yb is another example of collective strength lying in a second or higher lying excited $K=0$ band. At first glance the lowest band seems more collective; Riedinger *et al.*[33] deduce $B(E2;0^+\rightarrow2^{+\prime}0)=0.030(6)\ e^2b^2$ (1 spu) for the $2^{+\prime}$ state at 1134 keV. But the $0^{+\prime}$ state at 1069 keV is not Coulomb excited. This large $B(E2)$ value arises because[33] the $2^{+\prime}$ is in very close proximity to the $I^{\pi}K=2^{+}2$ state at 1146 keV, and thus the two states strongly mix. Now, Riedinger *et al.* measure $B(E2;0^{+\prime\prime}\rightarrow 2^+_{\mathrm{g.b.}}) =0.057(11)\ e^2b^2$ (2 spu) for the 0^+ state at 1228 keV. In the rotational limit $B(E2;0^+\rightarrow2^{+\prime\prime})$ would also have this value, but mixing with the ground band is strong. From the (d,d') reaction cross sections[7] we estimate $B(E2;0^+ \rightarrow2^{+\prime\prime})=0.030\ e^2b^2$ (1 spu), implying $Z_\beta\approx0.04$, for the $2^{+\prime\prime}$ state at 1306 keV.

In the ^{172}Yb, the low lying $I^{\pi}K=2^{+}0$ state at 1118 keV exhibits little collectivity; $B(E2;0^+\rightarrow2^+0)\approx0.2$ spu. This $K=0$ band mixes strongly with the ground band; $Z_\beta\approx0.036$. In ^{174,176}Yb no collective $K=0$ states are observed.

C. Octupole vibrational states with $I^{\pi}=3^-$

In each nucleus we observed at least one strong excitation which we attribute to the octupole vibrational states with $I^{\pi}=3^-$. The measured $B(E3)$ values and level energies are presented in Table II, along with predictions from the microscopic calculations by Neergård and Vogel.[56]

Experimentally, for ^{156,158}Dy, ^{162}Er, and ^{168}Yb, the $B(E3)$ values of the most collective states are

TABLE II. Comparisons of measurements and microscopic calculation of $B(E3;0^+_{\mathrm{g.s.}}\rightarrow3^-)$ for one-octupole-phonon 3^- states. The label α is the K quantum number corresponding to the largest component of the Coriolis-coupled wave function, since K is not a "good" quantum number for the mixed octupole states.

	Experiment			Theory[a]		
Nucleus	$I^{\pi}K$	E (keV)	$B(E3)$ ($e^2b^3\times10^{-2}$)	E (keV)	$B(E3)$ ($e^2b^3\times10^{-2}$)	α
^{156}Dy	(3^-1)	1367	22(7)	1264	20.4	0
				1491	0.7	2
	(3^-)	1408	0.9[b]	2014	0.6	1
				2064	2.5	3
^{158}Dy	3^-?	1398	21(5)	1364	14.8	2
				1521	0.3	1
				1889	0.2	3
				1990	0.1	0
^{162}Er	3^-?	1351	19(4)	1436	10.5	1
	3^-?	1623	<7.2	1749	2.4	2
				1905	0.4	3
				2001	0.5	0
^{164}Er	3^-?	1433	15(3)	1500	9.6	0
	3^-?	1568	9.1(34)	1659	0.3	2
				1829	0.3	3
				1974	0.0	1
^{168}Yb	3^-?	1478	22(4)	1552	6.4	1
	(3^-)	1595	9(2)	1707	1.5	2
				1795	0.0	3
				1989	0.4	0

[a]Reference 56.

[b]Estimate derived from (d,d') studies as discussed in the text.

nearly the same, as are the level energies. In ^{164}Er (and possibly in ^{168}Yb) two octupole states are excited. In ^{164}Er their summed strength is about that of the strongest states in the other nuclei. Estimates of $B(E3)$ values from a renormalization of the (d,d') results, as discussed in Sec. IV A, are given for the states at 1408 and 1623 keV, in ^{156}Dy and ^{162}Er, respectively.

The quantitative agreement with theory,[56] while not as good as in our Gd and Hf study,[8] is reasonable in ^{156,158}Dy but gets progressively poorer with increasing A in ^{162,164}Er and ^{168}Yb. Still, as predicted, the collectivity concentrates in the states with the lowest energies. Also, except for in ^{168}Yb, the collectivity decreases somewhat with increasing mass. Unfortunately, the K-quantum number assignments are not experimentally sound enough to test the calculations on this point. The overall agreement between theory and experiment is due to the inclusion of the Coriolis interaction, as also is the case for the actinide region[57] and for our Gd, Hf study.

D. Comparison between experimental and theoretical $B(E2)$ values to $I^\pi=2^+$ vibrational-like states

Good quantitative agreement between microscopic theory and experiment has been reached only in several cases in the rare-earth region although it has been a quarter of a century since the qualitative concepts of γ and β vibrations were introduced. Excellent quantitative agreement of theory with experiment has been achieved for ^{152}Sm, using a variable-moment-of-inertia approach[58] and for ^{150,152}Sm and ^{154,156}Gd, using the pairing-plus-quadrupole calculational method of Kumar.[59,60]

Calculations yielding as good an agreement for the nuclei we have studied are not yet available, especially for $K^\pi=0^+$ states. Some results are available for $K^\pi=2^+$ states, and in Table III we present a comparison of theory and experiment for level energies and $B(E2)$ values to $I^\pi K=2^+2$ states in the stable nuclei Dy, Er, and Yb. These theoreti-

TABLE III. Comparison between experimental and theoretical values of $B(E2)$ (in single particle units) and level energies of $I^\pi K=2^+2$ states in Dy, Er, and Yb. The theoretical values are the results of microscopic calculations using pairing-plus-quadrupole or surface-delta interactions, with Woods-Saxon or Nilsson-type single particle potentials. These values are taken from the compilation by Soloviev.[a]

	Level energies (MeV)				$B(E2;0^+0\to I^\pi K=2^+2)$(spu)			
		Theory				Theory		
Nucleus	Exp.	PPQ (WS)[b]	PPQ (N)[c]	SDI (N)[d]	Exp.	PPQ (WS)[b]	PPQ (N)[c]	SDI (N)[d]
^{156}Dy	0.891	0.92			7.2(5)	4.7		
^{158}Dy	0.946	1.1	0.84	1.05	5.9(4)	5.7	13.6	5.0
^{160}Dy	0.966	1.00	0.87	0.99	4.7(1)	6.2	12.1	5.3
^{162}Dy	0.888	0.83	0.83	0.83	4.6(1)	5.5	11.3	5.9
^{164}Dy	0.762	0.80	0.67	0.65	4.5(1)	6.1	11.9	6.7
^{162}Er	0.901	0.85	0.98		6.3(4)	5.0	11.3	
^{164}Er	0.860	0.86	0.96	0.93	5.6(3)	8.0	10.5	5.2
^{166}Er	0.786	0.80	0.79	0.77	5.4(1)	5.9	10.6	5.6
^{168}Er	0.821	1.10	1.10	0.97	4.8(1)	4.7	8.3	4.7
^{170}Er	0.932	1.30	1.40	1.19	3.6(1)	5.5	5.8	3.8
^{168}Yb	0.984	0.96		1.19	4.6(3)	3.9		3.1
^{170}Yb	1.146	1.14			2.8(5)	4.0		
^{172}Yb	1.466	1.50		1.59	1.2(2)	0.04		0.6
^{174}Yb	1.634	1.60	1.60	1.53	1.7(2)	2.6	1.2	2.7
^{176}Yb	1.261	1.10	1.50	1.08	1.7(1)	1.9	1.9	3.7

[a]Reference 61 and references therein.
[b]Pairing-plus-quadrupole interaction; Woods-Saxon single particle potential.
[c]Pairing-plus-quadrupole interaction; Nilsson single particle potential.
[d]Surface-delta interaction; Nilsson single particle potential.

cal values were compiled by Soloviev[61] for pairing-plus-quadrupole (PPQ) calculations using either the Woods-Saxon (WS) or Nilsson (N) single particle potential, and for the surface-delta (SDI) calculations using the Nilsson potential.

The predicted level energies are in reasonable agreement with the experiment, with perhaps the PPQ (WS) calculations yielding better overall results. There are serious discrepancies between experiment and theory for ^{168,170}Er. For $B(E2)$ values, both the PPQ (WS) and SDI calculations show agreement in magnitude with experiment. However, the PPQ (WS) values have incorrect trends in the Dy and Er isotopes. The overall agreement of these calculations with experiment is not as good as obtained[58-60] for the Sm and Gd nuclei.

E. Summary

We have studied $I^{\pi}=2^{+}$ and 3^{-} vibrational-like states in ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb with Coulomb excitation by α particles. We find that in general the 2^{+} states do behave similarly to those in the deformed even-A Gd and Hf nuclei. Except for the possible anomaly concerning the $K=0$ states in the Dy isotopes, the lightest isotope has the largest $B(E2)$ value to these states, which lie lowest in energy there. The $B(E2)$ values decrease with increasing neutron number. In ^{164}Er we find that the $I^{\pi}=2^{+}$ state at 1484 keV has five times the strength of the one at 1315 keV. In each nucleus at least one $I^{\pi}=3^{-}$ octupole state is excited. Comparisons of experimental $B(E2)$ and $B(E3)$ values to microscopic calculations show general agreement.

ACKNOWLEDGMENTS

We thank Dr. J. L. C. Ford, Jr. and Dr. J. Gomez del Campo for their assistance with the spectrograph and detector system. We also thank G. F. Wells, F. DiCarlo, E. G. Richardson, and others of the ORNL tandem Van de Graaff staff for accelerator operations and maintenance. We appreciate communications of data prior to publication, from Dr. F. K. McGowan and Dr. J. W. Mihelich. The work of R. M. Ronningen was partially supported by the ORNL. R. S. G is an Oak Ridge Associated Universities Summer Fellow. Research at Vanderbilt University was supported in part by the U. S. Department of Energy Contract No. DE-A505-76 ER05034, at Oak Ridge National Laboratory by Contract No. W-7405-ENG-26 with Union Carbide Corporation, and at the University of Tennessee by the U. S. D. O. E. Contract No. DE-A505-76 ER04936.

---

*Present address: Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824.

†On leave from Delft Technical University, Delft, The Netherlands.

‡On leave from University of Tokyo, Tokyo, Japan.

[1]R. M. Ronningen, R. B. Piercey, J. H. Hamilton, C. F. Maguire, A. V. Ramayya, H. Kawakami, B. van Nooijen, R. S. Grantham, W. K. Dagenhart, and L. L. Riedinger, Phys. Rev. C 16, 2218 (1977).

[2]Y. Yoshizawa, B. Elbek, B. Herskind, and M. C. Olesen, Nucl. Phys. 73, 273 (1965).

[3]E. Veje, B. Elbek, R. Herskind, and M. C. Olesen, Nucl. Phys. A109, 489 (1968).

[4]R. Bloch, B. Elbek, and P. O. Tjøm, Nucl. Phys. A91, 576 (1967).

[5]T. Grotdal, K. Nybo, and T. Thorsteinsen, Nucl. Phys. A110, 385 (1968).

[6]P. O. Tjøm and B. Elbek, Nucl. Phys. A107, 385 (1968).

[7]D. G. Burke and B. Elbek, K. Dan. Vidensk. Selsk., Mat. Fys. Medd. 36, No. 6 (1967).

[8]R. M. Ronningen, J. H. Hamilton, A. V. Ramayya, L. Varnell, G. Garcia-Bermudez, J. Lange, W. Lourens, L. L. Riedinger, R. L. Robinson, P. H. Stelson, and J. L. C. Ford, Jr., Phys. Rev. C 15, 1671 (1977).

[9]C. E. Bemis, Jr., P. H. Stelson, F. K. McGowan, W. T. Milner, J. L. C. Ford, Jr., R. L. Robinson, and W. Tuttle, Phys. Rev. C 8, 1934 (1973).

[10]I. Y. Lee, J. X. Saladin, C. Baktash, J. E. Holden, and J. O'Brien, Phys. Rev. Lett. 33, 383 (1974).

[11]W. K. Dagenhart and T. Wilson Whitehead, Nucl. Instrum. Methods 85, 215 (1970).

[12]A. Winther and J. de Boer, in *Coulomb Excitation*, edited by K. Alder and W. Winther (Academic, New York, 1966), p. 302.

[13]R. O. Sayer (private communication).

[14]A. Bohr and B. Mottelson, K. Dan. Vidensk. Selsk., Mat. Fys. Medd. 26, No. 16 (1953); G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, *ibid.* 29, No. 9 (1955).

[15]C. Baktash, J. X. Saladin, J. O'Brien, I. Y. Lee, and J. E. Holden, Phys. Rev. C 10, 2265 (1974).

[16]F. K. McGowan, W. T. Milner, R. O. Sayer, R. L. Ro-

binson, and P. H. Stelson, Nucl. Phys. A289, 253 (1977).

[17]F. Todd Baker, Alan Scott, R. M. Ronningen, T. H. Kruse, R. Suchannek, W. Savin, and J. H. Hamilton, Phys. Lett. 70B, 167 (1977).

[18]J. J. O'Brien, J. X. Saladin, C. Baktash, and J. G. Alessi, Phys. Rev. Lett. 38, 324 (1977).

[19]K. Alder and A. Winther, *Electromagnetic Excitation* (North-Holland, Amsterdam, 1975), p. 292.

[20]T. W. Burrows, Nucl. Data Sheets 18, 553 (1976).

[21]J. K. Tuli, Nucl. Data Sheets 12, 245 (1974).

[22]A. Buyrn, Nucl. Data Sheets 17, 97 (1976).

[23]A. Buyrn, Nucl. Data Sheets 11, 327 (1974).

[24]L. R. Greenwood, Nucl. Data Sheets 11, 385 (1974).

[25]J. Adams, A. Visvanathan, E. Funk, and J. Mihelich, Bull. Am. Phys. Soc. 18, 1405 (1973).

[26]R. N. Oehlberg, L. L. Riedinger, A. E. Rainis, A. G. Schmidt, E. G. Funk, and J. W. Mihelich, Nucl. Phys. A219, 543 (1974).

[27]F. K. McGowan and W. T. Milner, Phys. Rev. C 23, 1926 (1981).

[28]J. M. Domingos, G. D. Symons, and A. C. Douglas, Nucl. Phys. A180, 600 (1972).

[29]J. W. Mihelich (private communication).

[30]F. K. McGowan, W. T. Milner, R. L. Robinson, P. H. Stelson, and Z. W. Grabowski, Nucl. Phys. A297, 51 (1978).

[31]R. O. Sayer, P. H. Stelson, F. K. McGowan, W. T. Milner, and R. L. Robinson, Phys. Rev. C 1, 1525 (1970).

[32]H. J. Wollersheim and Th. W. Elze, Z. Phys. A 280, 277 (1977).

[33]L. L. Riedinger, E. G. Funk, J. W. Mihelich, G. S. Schilling, A. E. Rainis, and R. N. Oehlberg, Phys. Rev. C 20, 2170 (1979).

[34]R. M. Lieder, H. Beuscher, W. F. Davidson, A. Neskakis, C. Mayer-Böricke, Y. El Masri, P. Monseu, J. Steyaert, and J. Vervier, Phys. Lett. 49B, 161 (1974).

[35]Y. El Masri, R. Janssens, C. Michel, P. Monseu, J. Steyaert, and J. Vervier, Z. Phys. A 274, 113 (1975).

[36]Y. El Masri, J. M. Ferté, R. Janssens, C. Michel, P. Monseu, J. Steyaert, and J. Vervier, Nucl. Phys. A271, 133 (1976).

[37]J. V. Maher, J. J. Kolata, and R. W. Miller, Phys. Rev. C 6, 358 (1972).

[38]A. A. Aburazakov *et al.,* Izv. Akad. Nauk. SSSR Ser. Fiz. 32, 749 (1968) [Bull. Acad. Sci. USSR, Phys. Ser. 32, 688 (1969)].

[39]F. W. N. de Boer, Ph.D. thesis, Instituut voor Kernphysisch Onderzock, Amsterdam, 1974 (unpublished); F. W. N. de Boer, P. F. A. Goudsmit, B. J. Meijer, P. Koldewijn, J. Konijn, and R. Beetz, Nucl. Phys. A236, 349 (1974).

[40]R. Janssens, Y. El Masri, J. M. Ferté, C. Michel, J. Steyaert, and J. Vervier, Nucl. Phys. A283, 493 (1977).

[41]R. L. West, E. G. Funk, A. Visvanathan, J. P. Adams, and J. W. Mihelich, Nucl. Phys. A270, 300 (1976).

[42]I. Y. Lee, D. Cline, R. S. Simon, P. A. Butler, P. Colombani, M. W. Guidry, F. S. Stephens, R. M. Diamond, N. R. Johnson, and E. Eichler, Phys. Rev. Lett. 37, 420 (1976).

[43]Noah R. Johnson, D. Cline, S. W. Yates, F. S. Stephens, L. L. Riedinger, and R. M. Ronningen, Phys. Rev. Lett. 40, 151 (1978); S. W. Yates, I. Y. Lee, N. R. Johnson, E. Eichler, L. L. Riedinger, M. W. Guidry, A. C. Kahler, D. Cline, R. S. Simon, P. A. Butler, P. Colombani, F. S. Stephens, R. M. Diamond, R. M. Ronningen, R. D. Hitchwa, J. H. Hamilton, and E. L. Robinson, Phys. Rev. C 21, 2366 (1980).

[44]O. C. Kistner, E. der Mateosien, and A. W. Sunyar, Phys. Rev. C 17, 1417 (1978).

[45]J. L. Egido, H. J. Mang, and P. Ring, Nucl. Phys. A334, 1 (1980).

[46]R. Bengsston and S. Frauendorf, Nucl. Phys. A314, 27 (1979).

[47]Michael A. Oothoudt and Norton M. Hintz, Nucl. Phys. A213, 221 (1973).

[48]R. Graetzer, G. B. Hagemann, K. A. Hagemann, and B. Elbek, Nucl. Phys. 76, 1 (1966).

[49]A. Charvet, R. Duffait, A. Emsallem, and R. Chéry, Nucl. Phys. A156, 276 (1970).

[50]L. L. Riedinger, N. R. Johnson, and J. H. Hamilton, Phys. Rev. Lett. 19, 1243 (1967); L. L. Riedinger, Noah R. Johnson, and J. H. Hamilton, Phys. Rev. 179, 1214 (1969).

[51]J. J. Kolata and M. Oothoudt, Phys. Rev. C 15, 1947 (1977).

[52]Donald G. Fleming, Christian Günther, Gudrun Hagemann, Bent Herskind, and Per O. Tjøm, Phys. Rev. C 8, 806 (1973).

[53]L. K. Peker and J. H. Hamilton, Phys. Rev. C 16, 486 (1977).

[54]F. W. N. de Boer, P. F. A. Goudsmit, P. Koldewijn, and B. J. Meijer, Nucl. Phys. A169, 577 (1971).

[55]R. M. Ronningen, R. Hitchwa, R. B. Piercey, J. H. Hamilton, L. L. Riedinger, F. Venecia, E. G. Funk, and J. W. Mihelich (unpublished).

[56]K. Neergård and P. Vogel, Nucl. Phys. A145, 33 (1970); F. K. McGowan has furnished us with the numerical results, communicated to him by P. Vogel.

[57]K. Neergård and P. Vogel, Nucl. Phys. A149, 217 (1970).

[58]R. G. Stokstad, J. S. Greenberg, I. A. Fraser, S. H. Sie, and D. A. Bromley, Phys. Rev. Lett. 27, 748 (1971).

[59]K. Kumar, Nucl. Phys. A231, 189 (1974).

[60]J. B. Gupta, Krishna Kumar, and J. H. Hamilton, Phys. Rev. C 16, 427 (1977).

[61]V. G. Soloviev, in *Theory of Complex Nuclei* (Pergamon, Elmsford, New York, 1976), pp. 356 and 370, and references therein.