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Inelastic scattering and inelastic breakup of deuterons by nuclei

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Deuteron inelastic scattering and inelastic breakup calculations are formulated in the framework of a simple three-body description, by using an adiabatic approximation. Application is then made for 80 MeV deuterons, exciting the first 2^+ state in ^{58}Ni. Similarities and differences between the elastic and inelastic breakup processes are discussed. Comparison with the predictions of the folding model is also made.

[NUCLEAR REACTIONS Deuteron, elastic scattering, inelastic scattering, elastic breakup, inelastic breakup, three-body model, adiabatic approximation, proton-neutron coincidence, folding model, ^{58}Ni.]

I. INTRODUCTION

Evidence has been accumulated to show that the deuteron-breakup mode plays an important role in deuteron elastic scattering,[1–8] in stripping reactions,[4,9–11] and in two-step processes which involve deuteron propagation in intermediate channels.[12–14] Recently, we reported[15] on a calculation showing that the breakup effect was also very important in deuteron inelastic scattering, in that it reduced significantly the inelastic scattering cross sections. Most of the deuteron amplitude thus removed from the inelastic scattering channel is expected to go to the inelastic breakup channel, making the corresponding cross sections appreciable. One of the major objectives of the present paper is to give a quantitative estimate of the size of these cross sections.

It appears in order here to clarify the terminology. In the present paper, we mean by *deuteron inelastic scattering* the process in which the target is excited into *discrete* excited states, the deuteron that leaves the interaction region remaining as a bound deuteron. On the other hand, *inelastic breakup* is a process in which the target is left again in a *discrete* excited state, but the deuteron is broken up into a proton and a neutron, and thus does not remain as a deuteron asymptotically. (The *deuteron elastic scattering* and *elastic breakup* processes are understood similarly; in these processes, the target remains in its ground state.) Our *inelastic breakup* is thus to be distinguished from the *inelastic breakup* considered by Baur and others.[16,17] In their work, the main interest concerned the continuum spectrum of one member of the broken up pair; e.g., the proton singles cross section if the projectile was a deuteron. The target, or more precisely the target-plus-neutron system, can be left in any state.

We treat the deuteron plus target system as a three-body system, as is usually done.[1–15] This simplification largely neglects the many-body nature of the target, but, even with this simplification, it is still very difficult to treat our system rigorously as a genuine three-body system. This difficulty is enhanced, because, as shown below, the *d*-wave breakup mode, in which the *p-n* (proton-neutron) relative angular momentum l equals 2, plays a significant role. In the present paper, we thus use an *adiabatic approximation.*

The adiabatic approximation here means to neglect the deviation of the *p-n* relative energy from its value in the deuteron, i.e., from the deuteron binding energy. This approximation is expected to be acceptable when the deuteron incident energy is sufficiently high. The adiabatic approximation we use is, in principle, very similar to that in the pioneering work of Johnson and Soper (JS).[1] However, while we can include with relative ease the $l\neq0$ breakup modes in our calculations, JS considered only the $l=0$ mode. Note, nevertheless, that unless the deuteron energy is very high the two methods predict essentially the same cross sections for stripping reactions, because the $l \neq 0$ modes do not play a significant role in stripping reactions.

The formalism of the present calculation is presented in Sec. II, which is divided into subsections A and B. In Sec. II A, we explain the adiabatic coupled-channels (ACC) method, which treats the elastic scattering and the elastic breakup reactions, while in Sec. II B, we discuss the formulation of the adiabatic coupled-channels Born approximation (ACCBA) method, which treats the inelastic scattering and the inelastic breakup reactions. The results of the calculations are presented in Sec. III, and Sec. IV is devoted to concluding remarks.

II. FORMULATION OF THE CALCULATIONS

A. The adiabatic coupled-channels method

We assume a simple three-body model in which the deuteron-nucleus collision system consists of a proton, a neutron, and an inert target nucleus. In the center-of-mass coordinates of the whole system, the total Hamiltonian may then be written as

$$H = K_R + V(\vec{r},\vec{R}) + K_r + v_{pn}(\vec{r}) , \tag{1}$$

with

$$V(\vec{r},\vec{R}) = V_p\left[\vec{R}+\frac{\vec{r}}{2}\right] + V_n\left[\vec{R}-\frac{\vec{r}}{2}\right] . \tag{2}$$

Here $\vec{r}$ is the relative coordinate between p (proton) and n (neutron), and $\vec{R}$ is the coordinate of the c.m. of the $p+n$ system relative to the target. K_r and K_R denote, respectively, the kinetic energies associated with the $\vec{r}$ and $\vec{R}$ degrees of freedom. V_p is the interaction between p and the target and similarly for V_n. Finally, v_{pn} is the interaction between p and n.

The three-body wave function $\Psi^{(\pm)}(\vec{K}_i;\vec{r},\vec{R})$ satisfies the Schrödinger equation

$$\{K_R + V(\vec{r},\vec{R}) + K_r + v_{pn}(\vec{r}) - E\}\Psi^{(\pm)}(\vec{K}_i;\vec{r},\vec{R}) = 0 , \tag{3}$$

where $\vec{K}_i$ is the c.m. momentum of the incident channel. The adiabatic approximation to this equation neglects the excitation energies of the p-n relative motion. More precisely, the approximation is to replace the $K_r + v_{pn}(\vec{r})$ part of the Hamiltonian by ϵ_d, the deuteron binding energy. If this is done, Eq. (3) is reduced to

$$\{K_R + V(\vec{r},\vec{R}) + \epsilon_d - E\}\Psi^{(\pm)}(\vec{K}_i;\vec{r},\vec{R}) = 0 . \tag{4}$$

If the spin-orbit interactions are also neglected, we may expend $V(\vec{r},\vec{R})$ and $\Psi^{(\pm)}(\vec{K}_i;\vec{r},\vec{R})$ in multipoles (taking $\vec{k}$ as the p-n relative momentum) as

$$V(\vec{r},\vec{R}) = 4\pi\sum_{\lambda} V_\lambda(r,R)[Y_\lambda(\hat{r})Y_\lambda(\hat{R})]_{00} , \tag{5}$$

$$\Psi^{(\pm)}(\vec{K}_i;\vec{r},\vec{R}) = \frac{4\pi}{K_iR}\sum i^L(lmLM\,|\,JM_J)(l'm'L'M'\,|\,JM_J) \times \chi^{(\pm)J}_{l'L';lL}(K_i;r,R)Y_{L'M'}(\hat{R})Y^*_{LM}(\hat{K}_i)\,|\,Y_{l'm'}(\hat{r})\rangle\langle Y_{lm}(\hat{r})\,| \ . \tag{6}$$

In (6), $(lmLM\,|\,JM_J)$ is the Clebsch-Gordan coefficient. In (5), the square bracket means a vector coupling. Therefore, for example,

$$[Y_l(\hat{r})Y_L(\hat{R})]_{JM_J} = \sum_{m,M}(lmLM\,|\,JM_J)Y_{lm}(\hat{r})Y_{LM}(\hat{R}) .$$

We also note here that throughout the present paper we use for Y_{lm} the Wigner phase; thus our $Y_{lm} = i^lY^{\rm CS}_{lm}$, where $Y^{\rm CS}_{lm}$ are the surface harmonics defined by Condon and Shortley.[18] From (6), it will be clear that l (and l') and L (and L') denote, respectively, the angular momenta associated with the $\hat{r}$ and the $\hat{R}$ degrees of freedom. We finally note that $\Psi^{(\pm)}(\vec{K}_i;\vec{r},\vec{R})$ of (6) satisfies the following time reversal relation:

$$\Psi^{(+)*}(\vec{K}_i;\vec{r},\vec{R}) = \Psi^{(-)}(-\vec{K}_i;\vec{r},\vec{R}) . \tag{7}$$

If (5) and (6) are inserted into (4), the following coupled equations for $\chi^{(\pm)J}_{l'L';lL}$ result:

$$(K_R+V_0(r,R)+\epsilon_d-E)\chi^{(\pm)J}_{lL;lL}+\sum_{\substack{l'L'\\ \lambda\neq 0}} i^{l'+L'-l-L}(-)^{l+L'+J}\hat{\lambda}\hat{l}\hat{L}$$
$$\times(l0\lambda 0\,|\,l'0)(L\,0\lambda 0\,|\,L'0)W(lLl'L';J\lambda)V_\lambda(r,R)\chi^{(\pm)J}_{l'L';lL}(K_i;r,R)=0\ , \tag{8}$$

where $\hat{\lambda}=[2\lambda+1]^{1/2}$, and $W(lLl'L';J\lambda)$ is the Racah coefficient.

The above coupled equations are to be solved with the boundary condition that

$$\chi^{(+)J}_{l'L';lL}(K_i;r,R)\underset{R\to\infty}{\longrightarrow}\delta_{ll'}\delta_{LL'}e^{i\sigma_L}F_L(K_iR)+T^J_{l'L';lL}(r)e^{i\sigma_{L'}}(G_{L'}(K_iR)+iF_{L'}(K_iR))\ , \tag{9}$$

where F_L and G_L are regular and irregular Coulomb wave functions, and σ_L is the Coulomb phase shift. The asymptotic form of $\chi^{(-)J}_{l'L';lL}$ is given by the complex conjugate of the asymptotic form that appeared in (9).

At this stage it should be stressed that the coupled equations (8), and hence the boundary condition (9), are defined for the R degree of freedom, i.e., for the c.m. motion of the $p+n$ system. The relative coordinate r appears there just as a parameter. However, it is in the spirit of the adiabatic approximation to interpret $T^J_{l'L';lL}(r)$ as an operator in r space, and thus to take its matrix element between two wave functions that describe the p-n relative motions in the initial and final channels, thus obtaining the physical T-matrix element describing the transition between these two channels.

The equations that describe the p-n relative motion may be written as

$$\{K_r+v_{pn}(\vec{r})\}\psi_d(\vec{r})=\epsilon_d\psi_d(\vec{r})\ , \tag{10a}$$

$$\{K_r+v_{pn}(\vec{r})\}\psi_{\vec{k}}(\vec{r})=\epsilon_k\psi_{\vec{k}}(\vec{r})\ , \tag{10b}$$

with $\epsilon_k=\hbar^2k^2/m$, m being the nucleon mass. We may rewrite the wave functions that appear in (10) as

$$\psi_d(\vec{r})=[\phi_d(r)/r]Y_{00}(\hat{r})\ , \tag{11a}$$

$$\psi_{\vec{k}}(\vec{r})=[(2\pi)^{3/2}/(kr)]\times\sum_{l,m} i^l\phi_l(kr)Y^*_{lm}(\hat{k})Y_{lm}(\hat{r})\ . \tag{11b}$$

From (11a), it is clear that we assume that the deuteron is in a pure S state. The radial functions that appeared in (11) are taken to have the following orthonormality properties:

$$\int_0^\infty \phi_d^*(r)\phi_d(r)dr=1\ , \tag{12a}$$

$$\int_0^\infty \phi_0^*(kr)\phi_d(r)dr=0\ , \tag{12b}$$

$$\int_0^\infty \phi_l^*(kr)\phi_l(k'r)dr=\delta(k-k')\ . \tag{12c}$$

We are now ready to write the expression for the differential cross section for the elastic scattering. It is given as[11,19]

$$d\sigma^{\rm el}/d\Omega=|f_C(\theta)+K_i^{-1}\sum_L(2L+1)e^{2i\sigma_L}\times t^d_{0L;0L}P_L(\cos\theta)\,|^2 \tag{13a}$$

with

$$t^d_{0L;0L}=\int_0^\infty \phi_d^*(r)T^L_{0L;0L}(r)\phi_d(r)dr\ . \tag{13b}$$

Equation (13b) is an example to show what is meant by the projection of the $T^J_{l'L';lL}(r)$ matrix upon the eigenstates of the p-n relative motion. The $t^d_{0L;0L}$ coefficients represent physical T-matrix elements such as are needed to describe the elastic scattering cross section. In Eq. (13a), $f_C(\theta)$ is the Rutherford amplitude.

The angle integrated elastic scattering cross section [disregarding the contribution from $f_C(\theta)$] is given as

$$\sigma^{\rm el}=\sum_L\sigma^{\rm el}(L)\ , \tag{14a}$$

$$\sigma^{\rm el}(L)=[4\pi/K_i^2](2L+1)\,|\,t^d_{0L;0L}\,|^2, \tag{14b}$$

which leads to the reaction cross section

$$\sigma^r=\sum_L\sigma^r(L)\ , \tag{15a}$$

$$\sigma^r(L)=[4\pi/K_i^2](2L+1){\rm Im}(t^d_{0L;0L})-\sigma^{\rm el}(L)\ . \tag{15b}$$

This reaction cross section includes the elastic breakup cross sections. The triple differential cross section for the elastic breakup may be written as

$$\frac{d^3\sigma_b^{\rm el}}{d\Omega_n d\Omega_p dE_p}=\left[\frac{4\pi m^2}{\mu_d\hbar^2}\right]\left[\frac{k_pk_n}{k^2K_i^3}\right]\left|\sum_{l'LL'} i^{-l'-L'}\hat{L}e^{2i\sigma_{L'}}[Y_{l'}(\hat{k})Y_{L'}(\hat{K})]_{LO}t^k_{l'L';0L}\right|^2 , \tag{16a}$$

$$t^k_{l'L';lL}=\int_0^\infty \phi^*_{l'}(kr)T^L_{l'L';lL}(r)\phi_d(r)dr . \tag{16b}$$

In (16a), μ_d is the reduced mass of the deuteron relative to the target. Since $\vec{k}=(\vec{k}_p-\vec{k}_n)/2$ and $\vec{K}=\vec{k}_p+\vec{k}_n$, a choice of the two vectors $\vec{k}$ and $\vec{K}$ determines the angular and energy variables of the outgoing proton and neutron. It is clear that (16b) gives another example of projecting the $T(r)$ matrix upon the eigenstates of the p-n relative motion.

By integrating (16a) over the proton momentum k_p, we find a form for the double differential cross section:

$$\frac{d^2\sigma_b^{\rm el}}{d\Omega_n d\Omega_p}=\left[\frac{4\pi m}{\mu_d K_i^3}\right]\int\frac{k_nk_p{}^2}{k^2}\left|\sum i^{-l'-L'}\hat{L}e^{2i\sigma_{L'}}[Y_{l'}(\hat{k})Y_{L'}(\hat{K})]_{L0}t^k_{l'L';0L}\right|^2 dk_p . \tag{17}$$

The momentum spectrum of the elastic-breakup cross section is expressed as

$$\frac{d\sigma_b^{\rm el}}{dk}=[4\pi/K_i{}^2]\sum(2L+1)\,|\,t^k_{l'L';0L}\,|^2 . \tag{18}$$

An integration of (18) over k gives rise to a simple expression for the total elastic-breakup cross section as

$$\sigma_b^{\rm el}=[4\pi/K_i{}^2]\sum(2L+1)\int\phi_d^*(r)\,|\,T^L_{l'L';0L}(r)\,|^2\phi_d(r)dr$$
$$-[4\pi/K_i{}^2]\sum(2L+1)\left|\int\phi_d^*(r)T^L_{0L;0L}(r)\phi_d(r)dr\right|^2 . \tag{19}$$

The s-wave breakup cross section, i.e., the $l'=0$ term in (19), is proportional to the mean square deviation of $T^L_{0L;0L}(r)$. The second term on the rhs of Eq. (19) is just the total elastic cross section given in (14).

B. The adiabatic coupled-channels Born approximation method

In the present subsection, we give formulas that are to be used for obtaining the inelastic scattering and inelastic breakup cross sections. We treat the excitation in Born approximation, by using as a specific choice of the distorted wave the ACC wave function obtained in the way described in the preceding subsection. We shall thus call our method an ACCBA method.

The form factor to be used in the ACCBA calculation is given as a sum of form factors pertaining to the excitations by proton and neutron. It is written

$$F_{M_B,M_A}(\vec{r},\vec{R})=\langle\Phi_{I_BM_B}\,|\,\hat{V}_p(\vec{r}_p)+\hat{V}_n(\vec{r}_n)\,|\,\Phi_{I_AM_A}\rangle , \tag{20a}$$

$$\hat{V}_i(\vec{r}_i)=R_i(dV_i/dR_i)\sum\alpha^i_{\gamma\mu}Y^*_{\gamma\mu}(\hat{r}_i);\quad (i=p \text{ or } n) , \tag{20b}$$

$$\alpha_{\gamma\mu}=(b^\dagger_{\gamma\mu}+(-)^\mu b_{\gamma-\mu})\beta_\gamma/\hat{\gamma} . \tag{20c}$$

In (20), $\phi_{I_AM_A}$ and $\phi_{I_BM_B}$ stand, respectively, for the intrinsic wave functions of the target in the ground and the excited states, while R_i is the radius of the potential V_i, and $\alpha_{\gamma\mu}$ denotes the collective coordinate.

The form factor may be expanded as

$$F_{M_B,M_A}(\vec{r},\vec{R})=i^{I_A+\gamma-I_B}\sum(I_AM_A\gamma m_\gamma\,|\,I_BM_B)\sum_{\lambda\Lambda}F_{\lambda\Lambda}(r,R)[Y_\lambda(\hat{r})Y_\Lambda(\hat{R})]^*_{\gamma m_\gamma} , \tag{21}$$

in which the factor $F_{\lambda\Lambda}(r,R)$ has the following explicit form:

$$F_{\lambda\Lambda}(r,R)=i^{\lambda+\Lambda-\gamma}\sqrt{\pi}(\hat{\lambda}\hat{\Lambda}/\hat{\gamma})\sum(\lambda m\Lambda 0\,|\,\gamma m)G_{\gamma,m}G_{\lambda,m}$$
$$\times\sum_{i=p,n}g_i(r_i)P_{\gamma|m|}(\cos\theta_i)P_{\lambda|m|}(\cos\theta)d(\cos\theta) , \tag{22a}$$

$$G_{\gamma,m}=(-)^{[m+|m|]/2}[(\gamma-|m|)!/(\gamma+|m|)!]^{1/2}\,, \tag{22b}$$

$$g_i(r_i)=[\beta_\gamma R_i/\hat{\gamma}](dV_i/dR_i)\,, \tag{22c}$$

where θ is the angle between $\hat{r}$ and $\hat{R}$, θ_i is the angle between $\hat{r}_i$ and $\hat{R}$, and $P_{\gamma m}$ is the associated Legendre function.

Note that the method used here in transforming between various coordinates is somewhat different from the well known method of Austern *et al.*[20] With the latter method, the use of the macroscopic form factor of (20) results in a singularity of the form $R_i^{-\gamma}$. In order to avoid this, we used the method of Landowne *et al.*[21] This gave rise to an additional summation over m, which would not have appeared had the method of Austern *et al.* been used.

The inelastic scattering differential cross section may be written as

$$\frac{d\sigma^{\rm inel}}{d\Omega}=N\sum|T^{\rm inel}_{M_B,M_A}|^2;\;\; N=\frac{\mu_d{}^2}{(2\pi\hbar^2)^2}\frac{K_f}{K_i(2I_A+1)}\,, \tag{23a}$$

$$T^{\rm inel}_{M_B,M_A}=\int\psi_d^*(\vec{r})\Psi^{(-)*}(\vec{K}_f;\vec{r},\vec{R})F_{M_B,M_A}(\vec{r},\vec{R})\Psi^{(+)}(\vec{K}_i;\vec{r},\vec{R})\psi_d(\vec{r})d\vec{r}\,d\vec{R}\,. \tag{23b}$$

In (23), μ_d is the reduced mass of the deuteron with respect to the target, while K_f is the momentum in the excited channel.

The inelastic scattering cross section, given by (23), is reduced after somewhat lengthy but straightforward algebra to

$$\frac{d\sigma^{\rm inel}}{d\Omega}=N(\hat{I}_B/\hat{\gamma})^2\sum\Big|\sum(-)^{L_i}\hat{L}_i\hat{L}_f(L_i0L_fM\,|\,\gamma M)\times G_{L_f,-M}P_{L_f|M|}(\cos\theta)\int\phi_d^*(r)X^{L_i}_{0L_fL_f}(r)\phi_d(r)dr\Big|^2\,, \tag{24a}$$

$$X^{L_i}_{l_fL_fJ_f}(r)=[\hat{L}_i\hat{J}_f/(K_iK_f)]\sum i^{l'+l''-\lambda+L'+L''-\Lambda}\hat{l}'\hat{l}''\hat{L}'\hat{L}''(l'0l''0\,|\,\lambda0)(L'0L''0\,|\,\Lambda0)\times\begin{Bmatrix} l' & L' & L_i\\ l'' & L'' & J_f\\ \lambda & \Lambda & \gamma\end{Bmatrix}\int\chi^{(+)J_f}_{l''L'';l_fL_f}(K_f;r,R)F_{\lambda\Lambda}(r,R)\times\chi^{(+)L_i}_{l'L';0L_i}(K_i;r,R)dR\,. \tag{24b}$$

In (24a), the factor N is the same as that defined in (23a). In (24b) the { } factor is the nine-j symbol.

The expression of (24) is rather complicated. However, $\sigma^{\rm inel}$ obtained by integrating (24a) over the angle θ has the following very simple form:

$$\sigma^{\rm inel}=4\pi N(2I_B+1)\sum\left|\int\phi_d^*(r)X^{L_i}_{0L_fL_f}(r)\phi_d(r)dr\right|^2\,. \tag{25}$$

The triple differential cross section of the inelastic breakup process may be written as

$$\frac{d^3\sigma_b^{\rm inel}}{d\Omega_n d\Omega_p dE_p}=\frac{\mu_d m^2}{(2\pi)^5\hbar^6}\frac{k_pk_n}{K_i(2I_A+1)}\sum|T^{\text{inel-bu}}_{M_B,M_A}|^2\,, \tag{26}$$

with

$$T^{\text{inel-bu}}_{M_B,M_A}=\int\psi_{\vec{k}}^*(\vec{r})\Psi^{(-)*}(\vec{K}_f;\vec{r},\vec{R})F_{M_B,M_A}(\vec{r},\vec{R})\Psi^{(+)}(\vec{K}_i;\vec{r},\vec{R})\psi_d(\vec{r})d\vec{r}\,d\vec{R}\,. \tag{27}$$

A more explicit form to replace (26) is

$$\frac{d^3\sigma_b^{\text{inel}}}{d\Omega_n d\Omega_p dE_p}=\left[\frac{\mu_d m^2}{2(2\pi\hbar^2)^3}\right][k_p k_n/(K_i k^2)][\hat{I}_B/(\hat{I}_A\hat{\gamma})]^2$$
$$\times\sum_{M_\gamma}\left|\sum(-)^{L_i}\hat{l}_f\hat{L}_f\hat{L}_i(l_f m_f L_f M_f\mid J_f M_\gamma)(L_i 0 J_f M_\gamma\mid \gamma M_\gamma)G_{l_f,-m_f}G_{L_f,-M_f}\right.$$
$$\left.\times P_{l_f|m_f|}(\cos\theta_k)P_{L_f|M_f|}(\cos\theta_{K_f})\int \phi_{l_f}^*(kr)X_{l_f L_f J_f}^{L_i}(r)\phi_d(r)dr\right|^2 . \quad (28)$$

The inelastic breakup cross section as a function of k is obtained by integrating (28) over Ω_n and Ω_p:

$$\frac{d\sigma_b^{\text{inel}}}{dk}=4\pi N(2I_B+1)\sum\left|\int \phi_{l_f}^*(kr)X_{l_f L_f J_f}^{L_i}(r)\phi_d(r)dr\right|^2 . \quad (29)$$

If this is then integrated over k, we obtain the (total) inelastic breakup cross section as

$$\sigma_b^{\text{inel}}=4\pi N(2I_B+1)\sum_{L_i L_f}\left\{\sum_{l_f J_f}\int \phi_d^*(r)\,|X_{l_f L_f J_f}^{L_i}(r)|^2\phi_d(r)dr\right.$$
$$\left.-\left|\int \phi_d^*(r)X_{0 L_f L_f}^{L_i}(r)\phi_d(r)dr\right|^2\right\} . \quad (30)$$

Note that the second term of (30) is the (total) inelastic cross section. Equation (30) is in fact very similar to Eq. (19).

As we shall show in Sec. III, it is interesting to compare the results that are obtained by using the formulas given above with those obtained by using the so-called folding-model method. Following Watanabe,[22] we construct the folding-model potential as

$$V_{\text{FM}}(\vec{R})=\int \psi_d^*(\vec{r})(V_p+V_n)\psi_d(\vec{r})d\vec{r} , \quad (31)$$

where $\psi_d(\vec{r})$ was defined in (10a). By taking the scalar part of (31), we have the optical potential in the folding model, which can be used to construct the corresponding distorted waves $\chi_{\text{FM}}^{(\pm)}(\vec{K},\vec{R})$. The inelastic scattering T matrix of the folding model is then given as[23]

$$T_{M_B,M_A}^{\text{FM}}=\int \chi_{\text{FM}}^{(-)*}(\vec{K}_f,\vec{R})\left[\int \psi_d^*(\vec{r})F_{M_B,M_A}(\vec{r},\vec{R})\psi_d(\vec{r})d\vec{r}\right]\chi_{\text{FM}}^{(+)}(\vec{K}_i,\vec{R})d\vec{R} , \quad (32)$$

where $F_{M_B,M_A}(\vec{r},\vec{R})$ was defined in (20a). Finally the folding-model inelastic cross section is

$$d\sigma^{\text{inel}}/d\Omega=N\sum|T_{M_B,M_A}^{\text{FM}}|^2 . \quad (33)$$

This completes the formulation of the calculations of the present paper. In the next section, we shall present results of such calculations.

III. CALCULATIONS AND THE RESULTS

We applied the ACC and ACCBA methods to the scattering of 80 MeV deuterons by ^{58}Ni. The elastic scattering and the elastic breakup were treated by the ACC method, while the inelastic scattering ^{58}Ni$(d,d')^{58}$Ni $(2^+;1.45$ MeV) and the inelastic breakup ^{58}Ni$(d,pn)^{58}$Ni $(2^+;1.45$ MeV) were treated by the ACCBA method.

The Becchetti-Greenlees potential[24] was used for V_p and V_n. The potential parameters were chosen for $E_p=E_n=40$ MeV, i.e., for half the deuteron incident energy. The spin-orbit interactions were neglected. For the adiabatic inelastic scattering calculations, we took the deformation parameter to be $\beta_2=0.19$, as in Ref. 25. Using the above parameters and the DWBA method, we also have shown that good fits to data[25] are obtained for the inelastic proton scattering ^{58}Ni$(p,p')^{58}$Ni (2^+; 1.45 MeV) at $E_p = 40$ MeV.

As noted earlier, we assumed that the (bound state) deuteron was in a pure S state. However, for the p-n continuum system we allowed the d-wave ($l=2$) as well as s-wave ($l=0$) deuteron states. Correspondingly, the multipolarity λ of the potential [cf. Eq. (5)] took on the values 0, 2, and 4. The deuteron wave function $\phi_d(r)$ and the s-wave continuum wave function $\phi_0(kr)$ were calculated analytically by using the Yamaguchi-type separable potential with $\alpha = 0.2316$ fm^{-1} and $\beta = 1.45$ fm^{-1}; cf. Ref. 26. For the d wave, on the other hand, we chose a simple free wave form;

$$\phi_2(kr) = \sqrt{2/\pi}(kr)j_2(kr) .$$

Equation (8) for ACC was solved at intervals of 0.25 fm from $r=0$ to $r=20$ fm. Since the excitation energy of the target was much smaller than the bombarding energy, we believe it is justifiable to ignore the excitation energy and thus we used the same ACC solution for both the elastic and the inelastic (1.45 MeV) channels. Throughout the present work, the Coulomb breakup was not taken into account, since its contribution is expected to be very small.

We present first in Table I the angle-integrated cross sections for the various reaction modes. All the inelastic and inelastic breakup cross sections given there are for the 1.45 MeV 2^+ state. The cross sections in the first line are those obtained with the ACC and ACCBA methods, while those in the second line were obtained by using the folding-model method. Since the latter neglects the breakup process, a comparison of these two lines gives a rather clear idea on how the breakup process affects the various cross sections.

One sees first that the breakup process reduces drastically the inelastic cross section (from 16.9 to 10.7 mb), and the decrement 6.2 mb is practically equal to 6.4 mb, which is the sum of the s- and d-wave inelastic breakup cross sections (and is about two thirds of the inelastic cross section). The significance of the inelastic breakup is evident.

The breakup process also causes prominent effects in the elastic and reaction cross sections. The (folding-model) elastic cross section is reduced by 222 mb, and the reaction cross section is increased by 96 mb. The elastic breakup cross section, 143 mb, is about 9% of the reaction cross section. For the elastic breakup, the d-wave contribution is more than twice the s-wave contribution. For the inelastic breakup, however, the d-wave contribution is only 1.3 times the s-wave contribution.

In Fig. 1, we show the contributions of the various L_i, the incident angular momentum of the deuteron, to the angle integrated cross sections of Table

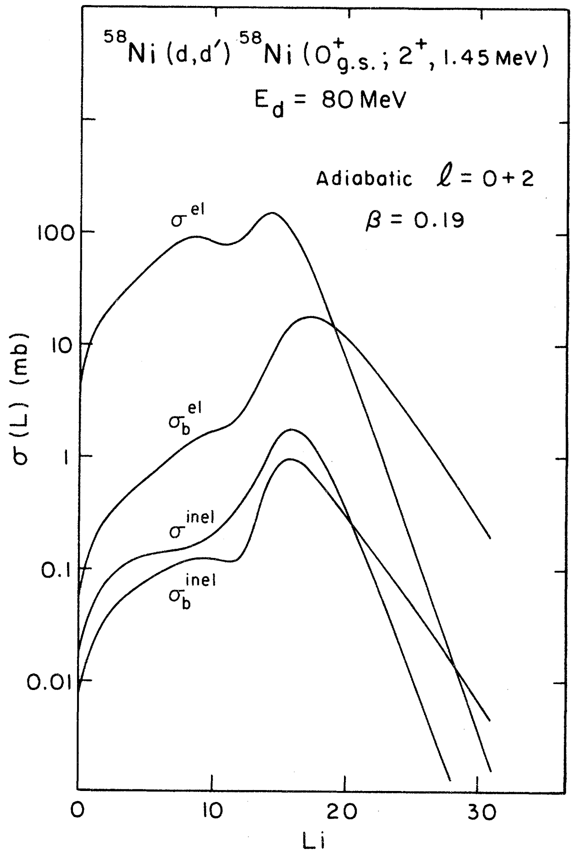

FIG. 1. Partial-wave cross sections of $d + {}^{58}$Ni scattering at $E_d = 80$ MeV as a function of the incident angular momentum of the deuteron, L_i. Calculations were done by ACC and ACCBA methods.

TABLE I. Integrated cross sections of $d + {}^{58}$Ni scattering at $E_d = 80$ MeV in units of mb. The cross sections in the first line are obtained by the ACC and ACCBA methods and those in the second line are obtained by the folding-model method.

	σ^{el}	σ^{react}	$\sigma_b^{\text{el}}(l'=0)$	$\sigma_b^{\text{el}}(l'=2)$	σ^{inel}	$\sigma_b^{\text{inel}}(l'=0)$	$\sigma_b^{\text{inel}}(l'=2)$
Adiabatic	1351.6	1533.0	45.9	97.5	10.7	2.8	3.6
Folding	1573.5	1437.3			16.9		

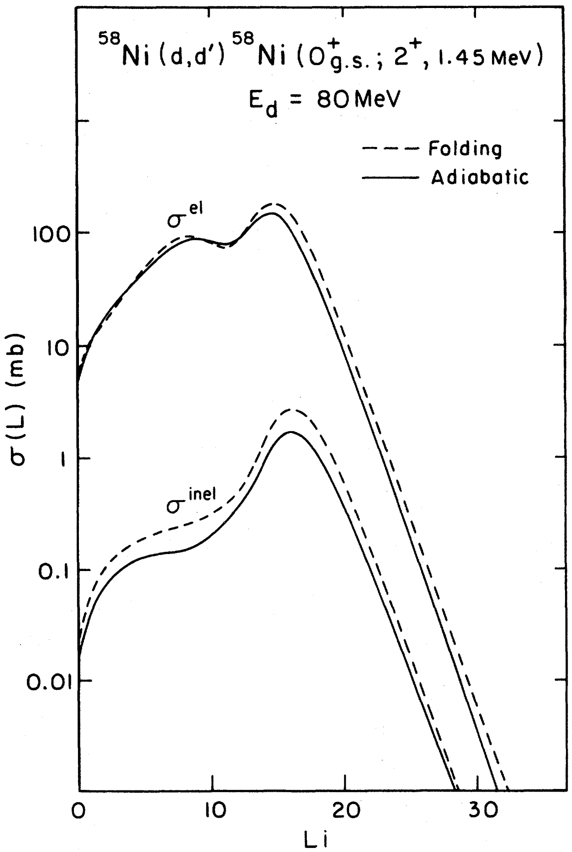

FIG. 2. Same as in Fig. 1 except that the dotted lines show the folding-model calculations.

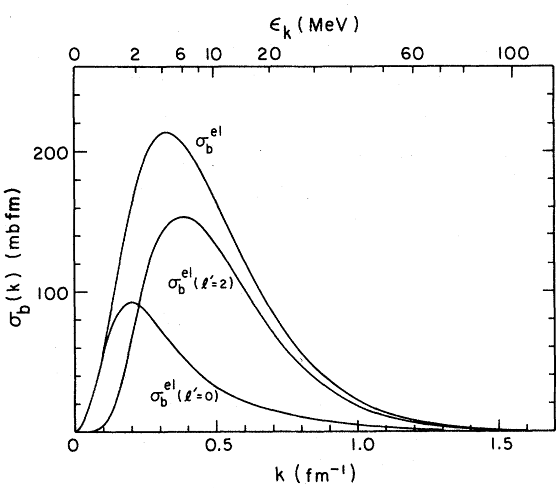

FIG. 3. Elastic breakup cross sections of $d+{}^{58}\mathrm{Ni}$ scattering at $E_d=80$ MeV as a function of the *pn* relative momentum k. σ_b^{el} is the sum of the *s*-wave breakup cross section $\sigma_b^{\mathrm{el}}(l'=0)$ and the *d*-wave breakup cross section $\sigma_b^{\mathrm{el}}(l'=2)$.

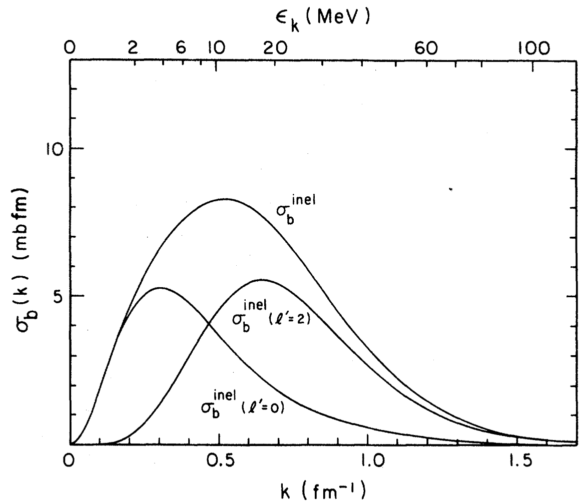

FIG. 4. Same as in Fig. 3 for the inelastic breakup.

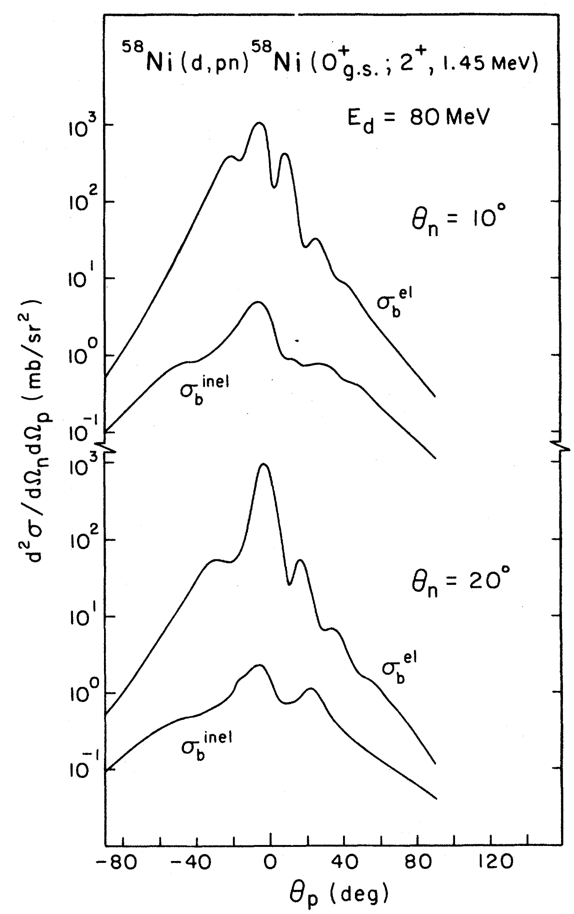

FIG. 5. Double differential cross sections of the elastic and inelastic breakup of $d+{}^{58}\mathrm{Ni}$ scattering at $E_d=80$ MeV.

I. Those given in Fig. 1 are results of ACC and ACCBA calculations. One first notices the similarity between the L_i dependences of $\sigma^{\rm el}$ and $\sigma^{\rm inel}$, and between $\sigma_b^{\rm el}$ and $\sigma_b^{\rm inel}$, in particular regarding the slopes in the large L_i region. It may also be worthwhile to notice that the value $L_i = 16$ that gives the peak contribution to $\sigma^{\rm inel}$ and $\sigma_b^{\rm inel}$ is slightly less than the corresponding $L_i = 17.5$ for $\sigma_b^{\rm el}$. The fact that the slopes for the large L_i for the breakup cross sections are only half as steep as those for the scattering cross sections is not unexpected, since breakup can take place even when the impact parameter is fairly large.

In Fig. 2, we show the L_i dependences of the effects of breakup upon $\sigma^{\rm el}$ and $\sigma^{\rm inel}$ by comparing the results of the folding-model calculations with those using the ACC and ACCBA methods. It is interesting to note that while the reduction (due to the breakup) of $\sigma^{\rm el}$ takes place only for waves with $L_i > 14$, the reduction for $\sigma^{\rm inel}$ occurs for all L_i.

We show in Fig. 3 the elastic breakup cross section as a function of k, the p-n relative momentum. The peaks of $\sigma_b^{\rm el}(l'=0)$, $\sigma_b^{\rm el}(l'=2)$, and their sum are located, respectively, at $\epsilon_k \simeq 2$, 6, and 4 MeV, where ϵ_k is the kinetic energy corresponding to k, as shown below Eq. (10). The smallness of these peak ϵ_k values shows that the adiabatic approximation is rather good here, although it becomes somewhat worse for the d wave.

A similar presentation of the k (or ϵ_k) dependences of the inelastic breakup cross sections is made in Fig. 4. Now the peak ϵ_k appears at about 4, 17, and 11 MeV, respectively, for the s wave, d wave, and the summed cross sections. An enhancement of the larger ϵ_k region, compared with the case of the elastic breakup, is clearly seen. Note that the adiabatic method violates conservation of energy, and this is why the cross sections do not die out beyond 80 MeV.

In order to give an idea of the appearance of the double differential cross sections, we present in Fig. 5 the proton angular distributions for the cases in which the neutron angle θ_n was taken, respectively, at 10° and 20°. It is seen in both figures that the proton distributions for the elastic breakup are sharply forward peaked, while the peaking is significantly reduced for the inelastic breakup, in particular for the larger θ_n. These features faithfully reflect the ϵ_k dependences of the cross sections presented earlier in Figs. 3 and 4.

IV. SUMMARY AND DISCUSSION

In the present paper, we have not attempted to discuss the fits to data of our calculations. Such a discussion was already given in a previous publication,[15] where rather good fits to data were achieved. The present calculations were done within the same framework as were the calculations of Ref. 15, and in this sense may be considered realistic.

We have found a few rather interesting features. We saw that the inelastic breakup accounted for nearly two thirds of the inelastic cross section. It is thus expected that the inelastic breakup cross section summed over all the inelastic excitations of the target constitutes a fairly large fraction of the total reaction cross section.

We showed that the breakup cross sections have rather long tails when seen as functions of the incident angular momenta. We showed also that the breakup effect uniformly reduces the (folding model) inelastic cross section for all the partial waves, while the reduction of the elastic cross section is limited to higher partial waves. It is interesting to compare these situations with that in stripping reactions where it is known that the reduction takes place only for lower partial waves.[9]

The d-wave breakup favors higher values of the relative p-n (linear) momentum, for both elastic and inelastic cases. This shows that the adiabatic approximation may get much less accurate for the d-wave breakup than it is for the s-wave breakup. It is desirable to test the validity of this approximation by comparing the results of the present calculations with those obtained with fewer approximations.

We have presented a few examples of the double differential cross sections, but unfortunately there are no data available to compare with. To obtain such data might be rather difficult, since neutrons in a few tens of MeV region would have to have their energies measured with a rather high accuracy.

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