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Determination of the spins and parities of low-lying states in ^{99}Mo by means of the ^{100}Mo($\vec{d}$,t)^{99}Mo and ^{98}Mo($\vec{d}$,p)^{99}Mo reactions

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Using the vector analyzing powers of the ($\vec{d}$,t) and ($\vec{d}$,p) reactions the spins of twenty-one states of ^{99}Mo have been determined. Below an excitation energy of 1 MeV, these spin assignments agree with those made on the basis of the ratio of S_{lj} values in (d,t) and (d,p) reactions leading to the same final states. New assignments have been made for seven states above 1 MeV. The results have been compared with calculations made by Roy and Choudhury for the even parity states. Fair agreement has been obtained.

[NUCLEAR STRUCTURE ^{99}Mo measured J, π, and spectroscopic factors, for states up to 2.2 MeV using ($\vec{d}$,t) and ($\vec{d}$,p) reactions.]

INTRODUCTION

The properties of the low-lying energy levels of the odd molybdenum isotopes have been investigated by many workers. In particular, the (d,p) and (d,t) reactions on the even isotopes have been done by Moorhead and Moyer,[1] Hjorth and Cohen,[2] Ohnuma and Yntema,[3] and Diehl *et al.*[4] Since the target nuclei have a J^{π} of 0^+, the values of l transfers to many states of the odd nuclei have been uniquely determined. Also, Moorhead and Moyer, and Diehl *et al.* used the ratios of the spectroscopic factors of the (d,p) and (d,t) reactions leading to the same final nuclear states to assign J values. This method depends on the expectation that the $l+\frac{1}{2}$ orbits have higher occupation numbers than the $l-\frac{1}{2}$ orbits in both target nuclei. No measurements of the vector analyzing powers of ($\vec{d}$,p) and ($\vec{d}$,t) reactions on these isotopes have yet been reported.

Shell-model calculations have been done by Bhatt and Ball[5] who calculated the energies of even parity states by considering the residual interactions of protons in the $g_{9/2}$ orbit and neutrons in the $2d_{5/2}$ orbit outside the semiclosed core of ^{90}Zr. In addition, Vervier[6] included particles in the $2p_{1/2}$ orbit, but these calculations were not extended beyond $A=96$.

Choudhury and Clemens[7] assumed particle-vibration coupling in the unified nuclear model and obtained good agreement for ^{95}Mo, but not for ^{97}Mo. Roy and Choudhury[8] calculated the energy levels and spectroscopic factors for the levels in ^{99}Mo using a quasiparticle-vibration coupling calculation in the unified nuclear model. Reasonably good agreement with experiment has been obtained by assuming that the phonon structure of ^{99}Mo is similar to that of ^{100}Mo as opposed to ^{98}Mo and taking the quasiparticle energies ϵ_j and occupation numbers from the ^{100}Mo(d,t)^{99}Mo reaction data of Hjorth and Cohen[2] and Diehl *et al.*[4] The comparison of energy levels and spectroscopic factors was carried out up to an energy of about 1.25 MeV. Many energy levels (with spins and parities) above 1.25 MeV have also been predicted, but have not previously been observed. In this work we reinvestigate the spins of ^{99}Mo, using the vector analyzing powers of the ($\vec{d}$,t) and ($\vec{d}$,p) reactions, as part of a larger study of the nuclear states of these isotopes. These reactions have been successfully used to distinguish between $l+\frac{1}{2}$ and $l-\frac{1}{2}$ angular momentum transfers by many authors.

EXPERIMENTAL

The experimental runs were carried out at the Tandem Accelerator Laboratory at McMaster University. Targets were made of ^{100}Mo and ^{98}Mo by evaporating the separated isotope on a backing of Zapon to a thickness of 80−100 μg/cm^2. An Enge split-pole spectrograph with a position-sensitive detector and particle identification system of the type developed by Michigan State University[9] was used.[10]

The (d,t) spectra were easily observed [Fig. 1(a)] because there were no interfering particles at the magnetic field setting used. To observe the (d,p) spectra a window was set to include only the proton peak in the proportional counter. A typical proton spectrum is shown in Fig. 1(b). The response of the position-sensitive counter (i.e., channel number versus radius of curvature) was determined by using elastically scattered deuterons from ^{100}Mo and stepping the magnetic field of the spectrograph to sweep the focused particles across the counter. This enabled the energies of the excited states to be determined to within ±3 keV. The tritons were observed with an energy resolution of 12 keV FWHM. The angular distribution of the tritons from the reaction ^{100}Mo$(d,t)^{99}$Mo was determined at a deuteron energy of 16 MeV using an unpolarized beam to obtain l values and spectroscopic factors (Fig. 2). The elastic scattering at 30° was monitored and the yield was used to normalize the reaction differential cross sections.

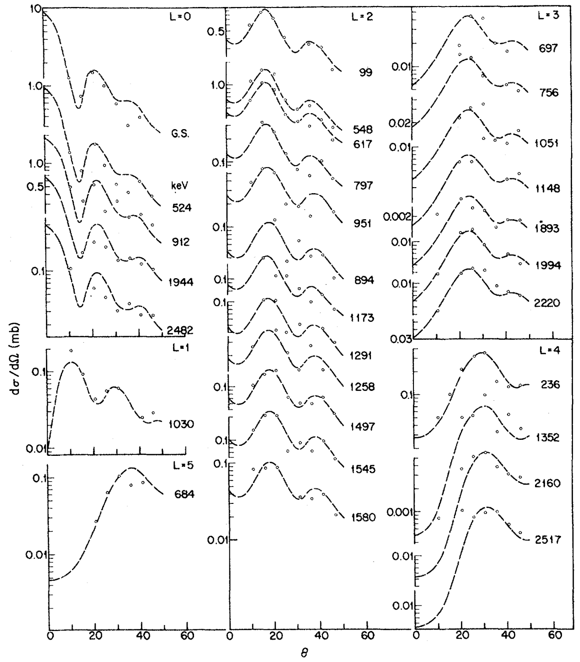

FIG. 2. The differential cross section as a function of center of mass angle for the ^{100}Mo$(d,t)^{99}$Mo reaction at a deuteron energy of 16 MeV. The curves are DWBA predictions.

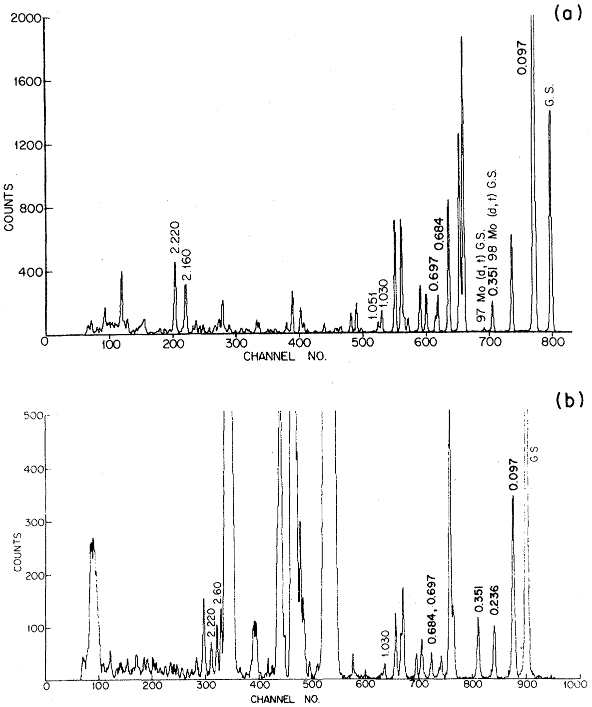

FIG. 1. (a) The spectrum of tritons recorded at 45° from the ^{100}Mo$(d,t)^{99}$Mo reaction with a deuteron energy of 16 MeV and (b) the spectrum of protons recorded at 30° from the ^{98}Mo$(d,p)^{99}$Mo reaction with a deuteron energy of 16 MeV.

The vector analyzing powers of the $(\vec{d},t)$ reaction were measured at beam energies of 16 and 12 MeV. Vector-polarized deuterons were obtained from the McMaster Lamb shift polarized source. The polarization was determined using the quench-ratio method and current of about 30 nA on target was used.

At an energy of 16 MeV the analyzing powers were very small for the higher excited states and were essentially useless above 0.5 MeV. At a deuteron energy of 12 MeV, the emerging tritons had energies close to the Coulomb barrier and A_y was large for excitation energies above 0.5 MeV. It was positive for $l-\frac{1}{2}$ and negative for $l+\frac{1}{2}$ transfers over a wide range of forward angles as shown in Fig. 3.

The vector analyzing power (VAP) of the ^{98}Mo$(\vec{d},p)^{99}$Mo reaction was also measured at a deuteron energy of 12 MeV, and these data, shown in Fig. 4, were used to verify the results obtained from the $(\vec{d},t)$ reaction, and to clarify uncertain results.

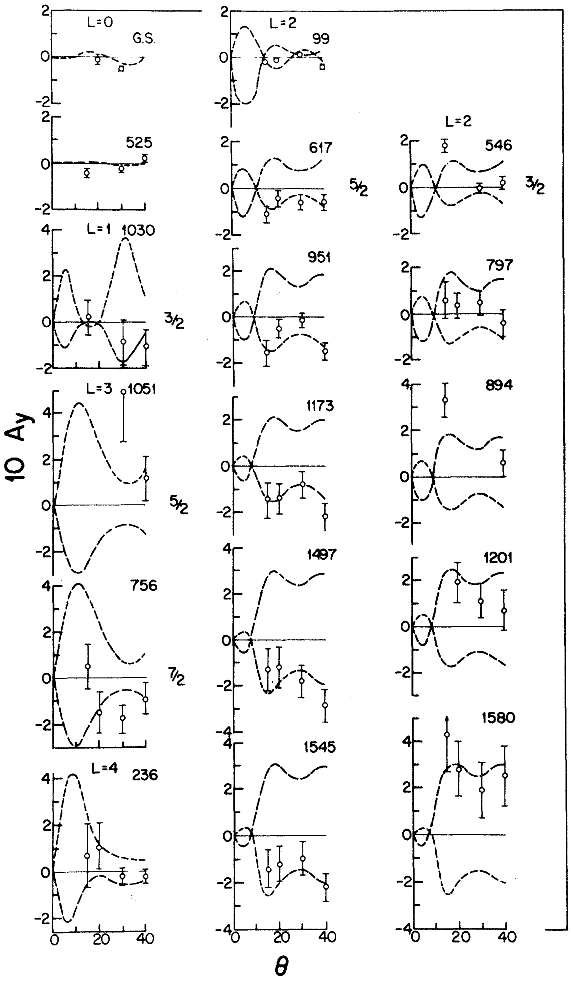

FIG. 3. The vector analyzing power as a function of laboratory angle θ of the ^{100}Mo$(d,t)^{99}$Mo reaction at $E_d = 12$ MeV. The curves are DWBA predictions.

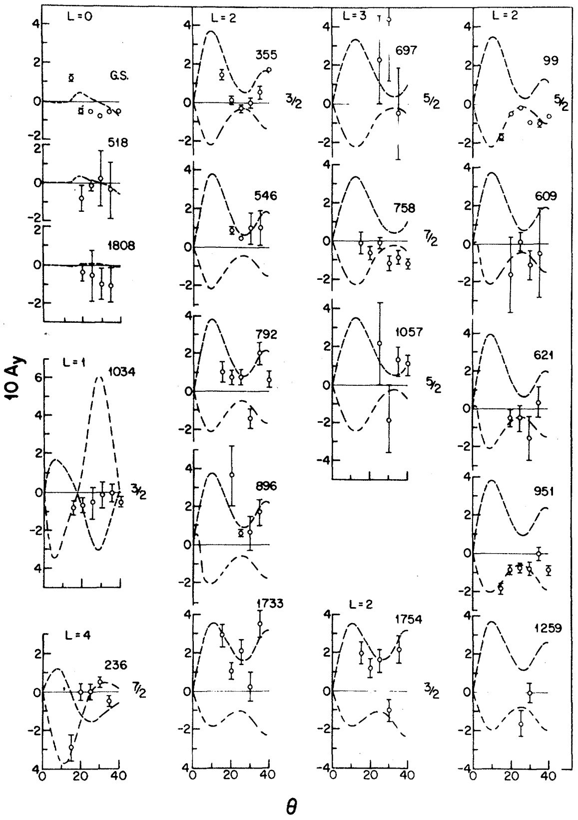

FIG. 4. The vector analyzing power as a function of laboratory angle θ of the ^{98}Mo$(d,p)^{99}$Mo reaction at $E_d = 12$ MeV. The curves are calculated using the DWBA formalism.

RESULTS AND ANALYSIS

The distorted wave calculations were done using the computer code DWUCK4.[11] The deuteron potentials were those of Lohr and Haeberli[12] and the triton potentials were those used by Hardekopf *et al.*[13] for ^{90}Zr. The proton potentials were those of Becchetti and Greenlees[14] and the bound neutron potential was calculated in each case to give the correct value of the binding energy. A list of these potentials is shown in Table I.

States below 1 MeV

The energies and parities of these states agree with those reported by Diehl *et al.*[4] for the ^{100}Mo$(d,t)^{99}$Mo reaction (Fig. 2). Our peak at 0.351 MeV is mainly due to the (d,t) reaction on ^{98}Mo

TABLE I. Optical-model parameters. All energies are in MeV; radii and diffuseness parameters are in fm.

Particle	Reaction	Energy	V	r_0	a_0	W	r_w	a_w	V_{so}	r_{so}	a_{so}	re
d	^{100}Mo$(dt)^{99}$Mo	16.00	111.03	1.05	0.86	10.12	1.43	0.78	7.0	0.75	0.5	1.30
d	^{100}Mo$(dt)^{99}$Mo	12.00	111.03	1.05	0.86	10.12	1.43	0.78	7.0	0.75	0.5	1.30
d	^{98}Mo$(d,p)^{99}$Mo	12.00	111.17	1.05	0.86	10.25	1.43	0.78	7.0	0.75	0.5	1.30
p	^{98}Mo$(d,p)^{99}$Mo	15	56.5	1.17	0.75	9.76	1.32	0.62	6.2	1.01	0.75	1.3
t	^{100}Mo$(d,t)^{99}$Mo	14,10	162	1.2	0.65	13.5[a]	1.6	0.87	6.0	1.15	0.51	1.3

[a] W = imaginary volume potential.

leading to the ground state of ^{97}Mo. Only one third of the intensity observed is due to the level in ^{99}Mo. This state is observed cleanly in the ^{98}Mo$(d,p)^{99}$Mo reaction and the cross section and VAP give $J^{\pi}=\frac{3}{2}^{+}$, in agreement with the suggestion of Diehl *et al.* and Moorhead and Moyer.

The 0.686 MeV state is assigned $l=5$ by Diehl *et al.* and Moorhead and Moyer. Ishimatsu *et al.*[15] give l values of 3 and 5, indicating that it is a doublet. It is clearly a doublet in Fig. 1(a), and the cross sections of the unfolded peaks are shown in Fig. 2. The lower energy peak is at 0.685 MeV and has $l=5$; the higher one is at 0.697 MeV with $l=3$. This latter has a spin of $\frac{5}{2}$ from the $(\vec{d},p)$ VAP measurements (Fig. 4). We were unable to assign a spin to the $l=5$ state.

The 0.756 MeV level is $\frac{7}{2}^{-}$ on the basis of both $(\vec{d},t)$ and $(\vec{d},p)$ VAP measurements, in disagreement with Diehl *et al.* and Moorhead and Moyer's tentative suggestions of $\frac{5}{2}^{-}$ and $\frac{7}{2}^{+}$, respectively.

TABLE II. The values of l and j transfers to states in ^{99}Mo from (dt) and (dp) reactions and the spectroscopic factors from the (dt) reaction.

Present work				Diehl *et al.*				Moorhead and Moyer (dp)		
Excitation energy	l	J	$S_{lj}(dt)$	Excitation energy	l	J	$S_{lj}(d,t)$	Excitation energy	l	J
0	0	$\frac{1}{2}$	0.250	0.0	0	$\frac{1}{2}$	0.328	0	0	$\frac{1}{2}$
0.099	2	$\frac{5}{2}$	2.25	0.98	2	$\frac{5}{2}$	2.889	0.097	2	$\frac{5}{2}$
0.236	4	$\frac{7}{2}$	1.37	0.236	4	$\frac{7}{2}$	2.26	0.233	4	$\frac{7}{2}$
0.351	2	$\frac{3}{2}$	0.19	0.350	2	$\frac{3}{2}$	0.129	0.348	2	$\frac{3}{2}$
0.524	0	$\frac{1}{2}$	0.167	0.526	0	$\frac{1}{2}$	0.442	0.529	0	$\frac{1}{2}$
0.548	2	$\frac{3}{2}$	0.406	0.549	2	$\frac{3}{2}$	0.789	0.552	2	$\frac{3}{2}$
0.617	2	$\frac{5}{2}$	0.227	0.615	2	$\frac{5}{2}$	0.463	0.619	2	$\frac{5}{2}$
0.684	5		0.527	0.686	5	$\frac{11}{2}$	0.863	0.688	5	$\frac{11}{2}$
0.697	3	$\frac{5}{2}$	0.052							
0.756	3	$\frac{7}{2}$	0.179	0.755	3	$\frac{5}{2}$	0.480	0.760	(4)	$(\frac{7}{2})$
0.797	2	$\frac{3}{2}$	0.069	0.792	2	$(\frac{3}{2})$?	0.155	0.798	(3)	$(\frac{7}{2})$
0.894	2	$\frac{3}{2}$	0.034	0.891	2	$\frac{3}{2}$	0.120	0.896	2	$\frac{3}{2}$
0.912	0	$\frac{1}{2}$	0.064	0.906	0	$\frac{1}{2}$	0.148	0.913	0	$\frac{1}{2}$
0.951	2	$\frac{5}{2}$	0.164	0.945	2	$(\frac{5}{2})$	0.291	0.952	3	$\frac{7}{2}$
1.030	1	$\frac{3}{2}$	0.020					1.033	1	$(\frac{3}{2})$
1.051	3	$\frac{5}{2}$	0.036							
1.148	3		0.019							
1.173	2	$\frac{5}{2}$	0.035							
1.201	2	$\frac{3}{2}$	0.033							
1.258	2	$\frac{5}{2}$	0.019					1.261	0	$\frac{1}{2}$
1.352	(4)							1.391	2	$\frac{3}{2}$
1.497	2	$\frac{5}{2}$	0.032					1.493	2	$\frac{3}{2}$
1.545	2	$\frac{5}{2}$	0.043					1.548		
1.580	2	$\frac{3}{2}$	0.021							
1.893	3		0.043							
1.944	0	$\frac{1}{2}$	0.05							
2.160	4		0.645							
2.220	3		0.049							

On the other hand, the 0.792 MeV level is quite definitely $\frac{3}{2}^+$ from our $(\vec{d},p)$ data, agreeing with Diehl's tentative assignment. Both the $(\vec{d},p)$ and $(\vec{d},t)$ VAP measurements give $\frac{5}{2}^+$ for the state at 0.951 MeV, which, presumably, is the same as Diehl's at 0.945 MeV. Our assignment agrees with Diehl's tentative assignment, but not with that of Moorhead and Moyer. The values of j^π assigned to the other states below 1 MeV in the present work agree with those assigned by Diehl *et al.*

States above 1 MeV

$l=0$ states

Apart from the ground state, the 0.526 MeV and the 0.912 MeV states, the other $l=0$ states found are at 1.944 and 2.482 MeV. The 1.944 MeV level was previously observed by Moorhead and Moyer, but that at 2.482 MeV is new.

TABLE III. Fullness parameters and energy centroids for the s and d orbits in ^{100}Mo.

^{100}Mo$(d,t)^{99}$Mo Orbit	V_j^2	Energy centroid ϵ_j (MeV)
$s_{1/2}$	0.75	0.236
$d_{5/2}$	0.46	0.252
$d_{3/2}$	0.15	0.665

$l=1$ states

The only $l=1$ state definitely identified is the one at 1.030 MeV and the spin is $\frac{3}{2}$ as found in both the $(\vec{d},p)$ and $(\vec{d},t)$ VAP measurements at 12 MeV.

TABLE IV. The experimentally obtained energies and (l,j) values for even parity states in ^{99}Mo and the predictions of Roy and Choudhury.

			States in ^{99}Mo		
	Present work			Roy and Choudhury	
E (MeV)	l	j	E (MeV)	l	j
0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$
0.099	2	$\frac{5}{2}$	0.100	2	$\frac{5}{2}$
0.236	4	$\frac{7}{2}$	0.220	4	$\frac{7}{2}$
0.351	2	$\frac{3}{2}$	0.350	2	$\frac{3}{2}$
			0.510	2	$\frac{5}{2}$
0.524	0	$\frac{1}{2}$	0.570	0	$\frac{1}{2}$
0.548	2	$\frac{3}{2}$			
			0.594	4	$\frac{7}{2}$
0.617	2	$\frac{5}{2}$	0.600	2	$\frac{3}{2}$
			0.700	2	$\frac{5}{2}$
			0.772	4	$\frac{7}{2}$
0.797	2	$\frac{3}{2}$	0.797	2	$\frac{5}{2}$
			0.838	2	$\frac{3}{2}$
			0.882	4	$\frac{7}{2}$
0.894	2	$\frac{3}{2}$	0.890	2	$\frac{3}{2}$
0.912	0	$\frac{1}{2}$	0.910	0	$\frac{1}{2}$
0.951	2	$\frac{5}{2}$	0.954	2	$\frac{5}{2}$
			1.130	2	$\frac{5}{2}$
1.173	2	$\frac{5}{2}$	1.170	2	$\frac{3}{2}$
			1.190	4	$\frac{7}{2}$
1.201	2	$\frac{3}{2}$			
			1.220	0	$\frac{1}{2}$

l=2 states

Eight such states have been found. Of these, the following are assigned by the $(\vec{d},t)$ reaction as follows: 1.173 $(\frac{5}{2}^+)$, 1.201 $(\frac{3}{2}^+)$, 1.497 $(\frac{5}{2}^+)$, 1.545 $(\frac{5}{2}^+)$, and 1.580 $(\frac{5}{2}^+)$. Our assignment for the state at 1.497 MeV $(\frac{5}{2}^+)$ disagrees with $\frac{3}{2}^+$ of Moorhead and Moyer. These authors observed the 1.545 MeV state but made no assignment. The states at 1.173 $(\frac{5}{2}^+)$ and 1.580 $(\frac{3}{2}^+)$ have not been previously reported. The state at 1.201 $(\frac{3}{2}^+)$ may be the 1.209 level reported by Cavallini *et al.*[16] in the decay of ^{99}Nb.

From the $(\vec{d},p)$ measurements, the assignments made are 1.259 $(\frac{5}{2}^+)$, 1.545 $(\frac{5}{2}^+)$, 1.733 $(\frac{3}{2}^+)$, and 1.754 $(\frac{5}{2}^+)$. Our level 1.259 $(\frac{5}{2}^+)$ has been reported for the first time, unless it is the 1.261 $(\frac{1}{2}^+)$ level reported by Moorhead and Moyer. Our 1.545 MeV level $(\frac{5}{2}^+)$ confirms the results already found for this level from the $(\vec{d},t)$ measurements. The level at 1.733 $(\frac{3}{2}^+)$ could be the 1.722 $(\frac{3}{2}^+)$ of Moorhead and Moyer, otherwise it is a new level. Finally, the 1.754 $(\frac{3}{2}^+)$ agrees with the above authors's tentative assignment of $\frac{3}{2}^+$.

l=3 states

The $(\vec{d},t)$ and $(\vec{d},p)$ VAP measurements at 12 MeV reveal the 1.050 MeV state to be $\frac{5}{2}^-$. The VAP for the 2.220 MeV state at 16 MeV is ambiguous (not shown). At 12 MeV bombarding energy, it is weakly excited; therefore, no *j* assignment can be made for this state.

DISCUSSION

Apart from the cases noted above, our VAP spin assignments agree with those derived from the ratios of the spectroscopic factors for (d,p) and (d,t) reactions leading to the same final nuclear states. There is, therefore, no sudden change in occupation numbers of the neutron orbits as one goes from ^{98}Mo to ^{100}Mo. Our spectroscopic factors for the (d,t) reaction (Table II) are somewhat smaller than those of Diehl *et al.*,[4] and so our fullness parameters are different. The values quoted here are closer to those of Hjorth and Cohen.[2] These differences can be accounted for by the optical model potentials used. The energy centroids, shown in Table III, agree well with those of Diehl *et al.*[4]

The predictions of Roy and Choudhury for the energies and (l,j) values for the even parity states in ^{99}Mo, up to an excitation energy of 1.220 MeV, are shown in Table IV. The values obtained from the present work are also listed. Four $l=0$ states were predicted and three were found. Four $l=4$ states were predicted, but only one was found. Eleven $l=2$ states were predicted. We found nine $l=2$ states but only four of these had the predicted j value.

The states above 1.220 MeV were only presented graphically in Ref. 8. The following four states have been correctly predicted with an energy uncertainty of ~40 keV: 1.258 $(\frac{5}{2}^+)$, 1.352 $(\frac{7}{2}^+$ or $\frac{9}{2}^+)$, 1.497 $(\frac{5}{2}^+)$, and 1.580 $(\frac{3}{2}^+)$.

Thus the calculations of Roy and Choudhury for the even-parity states of ^{99}Mo may be described as partially successful. It would be interesting to repeat these calculations using the values of ϵ_j and V_j^2 quoted here, even though the present values do not differ greatly from those already tried by these authors.

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[*] On leave from the University of Riyadh, Saudi Arabia.

[1] J. B. Moorhead and R. A. Moyer, Phys. Rev. 184, 1205 (1969).

[2] S. A. Hjorth and B. L. Cohen, Phys. Rev. 135, 920 (1964).

[3] H. Ohnuma and J. L. Yntema, Phys. Rev. 178, 1855 (1969).

[4] R. C. Diehl, B. L. Cohen, R. A. Moyer, and L. H. Goldman, Phys. Rev. C 1, 2132 (1970).

[5] K. H. Bhatt and J. B. Ball, Nucl. Phys. 63, 286 (1965).

[6] J. Vervier, Nucl. Phys. 75, 17 (1966).

[7] D. C. Choudhury and J. T. Clemens, Nucl. Phys. A125, 140 (1969).

[8] B. Bannan Roy and D. C. Choudhury, Phys. Rev. C 12, 1 (1975).

[9] R. G. Markham and R. G. H. Robertson, Nucl. Instrum. Methods 129, 131 (1975).

[10] For details of the system, see the internal Report MAL/80/1-2 by E. E. Habib and J. A. Cameron, Tandem Accelerator Laboratory, McMaster University.

[11] P. D. Kunz, private communication.

[12] J. M. Lohr and W. Haeberli, Nucl. Phys. A 232, 381 (1974).

[13] R. A. Hardekopf, L. R. Veeser, and P. W. Keaton, Jr., Phys. Rev. Lett. 35, 1623 (1975).

[14] F. D. Becchetti, Jr. and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[15] T. Ishimatsu, S. Hayashibe, N. Kawamura, T. Awaya, H. Ohmura, Y. Nakajima, and S. Mitarai, Nucl. Phys. A 185, 273 (1972).

[16] P. Cavalini, J. Bladiot, E. Monnand, and A. Moussa, Radiochem. Acta. 15, 105 (1971).