

Nuclear charge distribution and rms radius of ^{12}C from absolute elastic electron scattering measurements

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Elastic electron scattering cross sections for the nucleus ^{12}C have been measured in a momentum transfer range from 0.25 to 2.75 fm^{-1} . The data were analyzed in a model independent way with a Fourier-Bessel parametrization of the charge distribution. For the rms radius, the value $\langle r^2 \rangle^{1/2} = (2.464 \pm 0.012) \text{ fm}$ (no dispersion corrections applied) has been obtained, and agrees with those of other electron scattering experiments and with muonic atom experiments, but disagrees with data obtained from measurements of muonic x-ray transitions with a crystal spectrometer which show a larger rms radius. The extracted distribution disagrees with Hartree-Fock calculations in both the radial dependence and the rms radius. The behavior of the form factor in the diffraction minimum was investigated in detail. The experimentally determined cross sections in the minimum are always larger than those obtained from the Fourier-Bessel analysis. These deviations amounted to 5% for a primary energy of 320 MeV and to 2% for 240 MeV and may be interpreted as an indication of dispersion effects.

[NUCLEAR REACTIONS ^{12}C ; absolute (e,e) cross sections measured.
Charge distribution and rms radius deduced. Discussion of dispersion
effects, comparison with HF calculations.]

I. INTRODUCTION

During the past decades, elastic electron scattering measurements have been carried out to determine nuclear charge and magnetization distributions, while inelastic electron scattering experiments yield information about dynamic properties of nuclei by excitation of nuclear levels. It is customary to make such measurements relative to ^{12}C as a standard reference nucleus. Several laboratories have, therefore, undertaken absolute measurements¹⁻⁴ on ^{12}C . Most of these experiments only cover a momentum transfer range $q < 1 \text{ fm}^{-1}$, too small for a model independent evaluation of the charge distribution. In Stanford¹ in 1970, the charge distribution was determined by a measurement at high momentum transfers, but no data were taken at $q < 1 \text{ fm}^{-1}$. In this work we report ^{12}C cross sections taken in one laboratory which cover a wide range in q from 0.3 to 2.8 fm^{-1} .

The experimental setup including a fixed angle spectrometer, constructed for absolute cross section measurements, and the rotating target are described in Sec. II. Details about the measurements and determination of cross sections as well as their uncertainties are given in Sec. III. The method for the

evaluation of the charge distribution parameters and the influence of errors is presented in Sec. IV, and the results of this experiment are compared with those of other electron scattering measurements and of muonic x-ray transitions. Finally, in Sec. V, the results are interpreted and the extracted charge distribution is compared to Hartree-Fock (HF) calculations. A possible effect of dispersion contributions in the first minimum of the cross section will be discussed.

II. EXPERIMENTAL APPARATUS

The electron scattering facility at Mainz was described in detail by Ehrenberg *et al.*,⁵ Herminghaus *et al.*,⁶ and Glückert *et al.*⁷ In this section, only those aspects of the apparatus which are relevant to absolute measurements are explained.

A. Accelerator and beam handling system

The accelerator delivers a beam with an average current of up to 30 μA at energies from 80 to 350 MeV. The repetition rate of the linac is 100 Hz and

the pulse length about 3 μs . The energy is controlled by energy defining slits which are typically set to a width of 0.1%. Beam position monitors located in front of the energy defining system and also in front of the target are used to provide both momentum stability ($\pm 0.03\%$) and stability of the beam position (± 0.3 mm) at the target. The beam current is measured with an induction monitor (toroid) designed by Stephan.⁸ The precision of this monitor, for currents from 10 nA to 30 μA , is better than 0.3%. Its absolute calibration is checked repeatedly.

B. Spectrometers

Two spectrometers are used to detect the scattered electrons: a fixed angle spectrometer^{9,10} which was designed to allow precise absolute cross section measurements, and a 180° double focusing spectrometer for relative measurements.

1. 180° spectrometer

The double focusing 180° spectrometer has a radius of curvature of 1 m and can be set to scattering angles from 28° to 157° . The solid angle can be controlled by slits which are located at a distance of 460 mm from the target. With this geometry the scattering angle is known to an accuracy of $\pm 0.03^\circ$. Using the full opening of the slits, the scattering angle and solid angle of about 6 msr are defined by the walls of the vacuum chamber, allowing the definition of the scattering angle to be known within $\pm 0.02^\circ$. The 300 channel detector system¹¹ consists of overlapping plastic scintillators in coincidence with Čerenkov detectors, similar to the "Amsterdam System."¹² The channel width is 1 mm, which corresponds to a momentum bin of 2.5×10^{-4} . The relative efficiency of the channels is usually determined from a "white spectrum" measured in the quasielastic region of ^{12}C .

2. Fixed angle spectrometer

The fixed angle spectrometer was installed at a scattering angle of 28° and designed either to measure absolute electron scattering cross sections or, when carrying out relative measurements with the 180° spectrometer, to monitor the product of beam current times target thickness. Thus, experiments with inhomogeneous targets are feasible because the

counting rate of the fixed angle spectrometer is proportional to the actual target thickness.

The fixed angle spectrometer is of a quadrupole-quadrupole-dipole (QQD) type, a point-to-point focusing spectrometer as shown in Fig. 1. Its main parameters are given in Table I. The magnet-optical design and the shape of the collimators ensure that the counting rate of the spectrometer will be independent of fluctuations of the beam spot on the target. A distance as large as possible between target and entrance collimator was chosen in order to determine the solid angle precisely. Seven different sizes for the entrance collimators are available by remote control rotation of a wheel. In this way, the variation of the collimators is from one to sixty from the smallest to largest opening. Only the three largest collimators were used for absolute measurements, and their solid angle was known within an accuracy of $\leq 0.16\%$. The large distance from the target also allows the definition of the scattering angle to better than 0.01° . Slits of different widths mounted on a remotely controlled wheel are placed in the image plane 80 cm behind the dipole magnet to define the momentum acceptance. This acceptance $\Delta p/p$ can be varied from 0.5 to 12%. The smallest opening is mainly used for tuning the spectrometer; i.e., for adjusting the currents of the quadrupole and dipole magnets. Using a larger opening, a well defined portion of the spectrum is then observed by one Čerenkov detector as shown in Fig. 2. The efficiency of this single channel detector is 0.9997 ± 0.0003 . The detector can handle up to 3 counts per beam pulse, i.e., 300 counts per second with dead time losses smaller than 1%. These losses are measured on-line by delayed coincidences.

C. Alignment, angle, and energy calibration

The beam transport system with its magnets, collimators, and target holder was aligned by conven-

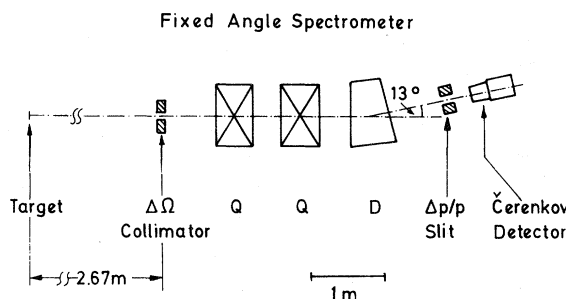


FIG. 1. Side view of the fixed angle spectrometer (schematic).

TABLE I. Parameters of the fixed angle spectrometer.

Quadrupoles:	
Distance from target	3.2 m
Length	200 mm
Aperture	80 mm
Dipole:	
Deflection angle	13°
Radius of curvature	1.8 m
Gap	60 mm
Optical properties:	
Linear dispersion	(1.93±0.01) mm/%
Momentum acceptance	±6%
Magnification in dispersion plane	-1.14
in nondispersion plane	-0.22
Distance of $\Delta\Omega$ collimator from target	2.678 m
Minimal solid angle $\Delta\Omega$	0.3786 μsr
Maximum solid angle $\Delta\Omega$	22.37 μsr

tional optical methods. The direction of the incoming electron beam was compared with the optical axis by a left-right electron scattering measurement with the 180° spectrometer. The scattering angle of this spectrometer was determined optically and calibrated with respect to a fixed scale. This calibration takes into account the small, reproducible

motion of the pivotal point of the magnet while rotating. The angular position of the spectrometer can be reproduced with an accuracy better than $\pm 0.01^\circ$.

The electron energy is determined from the spectrometer field setting which is controlled by a nuclear magnetic resonance (NMR) probe. The calibration of electron momentum versus NMR frequency has been obtained from the kinematics of electron scattering on a CH_2 target. By choosing the proper kinematics the energy of electrons scattered elastically from the proton is the same as for electrons scattered inelastically from carbon such that the respective peaks coincide. Thus the recoil energy transferred to the proton is calibrated by the well known excitation energy in ^{12}C and allows an exact calculation of the primary energy E_0 . Since the recoil energy is roughly proportional to E_0^2 , the relative uncertainty of the calibration function is smaller at higher energies. On the average, the total error of the energy calibration is $\pm 0.12\%$.

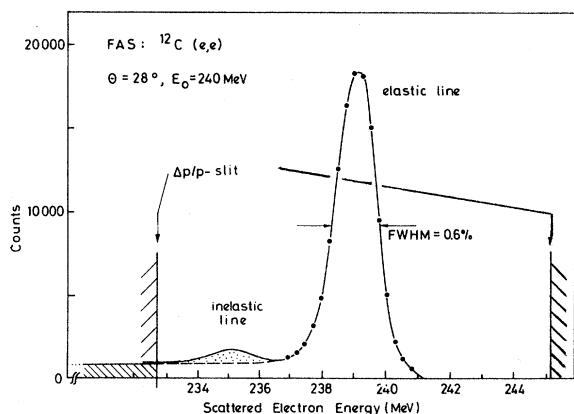


FIG. 2. Spectrum of electrons scattered from carbon as incident on the Čerenkov detector of the fixed angle spectrometer. The full circles represent a measurement performed under variation of the dipole field and with the slit set to $\Delta p/p = 0.5\%$. For the measurement of cross sections, the magnetic field is adjusted such that the elastic line is centered within the slits which are set to $\Delta p/p = 5\%$ as indicated by the arrows. The counting rate of the Čerenkov detector is then proportional to the area under the full line. To extract the cross section for elastic scattering, the losses due to radiative processes (hatched area) and the influence of inelastic levels (dotted area) have to be corrected.

D. Target

The precision of absolute cross section measurements depends directly on the knowledge of the actual target thickness. Thus either targets of high homogeneity are needed or sufficiently large areas of the target have to be exposed to the beam to average out inhomogeneities. A sheet of graphite, lathed to a thickness below 1 mm, always shows inhomogeneities of about 1–2% as found from x-ray absorption measurements.¹³ Therefore, a disk of

natural carbon (graphite) with a chemical purity of 99.98% was used; its diameter was 200 mm, the thickness 0.5 mm corresponding to 82 mg/cm². The average target thickness was calculated from weight and area measurements. This target was rotated and wobbled vertically during the exposure to the electron beam. The fact that the exposure time was large compared to the cycling time of the target resulted in an averaging out of inhomogeneities; thus the uncertainty of the effective target thickness is smaller than 0.2%.¹³

III. EXPERIMENTS AND DETERMINATION OF CROSS SECTIONS

Absolute elastic cross sections of ^{12}C were measured at beam energies from 100 to 300 MeV with the fixed angle spectrometer, covering a range of momentum transfer $q = (2E_0/\hbar c) \sin(\theta/2)$ from $q = 0.25 \text{ fm}^{-1}$ to $q = 0.75 \text{ fm}^{-1}$. Cross sections at higher momenta have been obtained with the 180° spectrometer at energies between 150 and 320 MeV and at angles from 40 to 130 degrees. These relative data cover a range in q from 0.55 to 2.75 fm⁻¹. They are normalized to the absolute data, obtained with the fixed angle spectrometer, in the overlapping region.

A. Absolute cross sections

The method of the absolute determination of the ^{12}C cross sections is based on a measurement of the elastically scattered electrons integrated up to a cut-off energy ΔE below the elastic peak. As can be seen from Fig. 2, the slit in front of the single channel Čerenkov detector of the fixed angle spectrometer defines the measured momentum range Δp of the scattered electrons; this corresponds to the cut-off energy ΔE . The magnetic field of the fixed angle spectrometer was adjusted in such a way that the elastic peak was located in the center of the slit. The missing counting rate due to radiative processes (hatched area) is taken into account by the usual radiative corrections.^{14,15} The small contribution of the inelastic levels (dotted area) has to be subtracted. This area is either determined from a measurement with the 180° spectrometer, set at 28°, or calculated from the known value of the inelastic cross sections. The reliability of these corrections has been checked by using different slit widths corresponding to $\Delta p/p$ ranging from about 4 to 10%. To cover the above mentioned momentum range at

the fixed angle of 28°, the energy was varied from 100 to 300 MeV. In this series of measurements, the reproducibility of the apparatus was checked by repeating measurements at the same energy. The resulting absolute cross sections are given in Table II.

B. Relative cross section measurements

Relative carbon cross sections were measured with the 180° spectrometer at energies of 150, 240, 300, and 320 MeV. The scattering angles were varied between 40° and 125°; thus, a momentum range from 0.55 to 2.75 fm⁻¹ was covered. In doing so, a sufficient overlap of the momentum transfer between the different energies was secured in order to be able to normalize the data with each other. To give an impression of the quality of the data, a carbon spectrum is presented in Fig. 3. This measurement was made under the most unfavorable conditions, namely in the first minimum of the cross section. The full line represents calculated line shapes taking into account the resolution function of the apparatus and radiative as well as straggling effects in the target. The observed deviations are mainly due to the ^{13}C content of the target and to an instrumental background consisting of a tail of the 4.4 MeV-peak of ^{12}C , the amplitude of the latter being about 75 times larger than that of the elastic one in its minimum. This background tail is

TABLE II. Absolute cross sections for elastic electron scattering from ^{12}C measured at $\theta = 28^\circ$ with the fixed angle spectrometer. $E - n$ denotes 10^{-n} .

E_0 (MeV)	$d\sigma/d\Omega$ (fm ² sr ⁻¹)	error/(%)
100.0	0.4624	0.45
100.0	0.4640	0.45
120.0	0.3054	0.46
140.0	0.2090	0.46
140.0	0.2075	0.46
150.0	0.1758	0.46
180.0	0.1083	0.46
180.0	0.1081	0.46
200.0	0.7916 $E - 1$	0.47
220.0	0.5886 $E - 1$	0.47
220.0	0.5881 $E - 1$	0.47
240.0	0.4412 $E - 1$	0.47
260.0	0.3274 $E - 1$	0.48
280.0	0.2450 $E - 1$	0.48
300.0	0.1850 $E - 1$	0.49
300.0	0.1842 $E - 1$	0.49

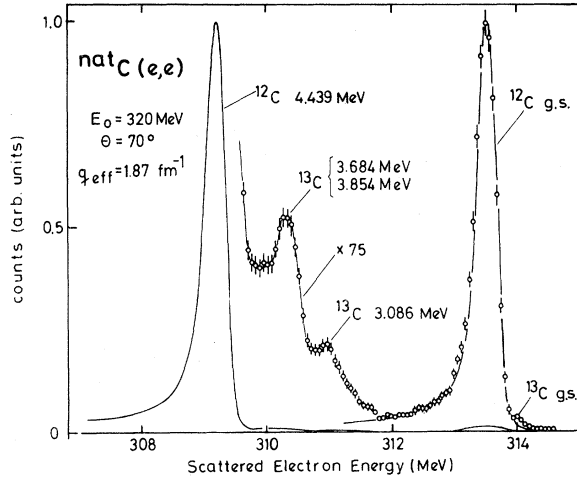


FIG. 3. Spectrum measured with the 180° spectrometer in the first minimum of the cross section for elastic scattering of electrons from carbon.

common to all peaks, and stems from electrons that are Møller-scattered from one detector to neighboring detectors and due to the tilted detector array, shows up mainly at the high-energy side of all peaks. This contribution as well as that of ^{13}C is corrected for, the corrections being relatively small outside of the minimum where the true counts in the peak of interest are large.

The cross sections were determined by a summation of the counts of all channels up to the cutoff energy ΔE and by the application of the appropriate radiative corrections given by Maximon¹⁴ and Isabelle.¹⁵ This method was cross checked against a curve fitting procedure, and excellent agreement between the two methods was found.

Near the minimum of the form factor, where the slope and curvature of the cross section curve is large with respect to energy and scattering angle, the effects of finite solid angle, energy spread of the electron beam, and straggling in the target became considerable. They have been corrected in an iterative way following the method of Lyman *et al.*,¹⁶ which was incorporated into the phase shift code MEFIT¹⁷ used in the present evaluation. The corrections were normally of the order of one percent or less; only near the first minimum they amounted to 12%. In order to check the validity of the correction, the measurements at 300 and 320 MeV were performed with horizontal apertures of 2.55° and 1.8°, respectively.

The overall stability of the measuring system was constantly monitored by the fixed angle spectrometer while taking relative data with the 180° spec-

trimeter. This means the counting rate of the fixed angle spectrometer had to remain constant under variation of the scattering angle for the 180° spectrometer, except for the change of the effective target thickness, which is easy to correct. The beam energy was evaluated for each spectrum separately and its stability was better than 2×10^{-4} . The final relative cross sections, normalized to the absolute data of Table II, are given in Table III. Here, all corrections discussed before have been carried out; i.e., these are cross sections that would be measured with infinitely small solid angle and target thickness.

All ^{12}C cross sections are plotted in Fig. 4 versus the effective momentum transfer q_{eff} . This figure shows the wide overlap of the different data sets as well as the drop of the cross sections over more than eight decades.

IV. CHARGE DENSITY DISTRIBUTION

The experimental cross section for elastic electron scattering on spin zero nuclei is, in Born approximation, related to the charge distribution $\rho(r)$ by

$$\left[\frac{d\sigma}{d\Omega} \right]_{\text{exp}} = \left[\frac{d\sigma}{d\Omega} \right]_{\text{point}} \cdot F^2(q), \quad (1)$$

where F is the form factor

$$F(q) = \frac{1}{Z} \int \rho(r) \frac{\sin(qr)}{qr} 4\pi r^2 dr. \quad (2)$$

Among the different methods to determine the nuclear charge distribution, the one introduced by Dreher *et al.*¹⁸ has been chosen. In this, the nuclear charge distribution has been developed into a Fourier-Bessel series

$$\rho(r) = \begin{cases} \sum_{n=1}^{\infty} a_n \frac{\sin(n\pi r/R)}{(n\pi r/R)} & \text{for } r < R \\ 0 & \text{for } r \geq R \end{cases} \quad (3)$$

where R is the cutoff radius of the charge distribution, assuming the charge is zero for $r \geq R$. Only the coefficients a_n with $n \leq n_{\text{max}}$ and

$$n_{\text{max}} = \frac{Rq_{\text{max}}}{\pi}$$

are well determined by the experiment measuring the form factor up to a momentum transfer q_{max} . The normalization has been chosen such that

$$\int \rho(r) 4\pi r^2 dr = Z. \quad (4)$$

TABLE III. Cross sections for elastic electron scattering from ^{12}C measured with the 180° spectrometer. They are normalized to the absolute cross sections given in Table II. $E-n$ denotes 10^{-n} .

θ (deg)	$d\sigma/d\Omega$ ($\text{fm}^2\text{sr}^{-1}$)	error/(%)	θ (deg)	$d\sigma/d\Omega$ ($\text{fm}^2\text{sr}^{-1}$)	error/(%)
$E_0 = 150.24$ MeV			$E_0 = 300.52$ MeV		
40	0.3087 $E-1$	0.47	50	0.1308 $E-3$	0.73
40	0.3057 $E-1$	0.47	55	0.3784 $E-4$	0.76
50	0.9061 $E-2$	0.46	60	0.9088 $E-5$	0.77
55	0.5118 $E-2$	0.46	60	0.9200 $E-5$	0.78
60	0.2967 $E-2$	0.46	60	0.9191 $E-5$	0.78
60	0.2944 $E-2$	0.46	60	0.9117 $E-5$	0.77
65	0.1721 $E-2$	0.46	60	0.9052 $E-5$	0.77
70	0.1018 $E-2$	0.46	60	0.9131 $E-5$	0.77
75	0.6059 $E-3$	0.54	65	0.1749 $E-5$	0.91
80	0.3602 $E-3$	0.47	70	0.1847 $E-6$	1.3
80	0.3627 $E-3$	0.47	73	0.3635 $E-7$	1.6
80	0.3634 $E-3$	0.47	75	0.3668 $E-7$	1.4
80	0.3617 $E-3$	0.54	76	0.4615 $E-7$	1.2
80	0.3603 $E-3$	0.46	77	0.6133 $E-7$	1.3
80	0.3630 $E-3$	0.62	78	0.7573 $E-7$	1.0
80	0.3613 $E-3$	0.62	85	0.1345 $E-6$	0.97
85	0.2179 $E-3$	0.47	90	0.1248 $E-6$	0.81
90	0.1324 $E-3$	0.54	95	0.9505 $E-7$	0.96
95	0.7954 $E-4$	0.47	100	0.6717 $E-7$	1.1
100	0.4851 $E-4$	0.70	110	0.2734 $E-7$	1.4
100	0.4844 $E-4$	0.54	120	0.9780 $E-8$	2.0
105	0.2942 $E-4$	0.55	130	0.3318 $E-8$	3.0
110	0.1796 $E-4$	0.48	$E_0 = 320.10$ MeV		
115	0.1105 $E-4$	0.48	50	0.6400 $E-4$	0.71
120	0.6713 $E-5$	0.49	55	0.1503 $E-4$	0.75
125	0.4123 $E-5$	0.63	60	0.2645 $E-5$	0.83
130	0.2531 $E-5$	0.50	60	0.2647 $E-5$	0.83
$E_0 = 240.17$ MeV			60	0.2639 $E-5$	0.83
55	0.3571 $E-3$	0.54	65	0.2490 $E-6$	1.1
65	0.5771 $E-4$	0.49	66	0.1363 $E-6$	1.2
75	0.7885 $E-5$	0.53	67	0.7240 $E-7$	1.4
75	0.7892 $E-5$	0.53	68	0.4410 $E-7$	0.89
75	0.7869 $E-5$	0.53	70	0.4788 $E-7$	1.5
75	0.7814 $E-5$	0.53	72	0.8195 $E-7$	1.0
80	0.2546 $E-5$	0.70	75	0.1340 $E-6$	0.88
85	0.6853 $E-6$	0.89	80	0.1675 $E-6$	0.78
90	0.1362 $E-6$	1.1	90	0.1040 $E-6$	0.87
90	0.1373 $E-6$	1.3			
94	0.2695 $E-7$	1.9			
96	0.1303 $E-7$	2.1			
96	0.1335 $E-7$	2.3			
97	0.1174 $E-7$	2.0			
98	0.1316 $E-7$	2.3			
102	0.2531 $E-7$	1.7			
105	0.3654 $E-7$	1.5			
110	0.4648 $E-7$	1.3			

This method yields the charge distribution including its error band and has the advantage that the various errors, which contribute to this band, can be easily distinguished. These are statistical errors and

systematic uncertainties as well as those due to the lack of data at high momentum transfers $q > q_{\text{max}}$. The latter is called the completeness error (see Sec. IV B).

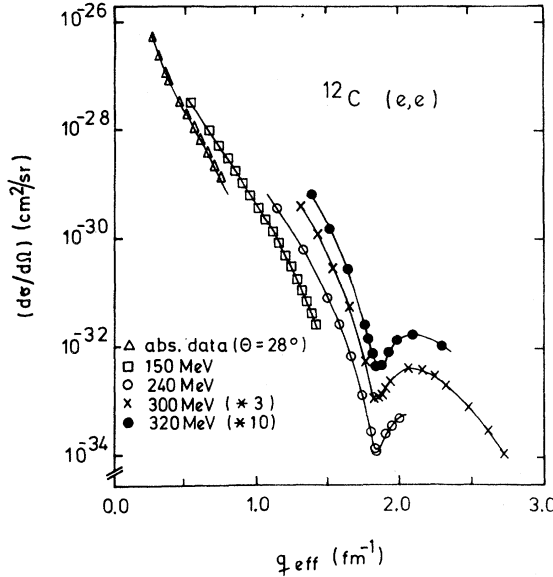


FIG. 4. Elastic electron scattering cross sections for ^{12}C plotted versus

$$q_{\text{eff}} = q \left[1 + \frac{3}{2} Z e^2 / (E_0 (\frac{5}{3})^{1/2} \langle r^2 \rangle^{1/2}) \right].$$

The absolute cross sections have been measured with the fixed angle spectrometer, the others with the 180° spectrometer.

A. Influence of instrumental uncertainties

A compilation of instrumental uncertainties, which have been discussed in Sec. II, is given in Table IV. Some of these uncertainties contribute randomly only, as, for instance, the reproducibility of the spectrometer position, whereas others, such as the error of the energy calibration factor, are truly systematic. The random parts of the uncertainties in energy and angle were converted into those of the cross sections in an iterative way, using the gradients of the cross sections with respect to energy and angle. The gradients being steepest near the form factor minimum, the random uncertainties there are largest and reach up to 0.6%. They have been added in quadrature to those of the counting statistics, thus forming the total statistical error as given in Tables II and III. This total statistical error is of the order of 0.5% for the absolute cross sections and about 1% for the relative data, with a limited increase in the minimum and at $q > 2.3 \text{ fm}^{-1}$ up to 3%. These errors were associated with the cross sections in the analysis; no additional contributions were added to improve upon the χ^2 of the fits.

TABLE IV. Instrumental uncertainties.

Parameter	Random	Sys-tematic
1. Primary energy	0.03%	0.12%
2. Target position		0.2 mm
3. Target angle	0.1°	0.2°
4. Target thickness		0.15%
5. Direction of incoming beam	0.005°	0.01°
6. Beam position on target		0.3 mm
7. Charge monitor calibration		<0.3%
8. Fixed angle spectrometer:		
(a) Solid angle		$\leq 0.16\%$
(b) Scattering angle	0.008°	0.01°
(c) Detector efficiency		0.03%
(d) Dead time correction		<0.1%
9. 180° spectrometer		
(a) Center of rotation		0.2 mm
(b) Angular position of spectrometer	0.001°	0.01°
(c) Scattering angle uncertainty due to 2, 6, 9(a), and 9(b) added in quadrature		0.06°

The influence of the truly systematic deviations, which are normally in one, but unknown, direction, has been investigated separately. The cross sections were altered (again using the respective gradients) from their nominal values to those valid at the systematically extreme values of angle and energy (see Table IV), successively. The uncertainty of the normalization was treated similarly. These uncertainties, converted into errors in the cross sections, were smaller than 0.4% for the absolute data and on the order of 1% for the relative data, but ranging up to about 4% in the minimum because of the steep gradients there. By comparison to the fits with the unchanged values, the influence of the truly systematic uncertainties on the charge distribution parameters was found.

B. Evaluation of the charge distribution

The cross sections given in Tables II and III have been analyzed with the fitting code MEFIT¹⁷ and using the Fourier-Bessel parametrization of the charge distribution. The completeness error has been calculated from the upper limit for the form factor at $q > q_{\text{max}}$ using the asymptotic estimate as given by Dreher *et al.*¹⁸

$$|F(q)| \leq c q^{-4} F_p(q), \quad (5)$$

where F_p is the form factor of the proton

$$F_p = \exp(-q^2 \langle r^2 \rangle / 6)$$

with¹⁰

$$\langle r^2 \rangle^{1/2} = 0.86 \text{ fm}.$$

The constant c has been estimated by matching the envelope in $F(q)$ at the last measured maximum of the form factor yielding $|F(q_{\text{max}} = 2.75 \text{ fm}^{-1})| = 0.0076$.

Some model dependence is introduced by the choice of the cutoff radius R ; therefore R was varied between 6 and 10 fm. Between $R = 6$ and 8 fm, the measured data were reproduced with equal quality. Only at $R > 9$ fm the adaption becomes so flexible that the method begins to fit statistical fluctuations. Consequently, the cutoff radius was chosen to be 8 fm. The variation of the rms radius with a change of R from 6 to 8 fm was less than 0.008 fm. Also, the resulting charge distribution did not show any significant change. In this whole procedure, oscillations leading to negative charge densities at large radii have been suppressed.¹⁷

The relative cross sections, measured with the 180° spectrometer, were normalized to those measured absolutely with the fixed angle spectrometer in a common fit. To do so, the normalization of the absolute data was fixed to be one and the normalization factors for the four relative data sets varied to minimize χ^2 , which resulted to 0.9. In this first step, the data in the vicinity of the diffraction minimum at 1.8 fm^{-1} have been omitted. The normalization factors found in this first step were then kept fixed in the final calculation where all measured cross sections were included. This led to a χ^2 per degree of freedom of about 2. The charge distribution parameters presented in this paper include all measured cross sections, although the cross sections in the minimum increase the χ^2 by a factor of 2. The influence on the charge distribution is small compared to the quoted errors, the rms radius decreasing by 1 am only. The relative deviation of the measured cross sections from those calculated from the best fit is shown in Fig. 5. This figure demonstrates the wide overlap of the individual data sets, their consistency with each other, and the absence of systematic trends except in the form factor minimum at 1.8 fm^{-1} . These systematic deviations will be discussed in Sec. V.

The Fourier-Bessel coefficients and their statistical errors are given in Table V. The resulting charge distribution with its error band is shown in Fig. 6(a); the individual contributions to the error band are plotted in Figs. 6(b) and (c) on an absolute and relative scale, respectively. The completeness

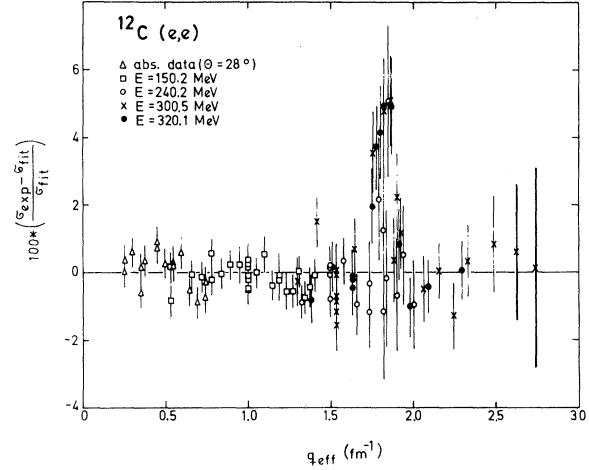


FIG. 5. Percentage deviations of the measured cross sections from those calculated from the best fit charge distribution. The pronounced deviations occur around the minimum of the cross sections at $q_{\text{eff}} = 1.8 \text{ fm}^{-1}$.

error dominates in the center of the nucleus, where the total error is largest. In the other parts, the influence of the different errors is about equal. Finally, it should be mentioned that not all distributions within the error band are allowed; the error band only represents the envelope of all possible distributions. This can easily be seen from the fact that these envelopes violate the normalization.

Any other allowed solution can be calculated from the error matrix; in Table V only the diagonal elements of this matrix are quoted. The rms radius deduced from this charge distribution is

TABLE V. Fourier-Bessel coefficients a_n as defined in Sec. IV for ^{12}C . Cutoff radius $R = 8 \text{ fm}$. The errors Δa_n represent the diagonal elements of the error matrix.

n	$a_n \text{ (fm}^{-3}\text{)}$	$\Delta a_n \text{ (fm}^{-3}\text{)}$
1	$1.5737 E-2$	$2.85 E-6$
2	$3.8896 E-2$	$1.75 E-5$
3	$3.7085 E-2$	$1.69 E-5$
4	$1.4795 E-2$	$1.01 E-5$
5	$-4.4830 E-3$	$8.87 E-6$
6	$-1.0057 E-2$	$2.73 E-5$
7	$-6.8695 E-3$	$8.88 E-5$
8	$-2.8813 E-3$	$4.35 E-4$
9	$-7.7228 E-4$	$7.4 E-4$
10	$6.6907 E-5$	$5.6 E-4$
11	$1.0636 E-4$	$3.3 E-4$
12	$-3.6864 E-5$	$1.9 E-4$
13	$-5.0134 E-6$	$1.0 E-4$
14	$9.4548 E-6$	$5.5 E-5$
15	$-4.7686 E-6$	$2.9 E-5$

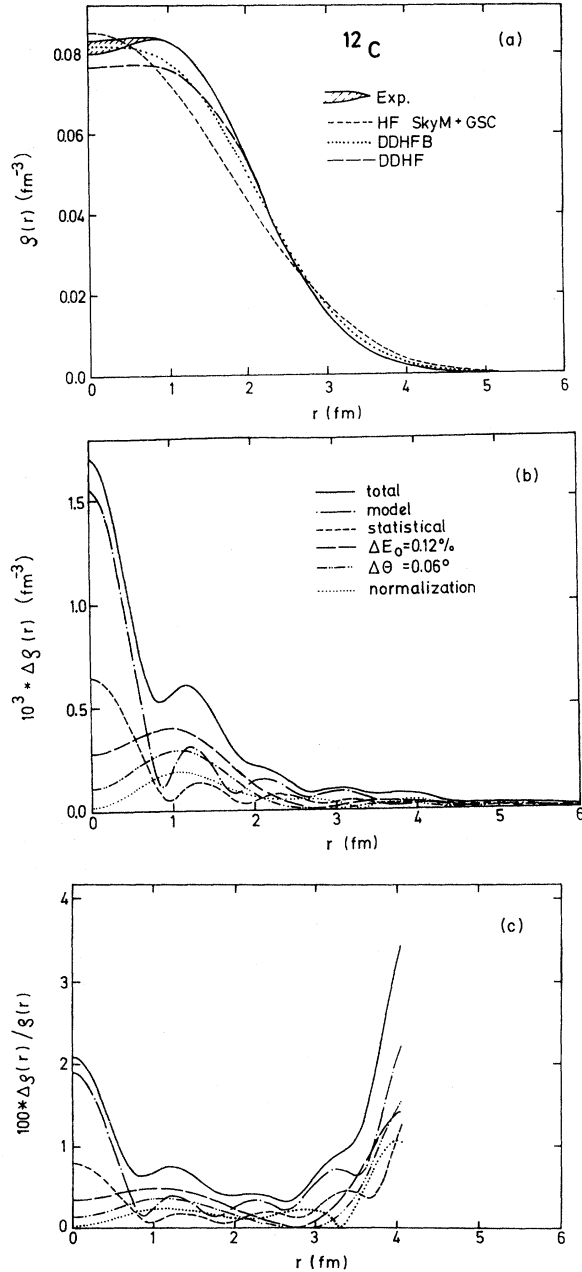


FIG. 6. Charge density distribution of ^{12}C . (a) The charge distributions extracted from the experiment in comparison with recent HF calculations. The calculation of Reinhard *et al.*, Ref. 32, has been performed using a modified Skyrme force and including ground state correlations (HF SkyM + GSC). The curve labeled DDHFB represents the results of Girod and Gogny, Ref. 30, whereas the density dependent HF calculation (DDHF) stems from Friar and Negele, Ref. 20. (b) The contribution of different uncertainties to the error band of the charge distribution. The individual contributions have been added in quadrature giving the total error. (c) same as (b) but shown on a relative scale.

$$\langle r^2 \rangle^{1/2} = 2.464(12) \text{ fm}.$$

The quoted uncertainty includes the statistical and completeness error as well as systematic uncertainties which contribute most.

C. Comparison with other measurements

The earlier electron scattering measurements of the Instituut voor Kernfysisch Onderzoek (IKO),² Darmstadt,³ the National Bureau of Standards (NBS),⁴ and Stanford¹ are compared to our results in Figs. 7(a) and (b) where the relative deviation of these cross sections with respect to our best fit is plotted.

The overall agreement with the data at low momentum transfer of IKO and Darmstadt is quite good, although the former measurements lead to a smaller rms radius which, however, still lies within the error limit. The NBS data show a slight decrease of the cross sections for increasing momentum transfer, resulting in a slightly larger rms radius which is again within the error limit. The rms radii quoted by these authors and those of a recent Moscow experiment¹⁹ are given in Table VI. All in all, no significant discrepancy exists between other low- q results and our own.

The Stanford data were the only ones at high momentum transfer available up to now. Before comparing these data—as shown in Fig. 7(b)—we had to correct them for finite solid angle of the spectrometer, etc.; these corrections decreased the experimental cross section by nearly 12% in the minimum. The data show excellent agreement with our results from $q = 1$ to 1.6 fm^{-1} ; however, in the first minimum at $q = 1.8 \text{ fm}^{-1}$, they still are higher by 13%. These deviations, already found by Friar and Negele²⁰ in an analysis of the IKO and Stanford data, led to assumptions about dispersion effects. This will be discussed in Sec. V.

Muonic atom transition energies yield, for light nuclei like carbon, only one radial moment, the so-called Barrett radius. The Barrett moment is given by

$$\langle r^k e^{-ar} \rangle = \frac{1}{Z} \int_0^\infty \rho(r) r^k e^{-ar} 4\pi r^2 dr \quad (7)$$

and can be used to define the equivalent nuclear charge radius $R_{k\alpha}$ of a homogeneously charged sphere according to the relation

$$\frac{3}{R_{k\alpha}^3} \int_0^{R_{k\alpha}} r^k e^{-ar} r^2 dr = \langle r^k e^{-ar} \rangle. \quad (8)$$

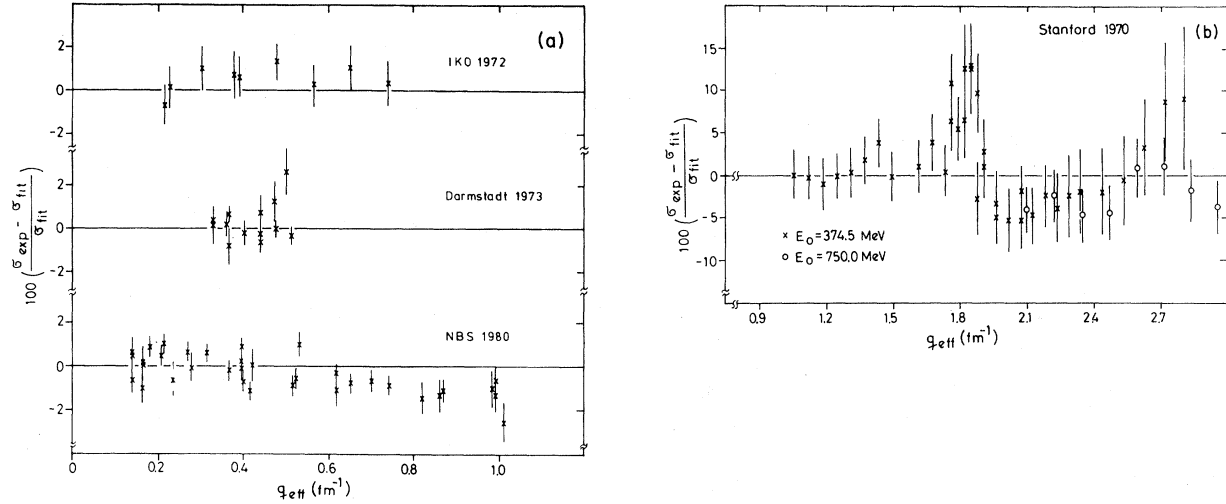


FIG. 7. Comparison with the measurements of other laboratories. The percentage deviations with respect to our best fit are plotted versus q_{eff} . (a) Data of IKO, Ref. 2; Darmstadt, Ref. 3; and NBS, Ref. 4. (b) Deviations of the Stanford data, Ref. 1. Note the different vertical scale and the suppressed zero of the q axis. Stanford data are only shown up to the momentum transfer this experiment has covered.

The determination of this radius is largely independent of the parametrization of the charge distribution. The following values have been chosen for ^{12}C as reported by Mallot and Rychel²¹

$$\alpha = 0.021 \text{ fm}^{-1}; k = 2.025.$$

In order to compare muonic data to electron scattering experiments, the ratio of the equivalent rms to the equivalent Barrett radius is determined from the charge distribution measured by elastic

electron scattering (Fig. 6). This ratio is given by²¹

$$V_2 = \frac{(5/3)^{1/2} \langle r^2 \rangle^{1/2}}{R_{k\alpha}} = 1.00256(25). \quad (9)$$

Schaller *et al.*²² have measured muonic x-ray transitions of ^{12}C with a Ge(Li) diode and report a center-of-gravity energy for the $2p-1s$ transition of 75262.5(5.0) eV. This value is corrected for the ^{13}C contribution. Using a nuclear polarization correc-

TABLE VI. Rms charge radius of ^{12}C . MHO, HO=(Modified) harmonic oscillator charge distribution. SOG=Sum of Gaussians.^b FB=Fourier-Bessel expansion of charge distribution. ETH=Eidgenössische Technische Hochschule.

Method	Remarks	$\langle r^2 \rangle^{1/2}$ fm	Reference, Year
(e,e) Stanford	MHO, $q = 1.1 - 4 \text{ fm}^{-1}$	2.460 ± 0.025^a	1, 1970
(e,e) IKO	HO, $q = 0.2 - 0.7 \text{ fm}^{-1}$	2.453 ± 0.008^a	2, 1972
(e,e) Darmstadt	HO, $q = 0.3 - 0.5 \text{ fm}^{-1}$	2.462 ± 0.022^a	3, 1973
(e,e) Stanford + IKO	SOG	2.468 ± 0.016^a	b, 1974
(e,e) Stanford + IKO	FB	2.447 ± 0.016^a	20, 1975
(e,e) Moscow	MHO, $q = 0.2 - 0.6 \text{ fm}^{-1}$	2.45 ± 0.025^a	19, 1979
(e,e) NBS + Stanford + IKO	FB, $q(\text{NBS}) = 0.2 - 1.0 \text{ fm}^{-1}$	2.472 ± 0.015^a	4, 1980
(e,e) Mainz	FB, $q = 0.3 - 2.8 \text{ fm}^{-1}$	2.464 ± 0.012^a	this work
(e,e) Mainz	FB, dispersion corrections applied	2.468 ± 0.012	this work
μ atom, Fribourg/SIN	Ge(Li), NP=1.5 eV	2.467 ± 0.017^c	22, 1980
μ atom, Fribourg/SIN	Ge(Li), NP=2.6 eV	2.4715 ± 0.016^c	24, 1982
μ atom, ETH Zürich/SIN	Crystal, NP=2.3 eV	2.4834 ± 0.0018^c	25, 1982

^aNo dispersion corrections applied.

^bI. Sick, Nucl. Phys. **A128**, 509 (1974).

^cNuclear polarization correction (NP) applied. The radial moment deduced from muonic x-ray energies has been converted to the rms radius with the help of the charge distribution from electron scattering.

tion of 1.5 eV and the above value of V_2 , they obtain a rms radius

$$\langle r^2 \rangle^{1/2} = 2.467(17) \text{ fm},$$

which is in good agreement with the value reported in this paper. Using a more recent calculation of Ericson²³ of 2.6 eV for the nuclear polarization correction, Schaller *et al.*²⁴ reported a revised value for the rms radius

$$\langle r^2 \rangle^{1/2} = 2.472(16) \text{ fm}.$$

Ruckstuhl *et al.*²⁵ at the Schweizerisches Institut für Nuklearforschung (SIN) have also measured muonic transition lines in ^{12}C with a previously unobtained precision using a crystal spectrograph. The $2p$ - $1s$ energy they measured is about 4 eV lower than that of Schaller *et al.*, and, with the 2.3 eV nuclear polarization correction of Rosenfelder,²⁶ they obtain

$$\langle r^2 \rangle^{1/2} = 2.4834(18) \text{ fm}.$$

This value for the rms radius is about one standard deviation larger than ours.

Applying a crude estimate for dispersion corrections to our electron scattering cross sections as described in Sec. V increases the rms radius by 4 am to

$$\langle r^2 \rangle^{1/2} = 2.468(12) \text{ fm}.$$

This reduces the deviation with respect to the rms radius deduced from the analysis of muonic x-ray energies measured with a crystal spectrometer where the corresponding correction, namely the nuclear polarization correction, has been applied.

V. INTERPRETATION OF RESULTS

The results of our experiment show two features; first the reduced deviation of the data in the minimum with respect to the Fourier-Bessel analysis, and second, the incompatibility of nuclear structure calculations with the measured charge distribution. Both findings will be discussed in the following.

A. Discussion of dispersion effects

The analyses of the measured elastic cross sections have been made neglecting virtual excitations of the nucleus during the scattering process. These virtual excitations, called dispersion effects, have been studied theoretically, and a compilation of

many relevant calculations has been given in a review article by Friar.²⁷ A complete calculation of this effect is rather difficult since all possible excitations should be taken into account. As all the levels of a nucleus and the corresponding excitation probabilities are not known, the dispersion correction is calculated with approximation methods, in which, for instance, the sum rule is used. Thus, the theoretical results differ widely, even in the sign of the effect. However, most calculations have in common that the fractional contribution of the dispersion correction to the cross sections is peaked in the diffraction minima. The first minimum of the cross section of ^{12}C is especially pronounced. Therefore, one would quite expect to find such an effect here.

To investigate the influence of dispersion effects on the analysis of our experiment, we used the fractional corrections of Friar and Rosen²⁸ as shown in Fig. 8. These calculations show a smooth increase of the effect proportional to q^2 with a superimposed peak at the position of the minimum. Correcting the measured cross sections for the contribution proportional to q^2 increased the rms radius by about 3 am. This is within the changes of 2 to 7 am calculated by Friar²⁷ using different approximations. We now consider the peaked contribution of the dispersion correction in the minimum. The calculations by Friar and Rosen²⁸ yielded a peaked contribution of about 1.5% for 747 MeV electrons. As

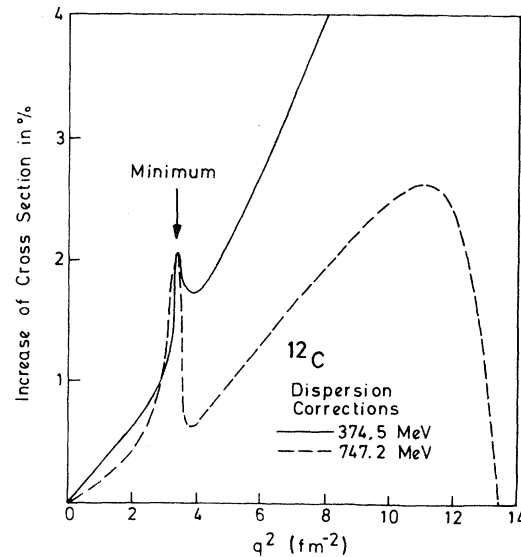


FIG. 8. Relative dispersion corrections calculated by Friar and Rosen, Ref. 28, as taken from Ref. 20. The cross sections calculated from the static charge distribution are increased by dispersion effects.

can be seen from Fig. 8 this contribution is energy dependent and decreases to 0.5% for 375 MeV. This tendency seems to be in agreement with our experiment, where the deviation from the Fourier-Bessel analysis in the first minimum is about 5% for 320 MeV and 2% for 240 MeV. As pointed out in Sec. IV B, omitting the data in the minimum increases the rms radius by 1 am. Thus, the total change of the radius due to dispersion effects would amount to about 4 am. Because of the uncertainties in the calculations and the smallness of the effect, all results presented in this paper are reported without these corrections unless otherwise stated.

One should remember, however, that experimental uncertainties are also largest near the minimum. Because of the finite solid angle and primary energy spread, the measured cross sections are averaged over these parameters and have to be unfolded as explained in Sec. III B. The maximum correction for the 300 MeV data, which were taken with $\Delta\theta = 2.55^\circ$, is 12.5%. A measurement at 320 MeV was performed to check the unfolding procedure; the aperture was $\Delta\theta = 1.8^\circ$, resulting in a maximum correction of 9%. Both data sets agree very well as shown in Fig. 5. Additional data near the minimum were taken at 240 MeV since at lower energies the gradient and curvature with respect to the angle are smaller, resulting in a maximum correction of about 7%. This measurement showed, on the average, less surplus in the minimum; but, as discussed before, the peaked contribution of the dispersion effects might also be smaller at lower energies.

To exclude numerical problems being responsible for the data surplus, we used the new phase shift code HADES²⁹ to calculate the cross section. The comparison with MEFIT did not give any indication of roundoff errors. Finally, there might still be some unknown instrumental background that increases the yield in the minimum, but has little influence elsewhere. Thus we cannot rule out that part of the deviations in the minimum still might be due to instrumental effects.

B. Comparison with nuclear structure calculations

In a naive model, the nucleus ^{12}C can be described as a cluster of three α particles, thus leading to an oblate intrinsic shape. The sequence of the 0^+ , 2^+ , and 4^+ levels roughly obeying the $J(J+1)$ rule for the energies within a rotational band gives further evidence for an intrinsically de-

formed ground state. On the other hand, the Hartree-Fock calculations of Friar and Negele²⁰ favor a spherical ground state, although a deformation cannot be ruled out since the binding energy is not strongly dependent on the quadrupole moment Q . As can be seen from Fig. 6, this calculation²⁰ describes the nuclear surface rather well, but yields a density about 7% too low in the center of the nucleus, the radius being 1.5% too high, and the total binding energy 10% too high.

The deformed calculations all yield larger rms radii due to the increased diffuseness of the angle-averaged density. The charge distribution calculated with an intrinsic deformation by Girod and Gogny³⁰ (see Fig. 6) reproduces the experimental density for $r=0$ but yields a larger diffuseness, and thus the rms radius is about 3% too high. Svenne and Mackintosh³¹ conclude that there is strong evidence for an oblate intrinsic state. They performed various HF calculations and investigated the influence of the spin-orbit force, which tends to decrease the deformation and the rms radius. With a relative strength for the spin-orbit force of $F_{so}=1.0$ the mass-rms radius of their spherical solution agrees with our results for the charge rms radius. For $F_{so}=1.5$ they are in agreement with our charge rms radius for the deformed solution and obtain a mass quadrupole moment $Q = -32.3 \text{ fm}^2$ which is about 50% too large compared to experimental findings (see, e.g., Table I of Ref. 31). Calculations of Reinhard and Drechsel³² with the inclusion of ground state correlations show a similar behavior. The spherical solution already has a large diffuseness which is even further increased by the ground state correlations.

Summarizing, we state that nearly all HF calculations yield, in the case of ^{12}C , more surface thickness than deduced from experiment. Since there is strong evidence for intrinsic deformation, the intrinsic skin thickness of ^{12}C must be significantly smaller than calculated up to now to yield an angle-averaged charge distribution which agrees with the experiment. An improvement in the description of the nuclear surface will simultaneously lead to better agreement with the experimentally determined rms radius.

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