Potential inversion for scattering at fixed energy

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An inversion scheme proposed recently by the authors is successfully applied to various cases of neutron-nucleus scattering. The results are compared with those of other inversion methods.

[NUCLEAR REACTIONS Inverse scattering problem at fixed energy, nonrational representation of scattering function; applied to complex nuclear potential.]

In this Communication we apply the solution to the inverse scattering problem at fixed energy, recently obtained by two of the authors,[1] to reconstruct the optical potentials for neutron scattering from the nuclei ^{16}O, ^{40}Ca, and ^{58}Ni over a wide range of energies. We also reconstruct, by this new method, the potential employed by Coudray[2] to test the Newton-Sabatier inversion scheme[3] at 10 and 50 MeV.

The method[1] solves the inverse scattering problem for a class of scattering functions which represent nonrational modifications of the rational functions of $\lambda^2(\lambda=l+\frac{1}{2})$ considered in the "Bargmann" inversion method of previous work.[4] It promises to be an efficient method for constructing potentials from scattering functions in practical cases.

The (in general, nonunitary) elastic scattering function $S(\lambda)$ at fixed energy is given as a ratio of two determinants $(n,m=1,\ldots,N)$,

$$S(\lambda)=S^{(N)}(\lambda)=S^{(0)}(\lambda)\frac{\|[(\sigma_{\beta_n}-\sigma_{\alpha_m})/(\beta_n{}^2-\alpha_m{}^2)]-[(\sigma_\lambda-\sigma_{\alpha_m}\sigma_{\beta_n})/(\lambda^2-\alpha_m{}^2\sigma_\lambda)]\|}{\|[(\sigma_{\beta_n}-\sigma_{\alpha_m})/(\beta_n{}^2-\alpha_m{}^2)]-[(\sigma_\lambda-\sigma_{\alpha_m})/(\lambda^2-\alpha_m{}^2)]\|}\ , \tag{1}$$

where

$$\sigma_\lambda=\exp[-i\pi(\lambda-1/2)]S^{(0)}(\lambda)\ , \tag{2}$$

and $S^{(0)}(\lambda)$ is the scattering function of a reference potential $V^{(0)}(r)$. In all but pathological cases one can arrange the fitting parameters α_m, β_n such that all $|\sigma_{\alpha_m}|\ll 1$ and all $|\sigma_{\beta_n}|\gg 1$; then we arrive at the simplified form

$$S(\lambda)\approx S^{(0)}(\lambda)\prod_{n=1}^{N}\frac{\lambda^2-\beta_n{}^2}{\lambda^2-\alpha_n{}^2} \tag{3}$$

for *real* λ. This is the simple, rational parametrization of our previous "Bargmann" inversion scheme.[4]

The local, l-independent complex potential $V(r)$ uniquely associated with the scattering function (1) is determined iteratively in the form

$$V(r)=V_N(r);\quad V_n(r)=V_{n-1}(r)+V^{(n)}(r),\qquad n=0,\ldots,N\ , \tag{4}$$

with $V_{-1}=0$ and

$$V^{(n)}(r)=\frac{2E}{\rho^3}\left[\frac{1-\rho(L_{\alpha_n}^{(n-1)}+L_{\beta_n}^{(n-1)})}{y^{(n-1)}}+\frac{1}{\rho(y^{(n-1)})^2}\right]; \tag{5}$$

here

$$y^{(n-1)}=(L_{\alpha_n}^{(n-1)}-L_{\beta_n}^{(n-1)})(\alpha_n{}^2-\beta_n{}^2)^{-1}\ ,\qquad n=1,\ldots,N\ , \tag{6}$$

where $L_\lambda^{(n)}$ is the logarithmic derivative of the "regular" solution to $V_n(r)$, which satisfies the Riccati

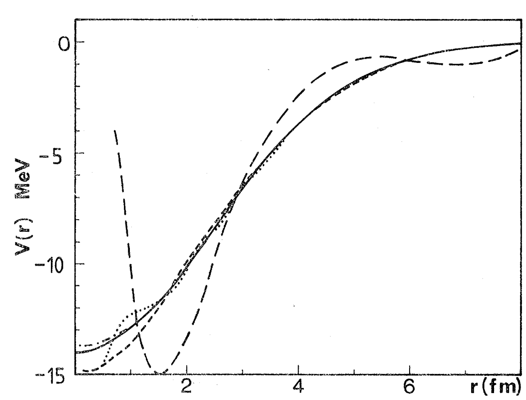

FIG. 1. Gaussian potential of Eq. (8) (——). Reconstruction from the scattering function: at 10 MeV, present inversion method (– – –) and Ref. 2 (— — —); at 50 MeV, present inversion method (—·—) and Ref. 2 (· · · ·).

TABLE I. Collision data and potential parameters of potentials 1 to 5 taken from Ref. 5.

Pot.	$E_{\rm lab}$ (MeV)	A_T	A_P	V	r_{01}	R_1	a_1	W	W_d	r_{02}	R_2	a_2
1	23.4	^{16}O	n	64.4134	0.9242	2.3288	0.8578	0.0	0.0			
2	52.5	^{16}O	n	35.136	1.2324	3.1055	0.7294	0.0	0.0			
3	52.5	^{16}O	n	35.136	1.2324	3.1055	0.7294	3.0831	0.0	1.3142	3.3116	0.4518
4	48.0	^{40}Ca	n	42.983	1.1435	3.9107	0.7391	5.2476	0.9730	1.5694	5.3673	0.4558
5	100.4	^{58}Ni	n	29.0173	1.1962	4.6303	0.7892	7.6992	0.0	1.4571	5.6403	0.5117

equation

$$\frac{dL_\lambda^{(n)}}{d\rho}+(L_\lambda^{(n)})^2+1-\frac{\lambda^2-\frac{1}{4}}{\rho^2}-\frac{V_n(r)}{E}=0 \qquad (7)$$

with appropriate boundary conditions[1] ($\rho=kr$).

As a first simple application of our method we fitted the function $S(\lambda)$ of Eq. (1) to the (numerically calculated) scattering function of the Gaussian potential

$$V(r)=-14\exp[-(r/3.5)^2] \ , \qquad (8)$$

employed by Coudray[2] for a test of the Newton-Sabatier inversion method[3] at the energies $E=10$ and 50 MeV. A good, although not exact, fit was obtained in both cases, using three pairs α_n, β_n ($N=3$). The results for the reconstructed potential of Eq. (4) are shown in Fig. 1 and compared with the input potential of Eq. (8) as well as with the results obtained by Coudray.[2] As in Ref. 2, we find that the agreement between input and reconstructed potentials improves with energy; nevertheless, even at the low energy of 10 MeV, the result of the present method appears quite acceptable. Only minor discrepancies show up at short distances. We expect that these could be removed by making an improved fit to the scattering function using $N>3$.

All the other applications are modeled on optical potentials for proton scattering from ^{16}O, ^{40}Ca, and ^{58}Ni taken from Ref. 5. However, we neglect the Coulomb potential and therefore effectively consider neutron scattering. The case of proton scattering will be dealt with in a future publication. The potentials to be reconstructed by inversion are all of the form

$$V(r)=-Vf_1(r)-iWf_2(r)-iW_dg_2(r) \ ,$$

where $f_{1,2}(r)$ is of the usual Woods-Saxon shape and $g_2(r)=-4a_2f_s(r)$. The parameters of these potentials are given in Table I. The energies vary from 23.4 to 100.4 MeV.

In Fig. 2 we show the results of the inversion for real input potentials. The fits of the parametrized scattering function $S(\lambda)$ of Eq. (1) to the input scattering function, although not perfect, are certainly within the customary experimental errors. The $\chi_S{}^2$ values of these fits are of the order of 10^{-2} to 10^{-3}. We note that the potential 1 ($N=3$, $\chi_S{}^2=3.5\times10^{-2}$) is fairly well reproduced by the inversion, with some discrepancy at short distances. In the case of potential 2 we have inverted two different fits to the scattering function, with $N=4$, $\chi_S{}^2=0.45\times10^{-2}$ and $\chi_S{}^2=0.69\times10^{-3}$, respectively. As expected, the

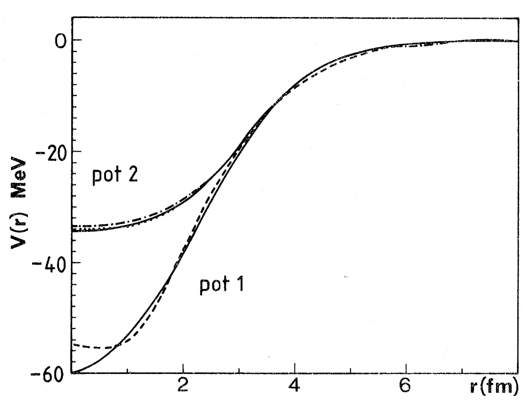

FIG. 2. Real potentials 1 and 2 at 23.4 and 52.5 MeV, respectively (——). Reconstruction of potential 1 (– – –); reconstruction of potential 2 with $\chi_S{}^2=0.45\times10^{-2}$ (–·–·) and $\chi_S{}^2=0.69\times10^{-3}$ (· · · ·).

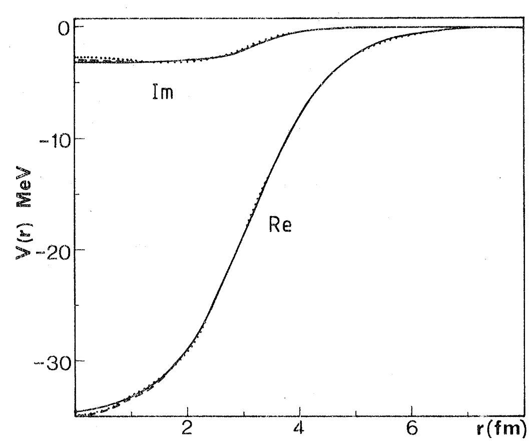

FIG. 3. Complex potential 3 for $n+{}^{16}$O at 52.5 MeV (——). Reconstruction with $\chi_S{}^2=0.47\times10^{-3}$ (· · · ·) and $\chi_S{}^2=0.16\times10^{-3}$ (– – –).

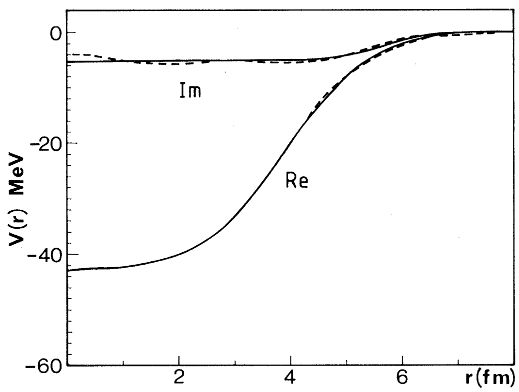

FIG. 4. Complex potential 4 for $n + {}^{40}Ca$ at 48.0 MeV (——). Reconstruction with $\chi_S{}^2 = 0.31 \times 10^{-2}$ (– – –).

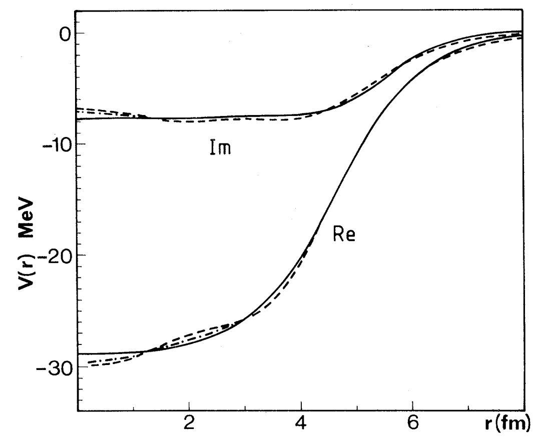

FIG. 5. Complex potential 5 for $n + {}^{58}Ni$ at 100.4 MeV (——). Reconstruction with $\chi_S{}^2 = 0.67 \times 10^{-3}$ (– – –) and $\chi_S{}^2 = 0.34 \times 10^{-3}$ (–·–·).

reproduction of the input potential improves when $\chi_S{}^2$ decreases, approaching perfect agreement. It is instructive to compare these results with those obtained by Münchow and Scheid,[6] who employed a modified Newton-Sabatier inversion method. In contrast to Ref. 6, the potentials yielded by the present scheme do not exhibit any violent oscillations at short distances nor any singularity at the origin (cf. also Ref. 7).

Figures 3 to 5 show the results of the inversion for various complex potentials. Figure 3 gives the potential 3 for $n + {}^{16}O$ scattering at 52.5 MeV, as well as its reconstructions using a three-pair fit to the scattering function ($N = 3$, $\chi_S{}^2 = 0.47 \times 10^{-3}$) and a six-pair fit ($N = 6$, $\chi_S{}^2 = 0.16 \times 10^{-3}$). In the latter case the reconstructed potential practically coincides with the input potential. In Fig. 4, we compare potential 4 for $n + {}^{40}Ca$ scattering at 48.0 MeV with its reconstruction using a fit to the scattering function with $N = 3$ and $\chi_S{}^2 = 0.31 \times 10^{-2}$. Even for this relatively large value of $\chi_S{}^2$ there is only a small discrepancy near the origin for the imaginary part. Finally, in Fig. 5 we compare the input and reconstructed potential 5 for $n + {}^{58}Ni$ scattering at 100.4 MeV corresponding to two different fits of the scattering function with $N = 3$, $\chi_S{}^2 = 0.67 \times 10^{-3}$ and $N = 4$, $\chi_S{}^2 = 0.34 \times 10^{-3}$, respectively. Again, agreement improves with decreasing $\chi_S{}^2$.

We have demonstrated that the present inversion scheme is quite successful for neutron-nucleus scattering over a wide range of energies. The results compare favorably with those obtained by other methods proposed in the literature.[2,6,7] Our reconstructed potentials are quite smooth and as expected, improve when the fit of the parametrized scattering function (1) to the exact scattering function of the input potential is made more accurate. There is no singularity at the origin, as in other methods. It is interesting to note that the parametrization (1) of the scattering function has the same *form* as the scattering function implied in the Newton-Sabatier scheme[3] (for a finite number of fitted phase shifts). However, it can be shown, in analogy to Ref. 8, that the scattering function of Eq. (1) is never identical to that of Ref. 3. This is to be expected, since we have seen that the reconstructed potentials of the scheme[3] and of the present method generally have quite different features. The present method is readily implemented numerically. The determination of the parameters α_m, β_n is efficiently carried out through a combined use of the approximate rational interpolation formula (3) and a least squares routine for the function (1). Once the parameters are found, the iterative calculation of the potential (4) is straightforward. Applications to charged particle, in particular, α particle and heavy-ion scattering, are now being investigated.

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