Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication may be no longer than 3½ printed pages and must be accompanied by an abstract and a keyword abstract. Page proofs are sent to authors, but, because of the rapid publication schedule, publication is not delayed for receipt of corrections unless requested by the author.

Fusion barriers for heavy-ion systems

S. K. Gupta and S. Kailas
Nuclear Physics Division, Bhabha Atomic Research Centre, Bombay 400085, India
(Received 29 March 1982)

Analytical expressions for the fusion barrier height and radius have been derived from a four-parameter empirical fusion cross section formula for heavy ions. The fusion barrier parameters calculated, using these expressions, are in good agreement with the literature values.

[NUCLEAR REACTIONS Fusion cross section excitation functions, fusion barrier parameters.]

In the last few years a vast amount of data of the fusion cross sections for various interacting heavy-ion systems as a function of bombarding energy have been measured. Significant theoretical developments have also taken place. A summary of the published literature on this topic is available from Ref. 1. One way of comparing the various model predictions (empirical and theoretical) will be to calculate the fusion barrier parameters and compare them amongst various models. It is of great interest to obtain the properties of the fusion barriers from an analysis of fusion reaction excitation functions. Recently we have successfully described,[2] by an empirical four-parameter formula, the fusion cross sections for projectiles ranging from ^{4}He to ^{40}Ar and for compound systems with mass numbers between 24 and 242. Using this empirical expression we have been able to predict the maximum fusion cross section and the energy at which the maximum occurs in agreement with the experimental data. In the present work, we have derived analytical expressions for the fusion barrier height and radius starting from the above mentioned empirical expression. A preliminary account of this work appears in Ref. 3.

We describe the fusion cross section for the interacting nuclei (Z_T,A_T) and (Z_p,A_p), using an empirical expression[2]

$$\sigma_{\rm fus}=10\pi\rho\left(\rho-\frac{z}{E}\right)\ (\text{in mb})\ , \qquad (1)$$

where

$$\rho=mE+b\ (\text{in fm})$$

and

$$z=1.44Z_TZ_p\ \text{MeV fm}\ .$$

E is the center of mass energy in MeV. Further,

$$b=(1.498\pm0.099)(A_T^{1/3}+A_p^{1/3})+(1.538\pm0.581)\ \text{fm}$$

and

$$m=\left[-(15.1\pm0.8)\left(\frac{A_T+A_p}{A_TA_p}\right)^{1/2}+(0.93\pm0.17)\right]\times10^{-2}\ \text{fm/MeV}\ .$$

The parameters of expression (1) were obtained by using a vast amount of fusion data and more details of this work are given in Ref. 2.

Following Bass,[4] one can write the fusion cross section by the classical expression

$$\sigma_{\rm fus}=10\pi R^2\left(1-\frac{V(R)}{E}\right)\ \text{mb}\ , \qquad (2)$$

leading to

$$10\pi R^2=\frac{d(E\sigma_{\rm fus})}{dE} \qquad (2a)$$

and

$$V(R) = E - \frac{E\sigma_{\text{fus}}}{10\pi R^2} \ . \tag{2b}$$

Here R (in fm) is the distance between the two interacting nuclei and $V(R)$ is the total potential for s wave at R. The maximum of V as a function of R is defined as the fusion barrier height (V_B) and the corresponding R is called as the barrier radius (R_B). Bass[4] has proposed a graphical method for determining the barrier parameters from fusion data. In the present work we have provided analytical expressions for R_B and V_B starting from the empirical expression for fusion cross sections.

Using expressions (1) and (2) we get

$$R^2 = 3\rho^2 - 2\rho b - mz \tag{3}$$

and

$$V(R) = E - \frac{\rho^2 E - \rho z}{R^2} \ . \tag{4}$$

In expressions (3) and (4), ρ and R are functions of E. Differentiating (4) with respect to E we get

$$\frac{dV}{dE} = 1 - \frac{2m\rho E - \rho^2 - mz}{R^2} + \frac{2}{10\pi R^3}\frac{dR}{dE} E\sigma_{\text{fus}} \ . \tag{5}$$

Using Eq. (3) it can be shown that the first two terms on the right hand side of Eq. (5) cancel each other. Hence

$$\frac{dV}{dE} = \frac{2}{10\pi R^3}\frac{dR}{dE} E\sigma_{\text{fus}} \ . \tag{6}$$

We can also write (6) as

$$\frac{dV}{dR} = \frac{2E\sigma_{\text{fus}}}{10\pi R^3} \ . \tag{6a}$$

In order to find V_B, we set dV/dR, given by Eq. (6a), equal to zero. The maximum occurs at $E = Z/\rho$ and the corresponding energy

$$E = V_B = \frac{-b + (b^2 + 4mz)^{1/2}}{2m} \ . \tag{7}$$

The corresponding barrier radius R_B is given as

$$R_B{}^2 = 2mz + \frac{b^2}{2} + \frac{b}{2}(b^2 + 4mz)^{1/2} \ . \tag{8}$$

Incidentally, the fusion cross section as represented by expressions (1) and (2) becomes zero at the barrier energy, which is consistent with the classical nature of these expressions. However, expressions (3) and (4) may still be valid at energies below V_B. As m and b are determined as global parameters, expressions (7) and (8) describe the barrier height and radius in an analytical form over the periodic table.

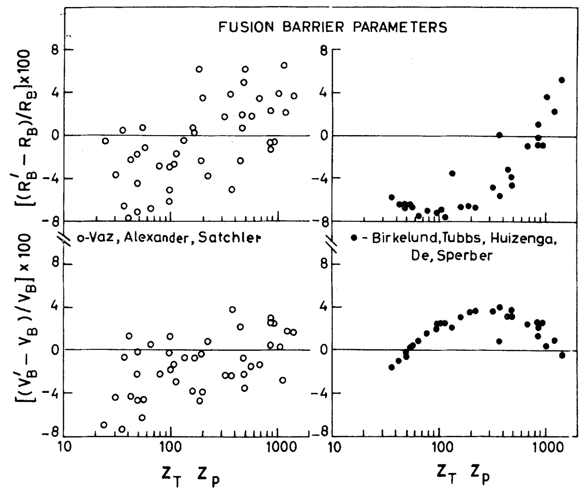

FIG. 1. Percentage deviations of the barrier parameters V_B and R_B, obtained from the present work from those, V_B' and R_B', predicted by Vaz *et al.* (Ref. 1) and Birkelund *et al.* (Ref. 5) are shown for various $Z_T Z_P$ systems.

Using expressions (7) and (8) we calculated the fusion barrier parameters V_B and R_B for a large number of interacting systems. We have compared our results with those of Birkelund *et al.*,[5] who have successfully described the fusion data with a classical trajectory model incorporating a long range friction force, and also with those of Vaz *et al.*,[1] who have fitted the fusion data by an empirical procedure in which they obtain the transmission coefficients from a Hill-Wheeler formula. Deviations of the literature values, V_B' and R_B', from our determination, V_B and R_B, are plotted as a function of $Z_T Z_P$ in Fig. 1. From the figure it can be seen that our values agree generally within ±4% for the barrier height and within ±8% for the barrier radius when compared with the values obtained from Refs. 1 and 5. As the empirical barrier parameters found in these references are in good agreement with the theoretical model predictions, our results can also be taken to be in accord with that of the theoretical models.

As the barrier parameters obtained by Vaz *et al.*[1] for various individual systems were compared with the predictions of our global empirical procedure, one notices deviations occurring on either side of the zero. However, as the values of Birkelund *et al.*[5] are based on the standard proximity nuclear potential plus the Bondorf Coulomb potential, they are smoothly varying over the periodic table. Our results based on global parameters deviate from their values systematically as one goes from lighter to heavier systems. Further, one notices more deviations in the comparison of R_B values. As the barrier is having a rather broad maximum, the value for the barrier radius (R_B) is not determined that accurately. This accounts for the observed deviations in R_B values.

The familiar technique of fitting $\sigma_{\rm fus}$ data with an expression of the type

$$\sigma_{\rm fus} = \pi R_B{}^2 \left(1 - \frac{V_B}{E} \right) , \tag{9}$$

and getting R_B from the intercept and V_B from the slope of the $\sigma_{\rm fus}$ vs $1/E$ plot, has several limitations, as the results from this procedure strongly depend on the range of energies over which the $\sigma_{\rm fus}$ data are fitted. Our procedure avoids this problem, in view of the functional form selected for fitting the $\sigma_{\rm fus}$ data.

To conclude, we feel that the simple analytical expressions defined in terms of universal parameters are fairly successful in predicting the barrier heights and radii for a large number of interacting heavy-ion systems, and the predictions from this empirical procedure are consistent with those obtained from more sophisticated techniques.

[1]L. C. Vaz, J. M. Alexander, and G. R. Satchler, Phys. Rep. 69, 373 (1981).
[2]S. Kailas and S. K. Gupta, Z. Phys. A302, 355 (1981).
[3]S. K. Gupta and S. Kailas, Nucl. Phys. Solid State Phys. (India) 24B, 37 (1981).
[4]R. Bass, Phys. Lett. 47B, 139 (1973); Nucl. Phys. A231, 45 (1974); Phys. Rev. Lett. 39, 265 (1977).
[5]J. R. Birkelund, L. E. Tubbs, J. R. Huizenga, J. N. De, and D. Sperber, Phys. Rep. 56, 107 (1979).