Energy-dependent separable potentials for low-energy K^+p elastic scattering

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Separable potentials reproducing the K^+p scattering data at energies up to the pion production threshold are presented. They are rank-one and energy dependent. Their interest lies mainly in their usefulness in a Faddeev treatment of the K^+d system.

[NUCLEAR REACTIONS K^+p; low-energy scattering; energy-dependent separable potentials.]

Although the problem of describing the KN interaction at low energies has deserved considerable attention in the last decades,[1–10] it is far from being solved. The scattering length and effective range for the isospin $I=1$ channel are reasonably well established through K^+p scattering experiments. For the $I=0$ channel, instead, the situation is much more unsatisfactory. The information on this channel must be drawn mainly from K^+d data,[2,11–19] available only above 342 MeV/c for elastic scattering and 252 MeV/c for breakup. Moreover, the analysis of these data requires a treatment of the three body problem, which usually is reduced to the impulse approximation.[2] This treatment causes large errors in the determination of the scattering length.[2,8] A more precise solution of the three body problem seems to be needed.

Faddeev equations provide an (in principle) exact method of relating three body data with two body parameters. Their efficient implementation makes use of separable potentials to represent the two body interaction. Hetherington and Schick[20] applied, for the first time, Faddeev-like techniques to the K^+d system. Their aim was mainly to analyze how rapidly the multiple scattering series converges to the Faddeev solution. For simplicity they used rank-one potentials of the Yamaguchi type for the KN system, both in the $I=0$ and 1 channels, and for the NN system. They noticed, however, that a rank-one potential of that kind cannot reproduce simultaneously the experimental scattering length and effective range in the $K^+N(I=1)$ channel.

As a first step towards a forthcoming Faddeev analysis of the $K^+N(I=0)$ interaction in the K^+d system, we are interested in a separable potential fitting the existing data on K^+p scattering at low energies. Of course, a rank-two potential of the Yamaguchi type could be found. However, doubling the rank of the potential means doubling the number of rows and columns of the matrix to be inverted in the solution of the Faddeev equations. This implies larger errors and enormously longer time of computations.

Recently, Garcilazo[21,22] has proposed, for the S-wave nucleon-nucleon interaction, an energy-dependent rank-one separable potential that improves, in a wide range of energies, the fit obtained with energy-independent ones. Such energy dependence does not introduce additional complications in the Faddeev equations, as compared to the energy-independent case. It has seemed to us interesting to find a similar potential that, for suitably chosen energy-dependent intensity, can reproduce the low-energy K^+p data. This is the purpose of this paper. It is known[3–7] that a purely S-wave interaction is sufficient to explain the K^+p data at energies below the pion production threshold. For this reason we limit ourselves to the S-wave case.

Let us consider a separable potential (units $\hbar=c=1$ are used):

$$V(p,p';E)=g(p)[\lambda(E)/2\mu]g(p') \ , \tag{1}$$

where $g(p)$ represents the form factor, $\lambda(E)$ the intensity depending on the energy E, and μ the K^+p reduced mass. From the Lippman-Schwinger equation

$$T(p,p';E)=V(p,p';E)+\int_0^\infty q^2\,dq\,\frac{V(p,q;E)T(q,p';E)}{E-q^2/2\mu+i\epsilon} \ , \tag{2}$$

replacing the potential given by Eq. (1) with a Yamaguchi[23] form factor

$$g(p)=(p^2+\beta^2)^{-1} \ , \tag{3}$$

one obtains for the T matrix

$$T(p,p';E)=\frac{\left[2\mu/\lambda(E)-\int_0^\infty q^2\,dq\,(q^2+\beta^2)^{-2}(E-q^2/2\mu+i\epsilon)^{-1}\right]^{-1}}{\mu(p^2+\beta^2)(p'^2+\beta^2)} \ . \tag{4}$$

From this expression, it turns out for the on-shell T matrix ($p = p' = (2\mu E)^{1/2} = p_0$) ,

$$T(p_0,p_0;E) = \mu^{-1}[2(p_0^2+\beta^2)^2/\lambda(E) - (p_0^2-\beta^2)\pi/2\beta + p_0\pi i]^{-1} , \quad (5)$$

from which the phase shift can be obtained through the relation

$$T(p_0,p_0;E) = -\mu^{-1}\pi^{-1}[p_0\cot\delta(E) - ip_0]^{-1} . \quad (6)$$

Combining Eqs. (5) and (6) one obtains

$$p_0\cot\delta(E) = -2(p_0^2+\beta^2)^2/\pi\lambda(E) + (p_0^2-\beta^2)/2\beta . \quad (7)$$

Substitution of $1/\lambda(E)$ by its Taylor series gives an effective-range expansion of the form

$$p_0\cot\delta(E) = \frac{-1}{a} + \tfrac{1}{2}rp_0^2 + \sum_2^\infty A_n p_0^{2n} , \quad (8)$$

where the coefficients are given by

$$-1/a = -2\beta^4/\pi\lambda(0) - \beta/2 , \quad (9)$$

$$\tfrac{1}{2}r = \frac{-4\beta^2}{\pi\lambda(0)} - \frac{\beta^4}{\pi\mu}\left[\frac{d}{dE}\frac{1}{\lambda(E)}\right]_{E=0} + 1/2\beta , \quad (10)$$

$$A_n = -\frac{2}{\pi n!}\left(\frac{1}{2\mu}\right)^n\left[\frac{d^n}{dE^n}\frac{(2\mu E+\beta^2)^2}{\lambda(E)}\right]_{E=0} , \quad n \geq 2 . \quad (11)$$

Experimental values of the phase shifts at energies up to the inelastic threshold can be fitted by Eq. (8) with only the first two terms in the right-hand side.[5,6] The most reliable values of the scattering length and effective range seem to be

$$a = 0.309 \pm 0.002 \text{ fm} , \quad (12a)$$

$$r = 0.32 \pm 0.02 \text{ fm} . \quad (12b)$$

Contrary to what happens to energy-independent potentials, our rank-one potential can reproduce both low-energy parameters merely by taking

$$[\lambda(E)]_{E=0} = \lambda_0 = \frac{4\alpha\beta^4}{(2-a\beta)\pi} , \quad (13)$$

$$\left[\frac{d}{dE}[\ln\lambda(E)]\right]_{E=0} = -2\mu C(\beta) , \quad (14)$$

where we have denoted

$$C(\beta) = \frac{-ar\beta^2+3a\beta-4}{(2-a\beta)\beta^2} . \quad (15)$$

To be useful in the description of the low-energy K^+p system our potential must produce, besides the correct a and r, very small values of the parameters A_n in order to be in agreement with the experimental evidence. To achieve this, we have still at our disposal the choice of the parameter β appearing in the Yamaguchi form factor, Eq. (3), and of the functional dependence $\lambda(E)$, with the restrictions imposed by Eqs. (13) and (14). There exists considerable freedom in the selection of $\lambda(E)$. For simplicity reasons we have chosen an exponential dependence

$$\lambda(E) = \lambda_0\exp[-2\mu C(\beta)E] , \quad (16)$$

which gives for the parameters of the effective-range expansion

$$A_n = -\frac{2C^{n-2}(\beta)}{\pi\lambda_0 n!}[n(n-1) + 2n\beta^2C(\beta) + \beta^4C^2(\beta)], \quad n \geq 2 . \quad (17)$$

The last step is to determine the value of β so as to obtain as small as possible values of A_n. In Fig. 1 we have represented, with the experimental values of a and r, the function $C(\beta)$ entering into the expression of the A_n in terms of β, and the intensity coefficient λ_0 given by Eq. (13). For values of β lower than $2/a$ the intensity coefficient λ_0 is positive, i.e., the potential is repulsive as it should be, according to the experimental values of the phase shifts. Values of β larger than $2/a$ should be discarded as they originate attractive potentials. At $\beta = 2/a$ the intensity coefficient becomes singular. As it is well known,[24] this represents no difficulty, since the meaningful physical quantities are well defined. As far as $C(\beta)$ cannot vanish for any real β, it is obvious that only one of the A_n can be made exactly equal to zero.

Inspection of Eq. (17) suggests to choose the value of β, in the range $0 \leq \beta \leq 2/a$ (repulsive potential), that makes minimum the absolute value of $C(\beta)$. That value of β and the corresponding ones of λ_0 and

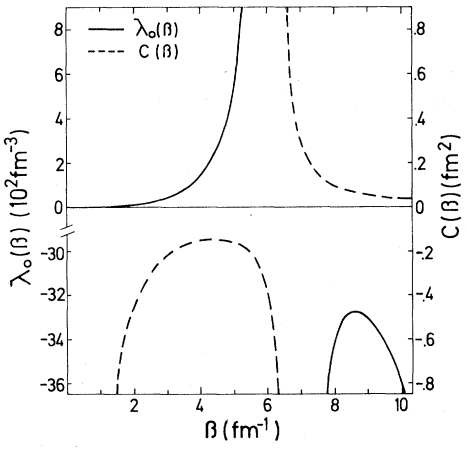

FIG. 1. Intensity coefficient $\lambda_0(\beta)$ and auxiliary function $C(\beta)$ (see text) in terms of the parameter β of the form factor of the energy-dependent potential.

$C(\beta)$ are

$$\beta^{I} = 4.406\,786 \text{ fm}^{-1} ,$$

$$C(\beta^{I}) = -0.148\,046 \text{ fm}^{2} ,$$

$$\lambda_0^{I} = 232.4499 \text{ fm}^{-3} .$$

We shall refer to this choice of β, and the resulting potential, as solution I. The values of the first A_n for this solution are

$$A_2^{I} = 0.001\,69 \text{ fm}^{3} ,$$

$$A_3^{I} = -0.000\,201 \text{ fm}^{5} ,$$

$$A_4^{I} = 0.000\,006\,81 \text{ fm}^{7}, \ldots .$$

Another possible criterium to determine β could be to require the vanishing of the A_n dominating at low energies, namely, A_2. We obtain in this way the solution II:

$$\beta^{II} = 4.740\,140 \text{ fm}^{-1} ,$$

$$C(\beta^{II}) = -0.151\,953 \text{ fm}^{2} ,$$

$$\lambda_0^{II} = 371.0553 \text{ fm}^{-3} .$$

For the coefficients A_n we have

$$A_2^{II} = 0 \text{ fm}^{3} ,$$

$$A_3^{II} = -0.000\,128 \text{ fm}^{5} ,$$

$$A_4^{II} = 0.000\,006\,01 \text{ fm}^{7}, \ldots .$$

We show in Fig. 2 the values of $p_0 \cot\delta(E)$ given by our potentials I and II, for energies up to the inelastic threshold. For comparison, the experimental data are represented by a straight line corresponding to the experimental values of a and r quoted in Eq. (12). To make evident the considerable improvement reached with respect to energy-independent rank-one separable potentials, we represent also in Fig. 2 the value of $p_0 \cot\delta(E)$ for the optimum one that fits the scattering length.

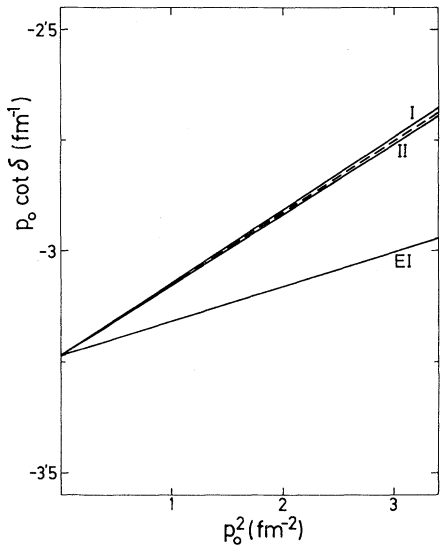

FIG. 2. Values of $p_0 \cot\delta$ calculated with the two potentials (I and II) presented in the text, and with the optimum energy-independent rank-one potential (*EI*) that fits the scattering length. The dashed line corresponds to the value of $-1/a + (1/2) r p_0^2$ for the experimental scattering length and effective range.

The energy-dependent potentials I and II presented here are by no means the only ones to fit the experimental values. Our choice of form factors (Yamaguchi) responds to the common practice. The energy dependence in the intensity has been chosen of exponential type for the sake of simplicity in the analysis of the higher terms in the effective-range expansion. Nevertheless, different form factors and energy dependence are not *a priori* discarded.

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