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# Simple expressions for polarization transfer observables in inelastic proton scattering

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Using the plane-wave impulse approximation we have derived simple expressions for the triple scattering parameters in inelastic proton transitions characterized by a single *lsj* value. The plane-wave impulse approximation gives an excellent account of the results of distorted-wave impulse approximation calculations leading to $1^+$, $T=1$ and $2^-$, $T=0$ states in $^{12}$C at $E_p=500$ MeV.

[ NUCLEAR REACTIONS Scattering theory, polarization transfer observables. ]

Medium-energy accelerators coupled with polarized-ion sources and magnetic spectrometers offer the exciting possibility of measuring new observables in inelastic proton scattering. At present the most extensive program in this area is at the Clinton P. Anderson Meson Physics Facility, where a flexible system of initial polarization orientation plus a versatile focal-plane polarimeter on the high-resolution spectrometer permits measurement of all the triple scattering parameters $D_{NN'}$, $D_{SS'}$, $D_{LL'}$, $D_{LS'}$, and $D_{SL'}$.[1]

Although computer codes are available which permit calculation of the $D_{ij}$'s in both the distorted-wave impulse approximation[2] (DWIA) and Glauber model,[3] the physics contained in extensive numerical computations is not always evident. The purpose of this Brief Report is to elucidate the new physics contained in the $D_{ij}$'s by deriving simple expressions for them in the plane-wave impulse approximation (PWIA). The simple equations are then compared to numerical results of the DWIA.

Employing the notation of Kerman, McManus, and Thaler[4] (KMT), the nucleon-nucleon ($NN$) scattering amplitude $M(q)$ is written

$$M(q)=A+B\sigma_{1\hat{n}}\sigma_{2\hat{n}}+C(\sigma_{1\hat{n}}+\sigma_{2\hat{n}})+E\sigma_{1\hat{q}}\sigma_{2\hat{q}}+F\sigma_{1\hat{p}}\sigma_{2\hat{p}}\ , \quad (1)$$

with

$$\hat{q}=\vec{q}/|q|,\quad \vec{q}=\vec{k}'-\vec{k}\ ,$$
$$\hat{n}=\vec{n}/|n|,\quad \vec{n}=\vec{k}\times\vec{k}'\ .$$
$$\hat{p}=\hat{q}\times\hat{n}\ .$$

For each amplitude of Eq. (1), $A=A_\alpha+A_\beta\vec{\tau}_1\cdot\vec{\tau}_2$, etc. Then, following the notation of Appendix III of KMT, the $NN$ amplitude is rewritten as

$$M(q)=M_0+\sum_\mu(-1)^\mu M_\mu^1\sigma_\mu^1\ , \quad (2)$$

with

$$\sigma_0^1=\sigma_{\hat{q}};\quad \sigma_1^1=-1/\sqrt{2}(\sigma_{\hat{n}}+i\sigma_{\hat{p}});$$
$$\sigma_{-1}^1=1/\sqrt{2}(\sigma_{\hat{n}}-i\sigma_{\hat{p}})\ .$$

The quantities $M_\mu$ can be determined from Eqs. (1) and (2):

$$M_0=A+C\sigma_{\hat{n}}\ ;$$
$$M_0^1=E\sigma_{\hat{q}};\quad M_1^1=-1/\sqrt{2}(C+B\sigma_{\hat{n}}-iF\sigma_{\hat{p}})\ ;$$
$$M_{-1}^1=1/\sqrt{2}(C+B\sigma_{\hat{n}}+iF\sigma_{\hat{p}})\ .$$

The nucleon-nucleus scattering amplitude is then

$$\overline{M}(q)=\langle m|M(q)e^{-i\vec{q}\cdot\vec{r}}|0\rangle\ . \quad (3)$$

At this point we make the simplifying assumption that only a single value of the orbital, spin, and total angular momentum transfer (*lsj*, respectively) is important in the inelastic transition. In addition, only unnatural parity transitions are considered (where $s=1$), since it is these cases where the new polarization observables are likely to yield the most interesting results. The nucleon-nucleus amplitude then becomes

$$\overline{M}_\mu=(-1)^{j-\mu}(2j+1)^{-1/2}(1\mu l0|j\mu)Q_{lj}M_\mu^1\ , \quad (4)$$

where $Q_{lj}$ is the nuclear matrix element defined by KMT. Then, using the equations

$$D_{ij}=\frac{\mathrm{Tr}(\overline{M}\sigma_i\overline{M}^+\sigma_j)}{\mathrm{Tr}(\overline{M}\,\overline{M}^+)}\ . \quad (5)$$

One arrives at the simple expressions for the triple scattering parameters in the $\hat{n}$, $\hat{q}$, and $\hat{p}$ coordinate system:

$$\sigma_0 D_{\hat{n}\hat{n}} = \bar{l}_1(C^2+B^2-F^2)-\bar{l}_2 E^2 \ ,$$
$$\sigma_0 D_{\hat{q}\hat{q}} = \bar{l}_1(C^2-B^2-F^2)+\bar{l}_2 E^2 \ ,$$
$$\sigma_0 D_{\hat{p}\hat{p}} = \bar{l}_1(C^2-B^2+F^2)-\bar{l}_2 E^2 \ , \qquad (6)$$
$$\sigma_0 D_{\hat{q}\hat{p}} = -\sigma_0 D_{\hat{p}\hat{q}} = 2\bar{l}_1 \mathrm{Im}(BC^*) \ ,$$
$$\sigma_0 = \bar{l}_1(C^2+B^2+F^2)+\bar{l}_2 E^2 \ ,$$

with

$$\bar{l}_1 = l+2, \ \ \bar{l}_2 = 2l+2 \ \text{for} \ j=l+1$$

and

$$\bar{l}_1 = l-1, \ \ \bar{l}_2 = 2l \ \text{for} \ j=l-1 \ .$$

For completeness, the analyzing power and polarization are

$$\sigma_0 A = \sigma_0 P = 2\bar{l}_1 \mathrm{Re}(BC^*) \ .$$

In terms of the quantities normally measured in laboratory experiments where polarization states are referred to coordinate systems containing the initial and final (primed) momenta [$L\,(L')$ parallel to $k\,(k')$, $N\,(N')$ parallel to $\vec{k}\times\vec{k}'$, and $S\,(S')$ parallel to $N\times L\,(N\times L')$]:

$$D_{LL'} = D_{\hat{q}\hat{q}}\cos(\theta+\theta_q)\cos\theta_q + D_{\hat{p}\hat{p}}\sin(\theta+\theta_q)\sin\theta_q - D_{\hat{q}\hat{p}}\sin\theta \ ,$$
$$D_{SS'} = D_{\hat{q}\hat{q}}\sin(\theta+\theta_q)\sin\theta_q + D_{\hat{p}\hat{p}}\cos(\theta+\theta_q)\cos\theta_q + D_{\hat{q}\hat{p}}\sin\theta \ ,$$
$$D_{LS'} = -D_{\hat{q}\hat{q}}\sin(\theta+\theta_q)\cos\theta_q + D_{\hat{p}\hat{p}}\cos(\theta+\theta_q)\sin\theta_q - D_{\hat{q}\hat{p}}\cos\theta \ , \qquad (7)$$
$$D_{SL'} = -D_{\hat{q}\hat{q}}\cos(\theta+\theta_q)\sin\theta_q + D_{\hat{p}\hat{p}}\sin(\theta+\theta_q)\cos\theta_q D_{\hat{q}\hat{p}}\cos\theta \ ,$$
$$D_{NN'} = D_{\hat{n}\hat{n}} \ .$$

In these expressions $\theta$ and $\theta_q$ are, respectively, the laboratory scattering and recoil angles. Equations (6) reduce even further under additional restrictions. Some of these of interest to understanding DWIA calculations are as follows: (1) $\theta = 0°$; $C=0$, $B=E$; (2) central (no spin-orbit or tensor interactions); $C=0$, $B=E=F$; and (3) pure tensor direct (no exchange); $C=0$, $2B=2F=E$. The normal spin-flip probability $S=\frac{1}{2}(1-D_{NN})$ under limit 2 has been derived previously, using a different approach.[5]

In Fig. 1 we compare the results of Eqs. (6) and (7) to DWIA calculations using the Love-Franey[6] interaction at $E_p = 500$ MeV. The $^{12}C(p,p')^{12}C(15.11$ MeV, $1^+$, $T=1)$ reaction was computed with the Cohen-Kurath[7] wave functions which allow $lsj = 011$, 101, 211, and 111 transfer, the latter arising from tensor exchange. Only $lsj = 011$ transfer was included from Eqs. (6) and (7). For the $^{12}C(p,p')^{12}C(2^-$, $T=0)$ reaction, a hypothetical wave function $(p_{3/2}s_{1/2})_{2^-}$ configuration was assumed. This configuration allows only $lsj = 112$ transfer.

It is obvious that the simple equations account well for the qualitative features of the DWIA. Distortion effects at these small momentum transfers $q \leqslant 1.2$ fm$^{-1}$ seem not to be important. To the extent that the equations with pure $lsj$ values predict the same $D_{ij}$'s as the DWIA, there is no sensitivity to the assumed transition density. This is, of course, not generally true.[8] For example, the other allowed $lsj$ values in the 15.11 MeV transition will contribute at larger momentum transfer and there may well be interference effects which depend sensitively on the assumed wave functions.

The major insight to be gained from Eqs. (6) and (7) is that, like the free $NN$ observables, the $D_{ij}$'s in nuclear transitions excited by $(p,p')$ are largely determined by various combinations of the effective $NN$ interaction amplitudes. The extent to which these amplitudes are different from the free amplitudes is a subject of great current interest.[9] It may also be, that at certain energies (particularly $E_p > 500$ MeV), even the free amplitudes are not sufficiently well deter-

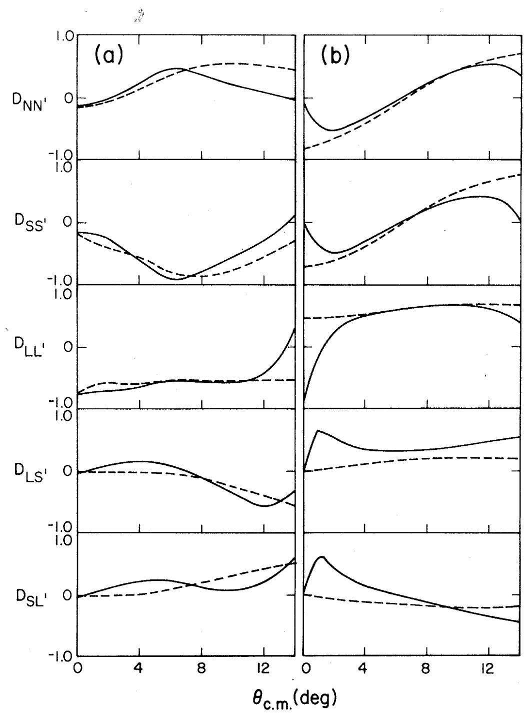


FIG. 1. Comparison of the results of Eqs. (6) and (7) (dashed curves) with distorted-wave impulse approximation calculations (solid curves) for $(p,p')$ reactions leading to the 15.11 MeV, $1^+$, $T=1$ state of $^{12}C$ (a), and a hypothetical $2^-$, $T=0$ state of $^{12}C$ at $E_p = 500$ MeV (b).

mined in the range of $q$ in which $(p,p')$ experiments are performed. Thus, the simple equations should be valuable in assessing success or failure of calculations in reproducing experimental data.

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