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Nuclear time delays extracted from proton-carbon bremsstrahlung data near the 1.7-MeV resonance

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Nuclear time delays of the order of 10^{-20} sec as a function of photon energy have been extracted from the experimental $p\ {}^{12}\mathrm{C}\gamma$ cross sections near the 1.7-MeV scattering resonance. Our extraction is based upon the Feshbach-Yennie approximation which includes both the principal term and the correction term.

[NUCLEAR REACTIONS Proton-carbon bremsstrahlung, extract nuclear reaction time delays from the experimental data.]

In two recent articles,[1,2] we have reported the results of our measurements of the proton-carbon bremsstrahlung ($p\ {}^{12}\mathrm{C}\gamma$) cross sections near the 1.7-MeV resonance and of our calculations of $p\ {}^{12}\mathrm{C}\gamma$ cross sections using two model-independent approximations. The most important conclusion obtained in these reports was that the structure due to resonance effects can be successfully described by the Feshbach-Yennie approximation. The agreement between experiment and theory suggests that we can use the Feshbach-Yennie theory to extract the nuclear time delay from the experimental data. In this paper, we present this time delay which we have extracted from our $p\ {}^{12}\mathrm{C}\gamma$ data.

The idea of using bremsstrahlung emission as a tool for measuring time delay was first proposed by Eisberg, Yennie, and Wilkinson,[3] whose classical treatment of the bremsstrahlung processes was extended later to a quantum mechanical treatment by Feshbach and Yennie.[4] In this paper, we shall follow the definition of time delay defined by these authors. Our extraction and calculations are based upon the Feshbach-Yennie approximation derived in Ref. 2.

For our $p\ {}^{12}\mathrm{C}\gamma$ case, the time delay is the elapsed time between the arrival of the incident proton ($t=0$) and the departure of the outgoing proton ($t=\tau$), i.e., the lifetime of the compound nucleus (${}^{13}\mathrm{N}^*$) formed in the collision, which is too short ($\tau \approx 10^{-20}$ sec) to be measured by conventional methods.

A group at Bologna[5] reported the first measurement of time delay in p-${}^{12}\mathrm{C}$ scattering near 1.7 MeV. This group used its measured bremsstrahlung γ-ray counting rate and elastic counting rate to determine the time delay.

Only the principal term of the Feshbach-Yennie approximation was considered by the group. However, as we have shown in Ref. 2, the correction term is not negligible in the region of the resonance, and we must take into account the correction term in the extraction of the time delay from the data. Unfortunately, the problem becomes much more complicated when the correction term is included in the calculation. To show this complication, let us now describe our method of extracting the time delay.

The bremsstrahlung cross section, $\sigma_\gamma \equiv d^3\sigma/d\Omega_q\, d\Omega_\gamma\, dK$, for the $p\ {}^{12}\mathrm{C}\gamma$ process

$$p(q_i^\mu) + {}^{12}\mathrm{C}(p_i^\mu) \to p(q_f^\mu) + {}^{12}\mathrm{C}(p_f^\mu) + \gamma(K^\mu)$$

(the four-momentum of each particle is given in the parentheses) can be written as

$$\sigma_\gamma = GJK[\tfrac{1}{2}\textstyle\sum(-M_\mu^\dagger M^\mu)]F \ . \qquad (1)$$

Here

$$G = e^2/(16\pi^3) \ ,$$

$$J = m^2/\{16\pi^2[(q_i\cdot p_i)^2 - m^2M^2]^{1/2}\} \ ,$$

$$F = \frac{[(p_i\cdot q_f)^2 - m^2M^2]^{3/2}}{M^2(p_i\cdot q_f)(p_f\cdot q_f) - M^2m^2(p_i\cdot p_f)} \ ,$$

e is the proton charge, m (M) is the proton (carbon) mass, and M_μ is the bremsstrahlung amplitude. The minus sign in $(-M_\mu^\dagger M^\mu)$ of Eq. (1) comes from the summation over the photon polarization, and the summation sign $\sum$ indicates a sum over the initial and final proton spins.

The elastic scattering cross sections,

$$\sigma_\alpha \equiv d\sigma_e(s_\alpha,t)/d\Omega_q \quad (\alpha = i,f) \ ,$$

for the corresponding $p\ {}^{12}\mathrm{C}$ elastic process,

$$p(q_i^\mu) + {}^{12}\mathrm{C}(p_i^\mu) \to p(\bar{q}_f^\mu) + {}^{12}\mathrm{C}(\bar{p}_f^\mu) \ ,$$

can be written as

$$\sigma_\alpha = J\Big(\tfrac{1}{2}\textstyle\sum[M_e^\dagger(s_\alpha,t)M_e(s_\alpha,t)]\Big)\bar{F} \ , \qquad (2)$$

where $M_e(s_\alpha,t)$ is the elastic scattering amplitude, $\bar{q}_f^\mu = \lim_{K\to 0} q_f^\mu$, $\bar{p}_f^\mu = \lim_{K\to 0} p_f^\mu$, $\bar{F} = \lim_{K\to 0} F$, and $t=(\bar{p}_f-p_i)^2=(\bar{q}_f-q_i)^2$. Since the elastic scattering amplitude can be evaluated at two different energies, two differential elastic scattering cross sections are defined in Eq. (2): σ_i is the cross section at the initial energy $s=s_i=(q_i+p_i)^2$, and σ_f is the cross section at the final energy $s=s_f=(q_f+p_f)^2$.

Actually, the cross section measured by our group is the ratio of the bremsstrahlung cross section σ_γ to the elastic scattering cross section σ_i. We denote this ratio by $\sigma_{\rm rel}$. Using Eqs. (1) and (2), we can write $\sigma_{\rm rel}$ in the form

$$\sigma_{\rm rel} = GK\left(\frac{F}{\bar{F}}\right)\frac{\sum(-M_\mu^\dagger M^\mu)}{\sum[M_e^\dagger(s_i,t)M_e(s_i,t)]} . \tag{3}$$

The elastic scattering amplitude $M_e(s_i,t)$ has the form

$$M_e(s_i,t) = \bar{u}(\bar{q}_f,\nu_f)T(s_i,t)u(q_i,\nu_i) ,$$

where T and u are the standard T matrix and the Dirac spinor, respectively. In order to simplify our notations, let us introduce two useful abbreviations:

$$\mathfrak{M}_{\alpha\beta} \equiv \sum[M_e^\dagger(s_\alpha,t)M_e(s_\beta,t)]$$

and

$$\mathcal{T}_{\alpha\beta} \equiv \tilde{T}(s_\alpha,t)\Lambda(\bar{q}_f)T(s_\beta,t)\Lambda(q_i) , \qquad (\alpha,\beta = i \text{ or } f) ,$$

where $\tilde{T} \equiv \gamma^0 T \gamma^0$ and $\Lambda(q) \equiv (\not{q}+m)/(2m)$. It is easy to show that $\mathfrak{M}_{\alpha\beta} = {\rm Tr}\mathcal{T}_{\alpha\beta}$. Using these new notations, σ_α ($\alpha = i$ or f) can also be written in the form

$$\sigma_\alpha = \tfrac{1}{2}J\mathfrak{M}_{\alpha\alpha}\bar{F} = \tfrac{1}{2}J({\rm Tr}\mathcal{T}_{\alpha\alpha})\bar{F} . \tag{4}$$

The time delay τ and the phase difference Φ can be defined in terms of $\mathfrak{M}_{\alpha\beta}$ or $\mathcal{T}_{\alpha\beta}$ as

$$\begin{aligned}\cos(\omega\tau) &\approx \cos\Phi \\ &= {\rm Re}(\mathfrak{M}_{if})/(\mathfrak{M}_{ii}\mathfrak{M}_{ff})^{1/2} \\ &= {\rm Re}({\rm Tr}T_{if})/[({\rm Tr}T_{ii})({\rm Tr}T_{ff})]^{1/2} ,\end{aligned} \tag{5}$$

where $\omega = K/\hbar \approx (s_i - s_f)/(2M\hbar)$.

The bremsstrahlung amplitude M_μ has the form[2]

$$M_\mu = \bar{u}(q_f,\nu_f)\left\{a_{f\mu}T(s_i,t) - a_{i\mu}T(s_f,t) + b_{f\mu}\frac{\partial T(s_i,t)}{\partial t} + b_{i\mu}\frac{\partial T(s_f,t)}{\partial t}\right\}u(q_i,\nu_i) , \tag{6}$$

where

$$a_{f\mu} = \frac{q_{f\mu}}{q_f\cdot K} + \frac{Zp_{f\mu}}{p_f\cdot K} - \frac{(1+Z)(p_f+q_f)_\mu}{(p_f+q_f)\cdot K} ,$$

$$a_{i\mu} = a_{f\mu} \text{ (changing } f \text{ to } i) ,$$

$$b_{f\mu} = \frac{2(\bar{q}_f-q_i)\cdot R}{p_f\cdot K}Zp_{f\mu} - \frac{2(\bar{p}_f-p_i)\cdot(R+K)}{q_f\cdot K}q_{f\mu} + 2(\bar{p}_f-p_i-Z\bar{q}_f+Zq_i)\cdot N_R\bar{p}_{f\mu} + 2(\bar{p}_f-p_i)_\mu ,$$

$$b_{i\mu} = b_{f\mu}\text{(changing } f \text{ to } i \text{ and } i \text{ to } f , \text{ but fixing } R \text{ and } N_R) .$$

In Eq. (6), Z is the atomic number of carbon, $R^\mu = (\bar{p}_f\cdot K)N_R^\mu$ and

$$N_R^\mu = \frac{m^2 p_i^\mu - (p_i\cdot\bar{q}_f)\bar{q}_f^\mu}{(p_i\cdot\bar{q}_f)(\bar{p}_f\cdot\bar{q}_f) - m^2(p_i\cdot\bar{p}_f)} .$$

Equations (6), (4), and (5) can be used to calculate $\sum(-M_\mu^\dagger M^\mu)/\mathfrak{M}_{ii}$. When the result is substituted into Eq. (3), we finally obtain

$$\sigma_{\rm rel} = -GK\left(\frac{F}{\bar{F}}\right)\left[(a_f\cdot a_f) + (a_i\cdot a_i)\left(\frac{\sigma_f}{\sigma_i}\right) - 2(a_i\cdot a_f)\left(\frac{\sigma_f}{\sigma_i}\right)^{1/2}\cos\Phi\right] - GK\left(\frac{F}{\bar{F}}\right)Y , \tag{7}$$

where

$$Y = {\rm Tr}\left\{(a_f\cdot b_f)\frac{\partial}{\partial t}\mathcal{T}_{ii} - (a_i\cdot b_i)\frac{\partial}{\partial t}\mathcal{T}_{ff} + 2(a_f\cdot b_i){\rm Re}(\mathcal{T}_{ifd}) - 2(a_i\cdot b_f){\rm Re}(\mathcal{T}_{\rm dif}) + (a_f\cdot a_f)\mathcal{T}_{iRi} + (a_i\cdot a_i)\mathcal{T}_{fRf} - 2(a_f\cdot a_i){\rm Re}(\mathcal{T}_{iRf})\right\}\Big/({\rm Tr}\mathcal{T}_{ii}) , \tag{8}$$

and we have used the abbreviations

$$\mathcal{T}_{ifd} \equiv \tilde{T}(s_i,t)\Lambda(\bar{q}_i)\left[\frac{\partial}{\partial t}T(s_f,t)\right]\Lambda(q_i) ,$$

$$\mathcal{T}_{\rm dif} \equiv \left[\frac{\partial}{\partial t}\tilde{T}(s_i,t)\right]\Lambda(\bar{q}_f)T(s_f,t)\Lambda(q_i) ,$$

and

$$\mathcal{T}_{\alpha R\beta} \equiv \tilde{T}(s_\alpha,t)(\not{R}/2m)T(s_\beta,t)\Lambda(q_i) \qquad (\alpha,\beta = i \text{ or } f) .$$

In Eq. (7) the first part, which includes three terms, is called the principal term and the second part, which includes Y, is called the correction term. As we can see from Eq. (8), Y, a very complicated function of T matrices, depends upon the elastic scattering cross sections and their derivatives with respect to t, upon Φ and its derivative, and also upon some terms which involve $\mathcal{K}$ [these terms come from the expansion of $\Lambda(q_f)=\Lambda(\bar{q}_f)=\mathcal{K}/2m$].

Equation (7) is the equation which can be used to determine Φ. The idea is to determine Φ from the elastic scattering and the bremsstrahlung data. Without the correction term (which involves Y) this can be done by measuring σ_i, σ_f, and $\sigma_{\rm rel}$. But if we have to take into account the correction term, the problem becomes very difficult because part of Y cannot be determined directly from the elastic data. In addition to this difficulty, we have to solve a complicated differential equation. To overcome these difficulties, we have introduced a hybrid method in which Y is calculated from the elastic scattering amplitude determined from the elastic data. Since Y is not part of the principal term, we expect that the error introduced by this approximation for the determination of Φ will be small.

Using the hybrid method described above, we have extracted first the values of $\cos\Phi$ for the incident proton energies of 1.88, 1.81, and 1.594 MeV, and then the time delay $\tau=\Phi/\omega$. However, we note that since the data have uncertainties, only the range of each data point, which lies within the regions $|\cos\Phi|\leq 1$ or $\tau\geq 0$, can be used in the extraction. These results are shown in Fig. 1. The solid curves in this figure are the results of theoretical calculations using Eq. (5). The resonance parameters used in these calculations are the best-fit parameters obtained from the elastic data by Armstrong *et al.*[6] For the case of 1.88 MeV, it is clear that the theoretical values for τ exhibit a bump of 1×10^{-20} sec around 100 keV of photon energy. The extracted values of τ from the experimental data around $K=100$ keV have also about the same order of 10^{-20} sec. As for the 1.81-MeV case, we can see from the figure that a maximum value of 1×10^{-20} sec is also predicted from Eq. (5), but a poor result is obtained from the experimental data. At the incident energy of 1.594 MeV, which is below the 1.7-MeV resonance, the value of τ is expected to be nearly zero since there is no bump in the bremsstrahlung spectrum. The agreement between the theoretical and experimental values of τ is satisfactory considering the statistical accuracy of the data.

The delay time of 1×10^{-20} sec is just about the lifetime of the 1735-keV level as predicted by the theoretical definition of the time delay given by the standard scattering theory.[7] This value is only about one-half the value obtained by the Bologna group, whose result is based upon the assumption that the external emission dominates the bremsstrahlung cross section with very little contribution from the internal emission of bremsstrahlung radiation. Although some differences exist between our analysis and that of the Bologna group, our result is in accord with the result of the Bologna group.

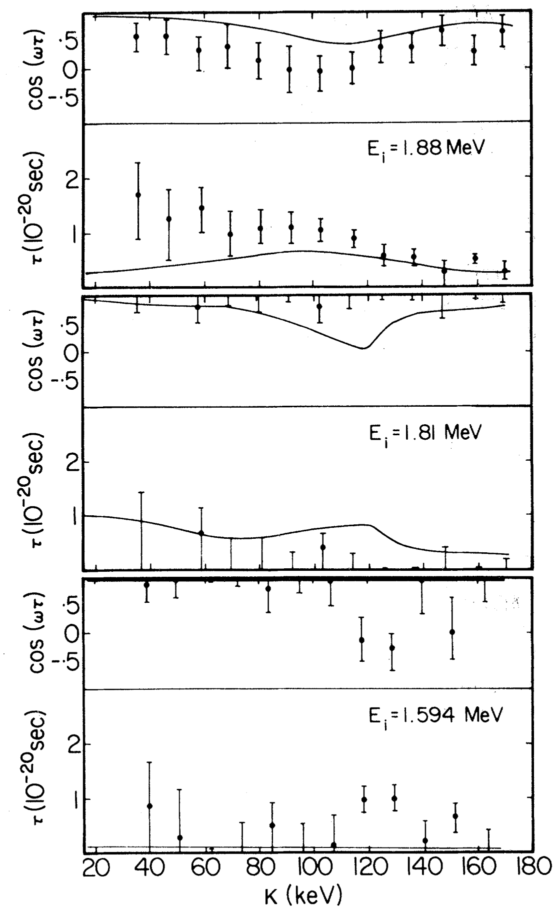

FIG. 1. $\cos(\omega\tau)$ and nuclear reaction time delays τ extracted from the $p\ ^{12}C\gamma$ data. The solid curves are the results of our calculations using Eq. (5).

In conclusion, we have extracted the nuclear time delay from our $p\ ^{12}C\gamma$ data near the 1.7-MeV resonance. Our extraction is based upon the Feshbach-Yennie approximation which includes both the principal term and the correction term. We have pointed out a difficulty when the correction term is taken into consideration and have introduced a hybrid method to overcome this difficulty. As a result, we have obtained the time delay as a function of photon energy for all three bombarding energies. For our 1.88-MeV case, the extracted values of the time delay exhibit a bump of about 1×10^{-20} sec around 100-keV photon energy, in satisfactory agreement with the theoretical prediction based upon the elastic scattering amplitude. Our result, together with that obtained by the Bologna group, supports the idea suggested by Eisberg, Feshbach, Wilkinson, and Yennie that bremsstrahlung processes can be used to measure nuclear time delay.

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