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Determination of the central part of the Λ-α interaction

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An investigation is made of the effect of the (central) Λ-nucleon potential $V_{\Lambda N}$ and of the density distribution ρ of the α particle on the central part of the $V_{\Lambda\alpha}$ interaction, in the framework of the rigid-core model, adopted originally in this context by Dalitz and Downs. Single and double Gaussian shapes for $V_{\Lambda N}$ and ρ are considered, the latter having the advantage that the charge form factor fits rather well the experimental results deduced from the electron scattering experiments including those at high momentum transfers. The differences observed in the values of $V_{\Lambda\alpha}$ depend mainly on the central Λ-nucleon potential assumed and they are very strong for small values of r. Differences also exist in this region because of the various densities used. The double Gaussian density has the effect of lowering the values of $V_{\Lambda\alpha}$ near the center of the α particle in almost all the cases of double Gaussian Λ-nucleon potentials.

[NUCLEAR STRUCTURE Hypernuclei; ${}^{5}_{\Lambda}$He; Λ-α interaction; Λ-N interaction, density of ^{4}He.]

I. INTRODUCTION

The knowledge of the Λ-α interaction is useful in hypernuclear structure calculations as the knowledge of the nucleon-α interaction is in problems of nuclear structure. Unfortunately, there are no Λ-α scattering experiments which could provide information on the Λ-α potential. A useful source of information is, however, the analysis of the ${}^{5}_{\Lambda}$He hypernucleus. The model adopted[1] for this purpose is the rigid-core one, originally used by Dalitz and Downs,[2] with the aim of determining the strength of the Λ-nucleon potential. On the basis of this model, Λ-α potentials have been constructed and used by certain authors.

Dalitz and Downs assumed Gaussian shapes for both the Λ-nucleon potential and the density distribution, and obtained in this way Λ-α potentials having the shape of a single Gaussian. A potential of this type was used, for example, by Tang and Herndon[3] though there is a small difference in the values of the parameters due to a difference in the experimental value of the binding energy of ${}^{5}_{\Lambda}$He and the range parameter they considered. Bodmer and Sampanthar[4] have considered the $V_{\Lambda\alpha}$ potential generated by a Gaussian density distribution and by a Yukawa Λ-nucleon potential. This leads again to an analytic expression for $V_{\Lambda\alpha}$ (in terms of the error and other functions). Furthermore, they considered the possibility of a contribution to the Λ-α potential from three-body ΛNN potentials of suitable shapes leading also to analytic expressions for $V_{\Lambda\alpha}$. Bodmer and Ali[5] have used the former contribution to $V_{\Lambda\alpha}$ in a study of ${}^{9}_{\Lambda}$Be treated as a three-body system. Their basic Λ-α interaction had been fitted for convenience by a superposition of two exponentials.

More recently Gibson, Goldberg, and Weiss[6,7] have used, in the framework of their analysis of light hypernuclear systems, either by means of the Hartree-Fock method or by means of the rigid-core model, Λ-nucleon potentials, both of single and double Gaussian shape (Volkov type), thus including a short range repulsion. They have also used in connection with the rigid-core model a density distribution for the α of single Gaussian shape, which leads to $V_{\Lambda\alpha}$ potentials of single and double Gaussian shape, respectively. Finally, the possibility of including a spin-orbit term in the $V_{\Lambda\alpha}$ potential has been considered by Alexander *et al.*,[8] Gibson and Weiss,[9] and Londergan and Dalitz.[10] Its origin comes from the spin-orbit force of the Λ-nucleon interaction, and in the work of Londergan and Dalitz is obtained from one-boson exchange models for this interaction.[11]

Since the source of information about the Λ-α potential is the ${}^5_\Lambda$He, it might be appropriate to recall the well-known overbinding problem related to this hypernucleus.[12(a),(b)] It had been realized long ago that the binding energy of the Λ in this hypernucleus obtained with various models and with a variety of Λ-nucleon potentials (deduced from the analysis of very light hypernuclei and the existing Λ-p scattering data) comes out considerably larger than the experimental value. The theoretical values of $B_\Lambda({}^5_\Lambda\mathrm{He})$ can exceed, for example, the experimental one by more than 2 MeV.[12(b)]

The reason for the observed discrepancy between the theoretical and the experimental value of $B_\Lambda({}^5_\Lambda\mathrm{He})$ is quite likely not to be just one and the clarification of the situation might require extensive and complicated calculations.[12(b)] One of the reasons is probably related to the simplifying assumptions about the (central) two-body Λ-nucleon potential, as, for example, the assumption about the same intrinsic range in both singlet and triplet states, which is usually made. Another reason should be the Σ-suppression effect to which Bodmer has drawn attention.[12(c)] This effect would reduce the Λ-binding energy most effectively in ${}^5_\Lambda$He. Among other reasons one can mention the suppression effect due to a tensor component or the inclusion of three-body ΛNN potentials. The results of the calculation so far, show, however, that both of these effects change the theoretical value of $B_\Lambda({}^5_\Lambda\mathrm{He})$ by a very small amount and are not able to allow for the observed discrepancy.[12(b)]

The same holds if a Majorana component is included in the Λ-N potential.[12(d)] It has been advisable in obtaining approximate Λ-α potentials to adjust the strength of the (attractive part of the) Λ-nucleon potential so that the experimental value for $B_\Lambda({}^5_\Lambda\mathrm{He})$ is obtained.[7] In this paper we are making a (re)adjustment of this strength in order to fit the experimental $B_\Lambda({}^5_\Lambda\mathrm{He})$, although we also give results without such a readjustment (see Secs. III and IV).

In the present study we neglect the spin-orbit term and we concentrate on the central part of the $V_{\Lambda\alpha}$ interaction which we further assume is generated by a two-body Λ-nucleon potential. As a matter of fact, it appears that most of the $V_{\Lambda\alpha}$ potentials used in practice, from the earlier[3,5,13] until recent[14] hypernuclear structure calculations, are of this type.

We attempt to make a rather detailed investigation of the effect, on the Λ-α potential, of the magnitude of the short range repulsion included in the Λ-nucleon one, and mainly of the density distribution for the α particle by allowing its shape to be consistent (in the framework of certain approximations) with the corresponding charge form factor of ^{4}He, deduced from the electron scattering experiments,[15,16] at the higher values of momentum transfer too. The latter effect has been completely overlooked in this context so far, to our knowledge.

In the next section we give a summary of the rigid core model on which the present analysis is based and in Sec. III we discuss in some detail the Λ-nucleon potentials and the density distributions of the α particle we use. In the final section, the numerical results are given and discussed.

II. SUMMARY OF THE RIGID-CORE APPROXIMATION

In the rigid-core model which was originally used for ${}^5_\Lambda$He by Dalitz and Downs,[2,12(a)] one considers the ${}^5_\Lambda$He as a two-body system consisting of a Λ and the α particle, the distortion of the ^{4}He core because of the presence of the Λ particle being neglected.

The radial wave function $\varphi(r)$ of the Λ-α relative motion is determined by solving numerically the Schrödinger equation

$$-\frac{\hbar^2}{2\mu_{\Lambda\alpha}}\frac{d^2\varphi(r)}{dr^2}+V_{\Lambda\alpha}(r)\varphi(r)=-B_\Lambda\varphi(r)\ , \quad (1)$$

where B_Λ is the separation (or binding) energy of the Λ from the α core. $\mu_{\Lambda\alpha}$ is the reduced mass of the Λ-α pair:

$$\mu_{\Lambda\alpha}=\frac{m_\alpha m_\Lambda}{m_\alpha+m_\Lambda}=\frac{4m_\Lambda}{4+m_\Lambda/m_N} \quad (2)$$

(m_Λ and m_N being the masses of the Λ particle and the nucleon, respectively) and $V_{\Lambda\alpha}$ the Λ-α potential given by the folding integral

$$V_{\Lambda\alpha}(r)=4\int V_{\Lambda N}(|\vec{r}-\vec{r}\,'|)\rho(|\vec{r}\,'|)d\vec{r}\,'\ . \quad (3)$$

In this expression $V_{\Lambda N}$ is the Λ-nucleon potential (taken to be central) and ρ the normalized to unity "body-density distribution" of the α particle, measured from the center of mass of the α.

Some interesting calculational details on a variational derivation of Eq. (1) are exhibited in the Appendix of Ref. 7, the relevant indications having been made to the authors of this reference by Bodmer. It turns out that Eq. (1) with $V_{\Lambda\alpha}$ given by Eq. (3) is the Euler-Lagrange equation if a trial wave function of the product type, namely $\Psi = X(\vec{s}_1, \vec{s}_2, \vec{s}_3)\varphi(r)$ (where $\vec{s}_1, \vec{s}_2, \vec{s}_3$ are the internal coordinates of ^{4}He), is assumed.

By taking into account the known value of B_Λ ($B_\Lambda \simeq 3.1$ MeV),[17] Eq. (1) can be used in order to obtain information on the strength of the Λ-N interaction (if a suitable range is chosen, appropriate for example, to 2π or the K exchange mechanism), as was done originally by Dalitz and Downs,[2] who varied the strength of the (Gaussian) Λ-N potential, until B_Λ in Eq. (1) became equal to the experimental value. The same (or similar) procedure has been used for the determination of the central part of the Λ-α interaction.[3,4,6,7]

III. Λ-NUCLEON POTENTIALS AND DENSITY DISTRIBUTIONS FOR THE α PARTICLE

Among the various shapes for the Λ-N interaction the Gaussian one suggests itself because of the considerable computational advantages. Instead, however, of using a single Gaussian, a sum of two at least Gaussian terms seems more appropriate, since in this way a short-range repulsion in the Λ-N potential may be included:

$$V_{\Lambda N}(r) = \sum_{i=1}^{N} V_i \exp(-r^2/a_i^2) \ . \tag{4}$$

On the other hand, a density distribution of Gaussian shape for the α particle, which is consistent with the oscillator shell model, may also be used. Such a density, however, although it reproduces the charge form factor $F_{\mathrm{ch}}(q)$ of the ^{4}He nucleus in the region of low momentum transfers q well, cannot reproduce the values of F_{ch} at higher q. As is well known,[15] there is a diffraction minimum in the charge form factor around $q^2 \simeq 10$ fm^{-2}. It appears, therefore, more appropriate to assume that the density distribution as well, is given by a sum of Gaussian terms

$$\rho(r) = \sum_{j=1}^{K} p_j \exp(-r^2/A_j^2). \tag{5}$$

With expressions (4) and (5) the convolution integral (3) can be calculated analytically in a very straightforward manner, as has been done in the analogous nuclear cases by using the well-known property that the convolution of two normalized Gaussians is a normalized Gaussian. The result for the Λ-α potential is the following:

$$V_{\Lambda\alpha}(r) = 4\pi^{3/2} \sum_{i=1}^{N} \sum_{j=1}^{K} V_i p_j (a_i^{-2} + A_j^{-2})^{-3/2} \times \exp[-r^2/(a_i^2 + A_j^2)] \ . \tag{6}$$

In the present paper we use Λ-nucleon potentials of single and double Gaussian form

$$V(r) = V_1 \exp\left[-\frac{r^2}{a_1^2}\right] + V_2 \exp\left[-\frac{r^2}{a_2^2}\right], \tag{7}$$

where V_2 is equal to or different from zero.

Apart from the potentials of Dalitz and Downs[2] which consist of a single Gaussian and will be denoted by DDI and DDII depending on whether the range corresponds to 2π or the K exchange mechanism, respectively, we use the potentials GI and GII of Gibson *et al.*[7] and also other potentials. The potential GI is again of single Gaussian shape, one difference from DI being that the range parameter has a slightly different value (in addition to a difference in the density and the experimental value of B_Λ which was used). The potential GII is of double Gaussian shape. Its repulsive part is assumed to have the same parameters as one of the Volkov-type potentials for the nucleon-nucleon interaction. By choosing also the value $a_1 = 1.21$ fm for the range parameter, Gibson *et al.* determined the value of $V_1 = -82.7$ MeV so that the empirical value of $B_\Lambda(^5_\Lambda\mathrm{He})$ is reproduced in solving Eq. (1). The density distribution they assumed was, as by Dalitz and Downs, of Gaussian shape, although the range parameter was somehow different.

The two potentials b and c of Bassichis and Gal we have also used were determined in Ref. 18. The range parameters a_1 and a_2 where chosen so that they give the intrinsic range parameters corresponding to two-pion exchange and K-meson exchange, respectively, while the values of the strengths were fixed by solving the S-wave scattering problem and comparing the results with the low-energy scattering data of Alexander *et al.*[19] It should be noted that the self-consistent calculations of Bassichis and

Gal have shown that with potential b, for example, a considerable overbinding of the Λ particle in ${}^{5}_{\Lambda}\mathrm{He}$ is observed.

We used in addition the potential of Ho and Volkov,[20] which is given by $V=\frac{1}{4}V_s+\frac{3}{4}V_t$, where the parameters of singlet and triplet parts of the potential (again of double Gaussian shape) were determined in a way similar to that adopted by Herndon and Tang[21] for their exponential with hard core potentials.

Three more Λ-nucleon potentials we have used are those reported recently by Zofka and determined by Sotona.[22] Two of them are of double Gaussian shape and the depths were chosen in a simple independent particle model so as to give for the Λ-binding energies in ${}^{17}_{\Lambda}\mathrm{O}$ and ${}^{41}_{\Lambda}\mathrm{Ca}$ the values 15 MeV (with harmonic oscillator constant $\alpha_\Lambda=0.58$ fm^{-1}) and 26.2 MeV (with $\alpha_\Lambda=0.52$ fm^{-1}), respectively. It should be noted that potentials $S\Lambda4$ and (particularly) $S\Lambda5$ are characterized by a very strong repulsion at short distances.

We have finally used two recent potentials of Verma and Sural,[14] which are again of double Gaussian shape. Their parameters were determined so as to reproduce the scattering lengths and effective ranges of Nagel's model B potential.[23] Among these two potentials, the potential three reproduces the experimental value of the Λ-binding energy in the hypertriton with the method of Ref. 14.

The parameters of all the potentials discussed above are given in Table I. In the same table the corresponding "readjusted" values of the strength of the attractive part of the Λ-nucleon potential, that is, the values which reproduce the experimental value of $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})\simeq3.1$ MeV with the rigid-core model, are also given. We should bear in mind that there is a small difference in certain values of V_1, which are expected to be the same with those reported by other authors. This is mainly due to the experimental value for $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ they consider. For example, Dalitz and Downs[2] use $B_\Lambda=2.9$ MeV and Gibson *et al.*[7] $B_\Lambda=3.08$ MeV. In the second column of this table the corresponding density distributions of ${}^{4}\mathrm{He}$ (discussed below), which are needed for the computation of the folding integral (3) are also given. In order to obtain the Λ-α potential in which we are interested, it is advisable to use these readjusted values of V_1. A number of the potentials discussed above are plotted in Fig. 1. They differ both in the short-range repulsion and in the attractive part.

Regarding the nucleon density distribution (assumed to be the same as the proton one), only a single Gaussian has been used to our knowledge. It is one of the main purposes of this paper to investigate the effect of a more general density distribution of double Gaussian (DG) form to the shape of the Λ-α potential. This is the following:

$$\rho_{\mathrm{DG}}(r)=\frac{\pi^{-3/2}}{1-6g}\left\{\frac{1-3g}{(b_1\sqrt{3/4})^3}\exp\left[-\frac{r^2}{(b_1\sqrt{3/4})^2}\right]-\frac{3g}{(b_1\sqrt{1/4+g^{2/3}/2})^3}\times\exp\left[-\frac{r^2}{(b_1\sqrt{1/4+g^{2/3}/2})^2}\right]\right\} \tag{8}$$

where $g=(1+2\gamma^{-2/3})^{-3/2}$ and $\gamma=(a/b_1)^3$.

This density was obtained in Ref. 24 by Fourier transforming the form factor of ${}^{4}\mathrm{He}$ in first-order Born approximation, derived originally by Czyz and Lesniak[25] (see also Refs. 26 and 27) by assuming a Jastrow-type many-body wave function

$$\Psi=\left[\prod_{i<j}f(r_{ij})\right]\Phi(r) \tag{9}$$

with a simple correlation function of the type

$$f(r)=[1-\exp(-r^2/a^2)]^{1/2}\ ,\quad 0<r<\infty\ , \tag{10}$$

and harmonic oscillator orbitals in the Slater determinant Φ. In deriving (8) the independent-pair approximation of the cluster expansion of the form factor was used.

The values of the oscillator parameter b_1 and the correlation parameter a were determined by fitting the corresponding expression of the charge form factor to the experimental values of Refs. 15 and 16, in which the more recent measurements at high momentum transfers have been given. The correction for the center of mass motion was taken into account by means of the usual Tassie and Barker factor[28] and the correction due to the proton charge form factor by means of the phenomenological expression:

$$f_p=A_{p_1}e^{-a_{p_1}{}^2q^2/4}-(A_{p_1}-1)e^{-a_{p_2}{}^2q^2/4}\ . \tag{11}$$

The free parameters were determined in Ref. 24 by fitting to the experimental results[29] (in the range $0<q^2<62$ fm^{-2}) and their values are the following: $a_{p_1}=0.721\,99$ fm, $a_{p_2}=0.352\,46$ fm, and $A_{p_1}=0.633\,87$.

TABLE I. Parameters of Λ-nucleon potentials.

Potential	Density	V_1 (Original) MeV	V_1 (Readjusted) MeV	a_1 (F)	V_2 (MeV)	a_2 (F)
Dalitz-Downs I	ρ_{GI}	−37.60	−38.15	1.0337		
(DDI)	ρ_{GII}		−39.79	1.0337		
(Ref. 2)	ρ_{DG}		−37.40	1.0337		
Dalitz-Downs II	ρ_{GI}	−155.78	−158.27	0.5907		
(DDI)	ρ_{GII}		−167.46	0.5907		
(Ref. 2)	ρ_{DG}		−156.29	0.5907		
Bassichis and Gal	ρ_{GI}	−55.2	−40.67	1.0310	9.2	0.5908
b	ρ_{GII}		−42.21	1.0310	9.2	0.5908
(Ref. 18)	ρ_{DG}		−39.82	1.0310	9.2	0.5908
Bassichis and Gal	ρ_{GI}	−59.8	−44.98	1.0310	27.6	0.5908
c	ρ_{GII}		−46.55	1.0310	27.6	0.5908
(Ref. 18)	ρ_{DG}		−44.20	1.0310	27.6	0.5908
Gibson *et al.* I	ρ_{GI}	−38.2	−36.78	1.05		
(GI)	ρ_{GII}		−38.34	1.05		
(Ref. 7)	ρ_{DG}		−36.05	1.05		
Gibson *et al.* II	ρ_{GI}	−82.7	−82.23	1.21	145.0	0.82
(GII)	ρ_{GII}		−82.87	1.21	145.0	0.82
(Ref. 7)	ρ_{DG}		−81.79	1.21	145.0	0.82
Ho-Volkov	ρ_{GI}	−64.75	−57.46	1.2	69.75	0.85
(H-V)	ρ_{GII}		−58.25	1.2	69.75	0.85
(Ref. 20)	ρ_{DG}		−56.93	1.2	69.75	0.85
Sotona $S\Lambda 1$	ρ_{GI}	−25.25	−27.14	1.2		
(reported by Zofka)	ρ_{GII}		−28.18	1.2		
(Ref. 22)	ρ_{DG}		−26.60	1.2		
Sotona $S\Lambda 4$	ρ_{GI}	−203.2	−212.28	0.8	949.6	0.4
(reported by Zofka)	ρ_{GII}		−214.73	0.8	949.6	0.4
(Ref. 22)	ρ_{DG}		−210.58	0.8	949.6	0.4
Sotona $S\Lambda 5$	ρ_{GI}	−878.8	−903.08	0.6	5432.0	0.3
(reported by Zofka)	ρ_{GII}		−908.34	0.6	5432.0	0.3
(Ref. 22)	ρ_{DG}		−898.73	0.6	5432.0	0.3
Verma and Sural	ρ_{GI}	−60.78	−50.47	1.25	71.31	0.82
potential 2	ρ_{GII}		−51.15	1.25	71.31	0.82
(Ref. 14)	ρ_{DG}		−50.01	1.25	71.31	0.82
Verma and Sural	ρ_{GI}	−141.1425	−128.34	1.10	197.96	0.82
potential 3	ρ_{GII}		−129.19	1.10	197.96	0.82
(Ref. 14)	ρ_{DG}		−127.75	1.10	197.96	0.82

In the theoretical expressions for $F_{ch}(q)$ the so-called Darwin-Foldy[30,31] factor which is usually given by $f_{DF} \simeq 1-(1/8m^2)q^2$ had been taken into account in the Chandra and Sauer[32] approximation

$$f_{DF} \simeq \exp(-q^2/8m^2) . \qquad (12)$$

The best fit values of the parameters obtained in the way described previously are the following: $b_1 = 1.2008$ fm and $a = 0.73295$ fm, and the overall fit is satisfactory.[24] With these values expression (8) takes the form

$$\rho_{DG} = 0.20726 \exp[-(r/1.0399)^2] - 0.16410 \exp[-(r/0.6883)^2] . \qquad (13)$$

Other double-Gaussian densities for ^{4}He have been considered repeatedly in the past.[33–37]

One should be reminded that unlike the situation with the single Gaussian density distribution, the double Gaussian one (13) has a rather pronounced dip at the origin and this feature is expected to influence the shape of the Λ-α potential. The density ρ_{DG} is plotted in Fig. 2 together with two single

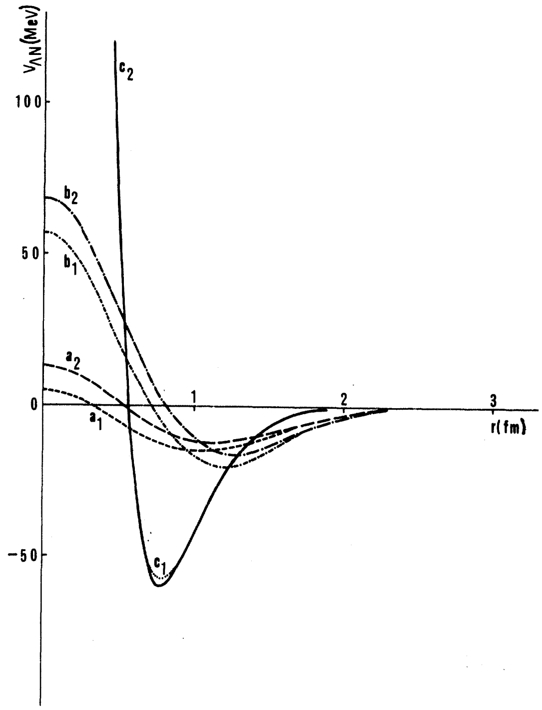

FIG. 1. Plots of certain Λ-nucleon potentials: Ho-Volkov (a), Verma *et al.* (set 3) (b), Sotona $S\Lambda4$ (reported by Zofka) (c). The odd-numbered are the original while the even numbered the readjusted ones (with the density $\rho_{\rm DG}$) (see Sec. III).

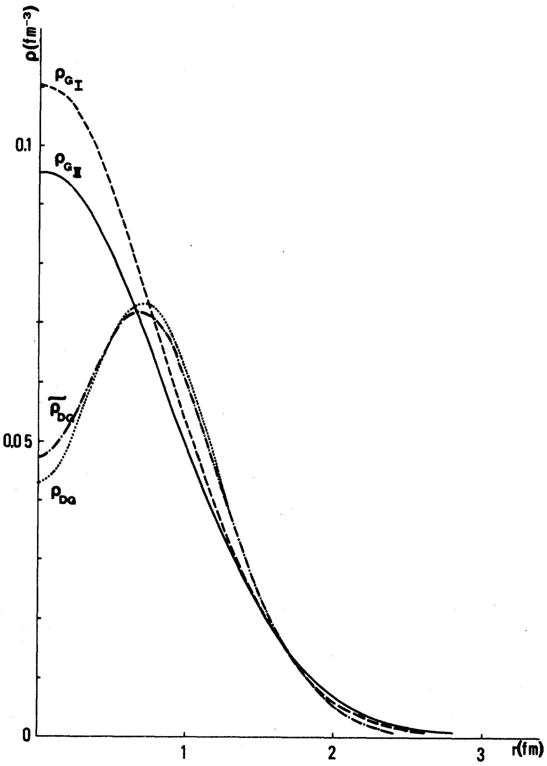

FIG. 2. Normalized to unity "body-density" distributions for the α particle, measured from the center of mass of the α. $\rho_{\rm GI}$ is the one used by Downs and Dalitz, $\rho_{\rm GII}$ that used by Gibson *et al.*, while $\rho_{\rm DG}$ and $\tilde{\rho}_{\rm DG}$ are those discussed in Sec. III.

Gaussian densities which have been used in the past. These densities are the one used by Dalitz and Downs[2]:

$$\rho_{\rm GI}=0.110\,49\exp[-(r/1.176)^2]\ , \tag{14}$$

and the one used by Gibson *et al.*,[7]

$$\rho_{\rm GII}=0.095\,57\exp[-(r/1.234)^2]\ . \tag{15}$$

In the same figure the density distribution $\tilde{\rho}_{\rm DG}$ is also plotted. The only difference between $\tilde{\rho}_{\rm DG}$ and $\rho_{\rm DG}$ is that in deriving this density the Darwin-Foldy factor was not included in the expression of the form factor fitted to the data. The expression for $\tilde{\rho}_{\rm DG}$ is

$$\tilde{\rho}_{\rm DG}=0.195\,42\exp[-(r/1.0552)^2] \\ -0.148\,29\exp[-(r/0.6960)^2]\ , \tag{16}$$

since $b_1=1.2184$ fm and $a=0.731\,17$ fm.

It is seen from Fig. 2 that it is quite similar to $\rho_{\rm DG}$, although somehow less deep at the origin and with a slightly bigger maximum. It should be clear that all the above density distributions are body density distributions, that is, of point protons (measured from the center of mass of the α). The corresponding charge density distributions have different shapes. They have neither a sharp maximum at the origin as do the $\rho_{\rm GI}$ or $\rho_{\rm GII}$ nor a dip like $\rho_{\rm DG}$, but they are flat in the region of small r, where their bigger values are.

IV. RESULTS AND DISCUSSION

In this section we report and discuss the results for the Λ-α potentials obtained with the Λ-nucleon potentials and density distributions for the α particle given in the previous section.

We may first remark that the Λ-α potentials considered in this paper can be written for $K=2$ in the form

$$V_{\Lambda\alpha}=\sum_{i=1}^{2N} C_i e^{-r^2/d_i^2}\ , \tag{17}$$

where the constants C_i and d_i are given by the following expressions:

$$C_i=\begin{cases}4\pi^{3/2}V_i p_1(a_i{}^{-2}+A_1{}^{-2})^{-3/2}\,, & \text{for } i=1,2,\ldots,N\\ 4\pi^{3/2}V_{i-N}p_2(a_{i-N}{}^{-2}+A_2{}^{-2})^{-3/2}\,, & \text{for } i=N+1,\ldots,2N\end{cases} \tag{18}$$

and

$$d_i=\begin{cases}(a_i{}^2+A_1{}^2)^{1/2}\,, & \text{for } i=1,2,\ldots,N\\ (a_{i-N}{}^2+A_2{}^2)^{1/2}\,, & \text{for } i=N+1,\ldots,2N\,.\end{cases} \tag{19}$$

TABLE II. Parameters of Λ-α potentials.

Potential	Density	C_1 (MeV)	d_1 (F)	C_2 (MeV)	d_2 (F)	C_3 (MeV)	d_3 (F)	C_4 (MeV)	d_4 (F)
Dalitz-Downs I	ρ_{GI}	−43.93	1.5656						
(DDI)	ρ_{GII}	−42.14	1.6098						
	ρ_{DG}	−68.03	1.4663	25.70	1.2419				
Dalitz-Downs II	ρ_{GI}	−57.28	1.3158						
(DDII)	ρ_{GII}	−53.92	1.3681						
	ρ_{DG}	−97.76	1.1960	51.45	0.9070				
Bassichis and Gal	ρ_{GI}	−46.62	1.5638	3.44	1.3194				
b	ρ_{GII}	−44.50	1.6080	2.96	1.3681				
	ρ_{DG}	−72.16	1.4644	5.76	1.1960	27.30	1.2396	−3.03	0.9070
Bassichis and Gal	ρ_{GI}	−51.57	1.5638	9.99	1.3158				
c	ρ_{GII}	−49.08	1.6080	8.89	1.3681				
	ρ_{DG}	−80.09	1.4644	17.27	1.1960	30.31	1.2396	−9.09	0.9070
Gibson *et al.* I	ρ_{GI}	−43.48	1.5764						
(GI)	ρ_{GII}	−41.74	1.6203						
	ρ_{DG}	−67.14	1.4778	25.13	1.2555				
Gibson *et al.* II	ρ_{GI}	−121.34	1.6872	108.57	1.4335				
(GII)	ρ_{GII}	−113.76	1.7283	98.33	1.4816				
	ρ_{DG}	−185.23	1.5955	178.70	1.3243	64.01	1.3920	−77.65	1.0706
Ho-Volkov	ρ_{GI}	−83.76	1.6800	56.11	1.4508				
(H-V)	ρ_{GII}	−78.95	1.7213	50.93	1.4984				
	ρ_{DG}	−127.57	1.5879	91.79	1.3431	44.28	1.3834	−39.02	1.0937
Sotona $S\Lambda 1$	ρ_{GI}	−39.57	1.6800						
(reported by Zofka)	ρ_{GII}	−38.19	1.7213						
	ρ_{DG}	−59.61	1.5879	20.69	1.3834				
Sotona $S\Lambda 4$	ρ_{GI}	−151.16	1.4221	126.91	1.2419				
(reported by Zofka)	ρ_{GII}	−138.26	1.4706	111.37	1.2972				
	ρ_{DG}	−247.83	1.3120	228.11	1.1142	109.32	1.0553	−143.57	0.7960
Sotona $S\Lambda 5$	ρ_{GI}	−339.25	1.3200	328.36	1.2134				
(reported by Zofka)	ρ_{GII}	−303.79	1.3721	286.44	1.2699				
	ρ_{DG}	−582.36	1.2006	600.55	1.0823	303.90	0.9131	−412.96	0.7508
Verma and Sural	ρ_{GI}	−78.03	1.7161	53.39	1.4335				
potential 2	ρ_{GII}	−73.74	1.7565	48.36	1.4816				
	ρ_{DG}	−117.95	1.6260	87.88	1.3243	40.06	1.4270	−38.19	1.0706
Verma and Sural	ρ_{GI}	−163.70	1.6101	148.23	1.4335				
potential 3	ρ_{GII}	−152.25	1.6531	134.24	1.4816				
	ρ_{DG}	−254.50	1.5138	243.97	1.3243	92.75	1.2976	−106.01	1.0706

When the density distribution is of a single Gaussian form ($K=1$), the coefficients C_i for $i=N+1,\ldots,2N$ are zero.

The values of C_i and d_i for each of the three densities ρ_{GI}, ρ_{GII}, and ρ_{DG}, and each of the various Λ-nucleon potentials, are given in Table II. The values of V_1, readjusted so that the experimental value of $B_\Lambda({}^5_\Lambda\mathrm{He})\simeq 3.1$ MeV is reproduced with the rigid-core model, were used in obtaining the results of this table. If the original Λ-nucleon potentials were used, the $V_{\Lambda\alpha}$ would not be consistent in general with the Λ-α rigid-core model (with the exception, of course, of those for which the V_1 was adjusted originally with this model). It also does not appear that these Λ-N potentials reproduce (at least most of them do not) the experimental value of the binding energy of the Λ in ${}^5_\Lambda$He if the calculations are performed with other models.

Some of the derived $V_{\Lambda\alpha}$ potentials are shown in Figs. 3–5. It is seen from these figures that for the potential DDI, which is of single Gaussian shape,

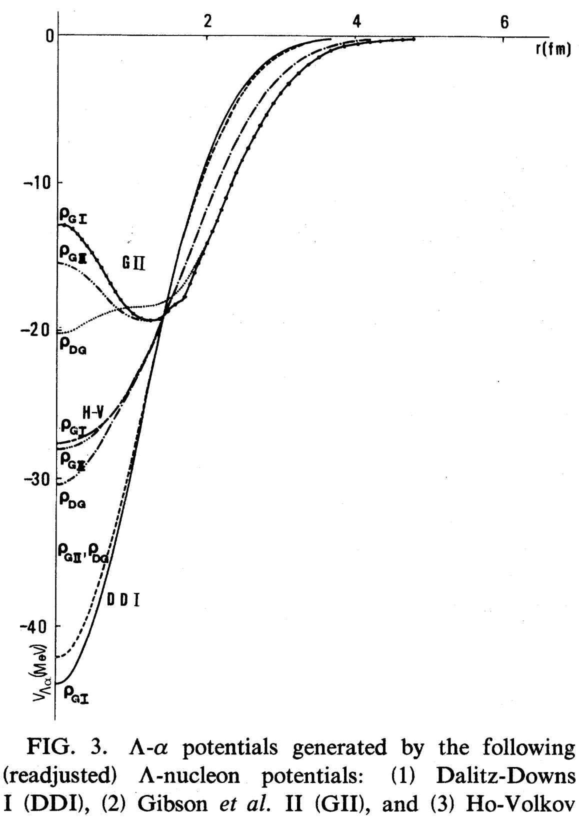

FIG. 3. Λ-α potentials generated by the following (readjusted) Λ-nucleon potentials: (1) Dalitz-Downs I (DDI), (2) Gibson *et al.* II (GII), and (3) Ho-Volkov (H-V). The various $V_{\Lambda\alpha}$ curves corresponding to the "same" Λ-nucleon potential (that is, with the same parameters apart from the readjusted value of V_1) originate from the different densities used for the α particle (indicated on each curve).

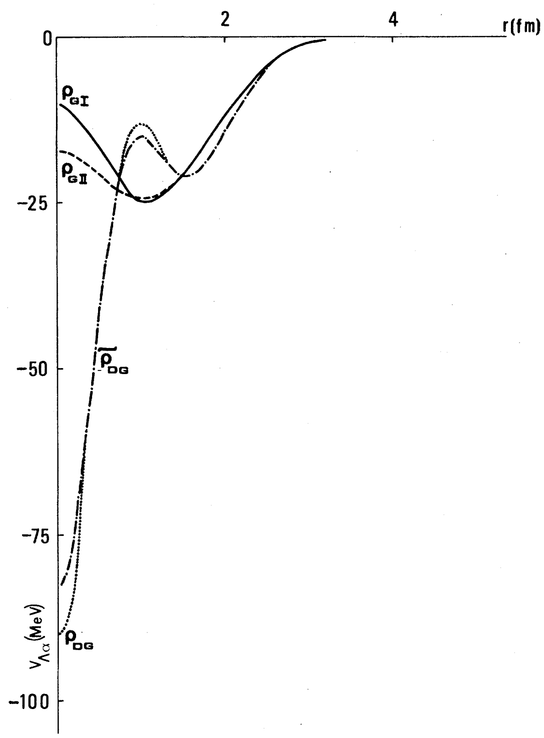

FIG. 4. $V_{\Lambda\alpha}$ potentials derived from the (readjusted) $V_{\Lambda N}$ potential $S\Lambda5$ of Sotona (reported by Zofka). The various curves correspond to the densities used for the α particle.

the difference in the $V_{\Lambda\alpha}$ values due to the various densities is quite small, mainly for large values of r. The same holds for the remaining ΛN potentials of single Gaussian shape. This behavior changes, however, in the case of ΛN potentials of double Gaussian shape. The differences are small at sufficiently large values of the Λ-α separation distance r, but at smaller values of r, larger differences appear, particularly in the case where ρ_{DG} is used. It is a common feature of almost all the $V_{\Lambda\alpha}$ potentials generated by means of the double Gaussian density and double Gaussian Λ-N potentials (having a short range repulsion), to have deeper values near the origin compared with the corresponding values of the $V_{\Lambda\alpha}$, obtained with the same Λ-nucleon potential, but with the single Gaussian densities. This effect is more pronounced in the cases of the potentials $S\Lambda4$ and $S\Lambda5$ which have a repulsion much stronger than has been customary so far.

Different Λ-α potentials are derived if the original values of V_1 in the Λ-nucleon potentials are used instead of the readjusted ones, as was pointed out. The magnitude of the difference (for a particular value of r) depends on the specific Λ-nucleon

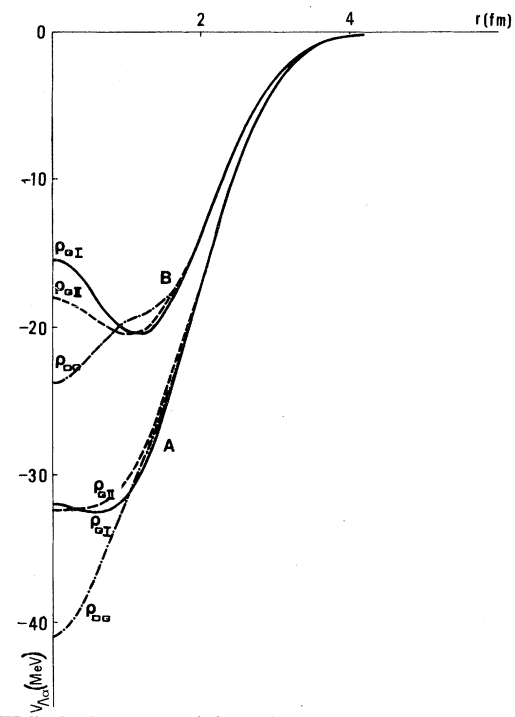

FIG. 5. $V_{\Lambda\alpha}$ potentials derived by using either the original Λ-N potential of Verma *et al.* (set 3) (A), or the readjusted one (B). The different curves derived with each of them correspond to the densities used for the α particle.

potential and on the density which was used in computing the folding integral (3). For purposes of comparison we also give the parameters of the Λ-α potentials derived from the original Λ-nucleon ones. These parameters are obviously the same as those obtained with the readjusted V_1 apart from C_1 and C_2 (or C_3), as one can see from expression (18). They are given in Table III and are denoted by $\widetilde{C}_1$, $\widetilde{C}_2$, and $\widetilde{C}_3$, respectively. Small differences in their values from the corresponding values of Table I, in cases where we would expect there to be no differences, are mainly due to small differences in the experimental values of $B_\Lambda({}^5_\Lambda\text{He})$ which were used.

It follows from the values given in Table I that the difference $|[V_1 \text{ (original)} - V_1 \text{ (readjusted)}]|$ varies considerably, its maximum value being 28% of the value of $|V_1 \text{ (original)}|$ (corresponding to the case of the Bassichis and Gal b Λ-nucleon potential and the ρ_{DG}). These differences, however, which for some potentials are quite small, may lead to much larger differences in the values of $V_{\Lambda\alpha}$. This situation is illustrated in Fig. 5, in which plots are given of the two types of $V_{\Lambda\alpha}$ potentials derived from the original and readjusted Λ-nucleon potential of Verma and Sural (set 3) and the various densities. The difference in the values $|V_{\Lambda\alpha} \text{ (original)} - V_{\Lambda\alpha} \text{ (readjusted)}|$ at the origin, obtained with the ρ_{GI}, ρ_{GII}, and ρ_{DG}, are 51%, 44%, and 42%, respectively, of the value of $|V_{\Lambda\alpha}(0) \text{ (original)}|$. The differences, however, for the other cases (apart from the $S\Lambda5$ with the ρ_{GII}, for which it is too big) are smaller. For example, for the GII and the Ho-Volkov potentials with the ρ_{DG}, the corresponding percentages are 6% and 27%, respectively.

It must be clear that the $V_{\Lambda\alpha}$ potentials given in this paper should not be considered as representing in general a sufficiently satisfactory approximation of the central part of the actual $V_{\Lambda\alpha}$ interaction, although an effort has been made for a partial improvement in comparison with older ones. The present work should rather be considered mainly as an attempt to investigate certain ambiguities in the determination of the central part of $V_{\Lambda\alpha}$. The results reported in this section show that the well-known incomplete knowledge of the Λ-nucleon potential, coupled with the uncertainties about the shape of the density distribution at short distances from the center of mass of the α, lead in certain cases to considerable differences in the values of $V_{\Lambda\alpha}$, mainly at very small r.

Among the ΛN potentials used, those consisting of two Gaussian terms and having a short range repulsion should be considered more appropriate. On the other hand, a double Gaussian density distribution like the ρ_{DG} which was employed appears preferable in view of the more recent measurements of the charge form factor of ^{4}He. It should be clear, however, that the problem of the accurate determination of the density distribution of the α is a difficult one and ambiguities exist, mainly at small r, because of the serious approximations involved in the theoretical expression of the form factor (in which, however, there are two adjustable parameters), and also because of the restriction of the measurements in a limited range of values of momentum transfer (although this is now quite wide) and errors in them. As an illustration (which does not appear, however, to lead to an important ambiguity), we might consider the inclusion (or not) of the Darwin-Foldy factor in the theoretical expression of the charge form factor F_{ch}. This is a small correction unless the value of the momentum transfer is very high. As was pointed out in Sec. II, the density which results by omitting this factor (as has been done in many analyses so far), without changing anything else in the expression of F_{ch}, leads to the density $\tilde{\rho}_{\text{DG}}$ which is plotted in Fig. 2. It is seen that there is a difference, although small

TABLE III. Parameters of $V_{\Lambda\alpha}$ potentials differing from those of Table II because of the use of the original values of V_1.

Potential	Density	$\tilde{C}_1$ (MeV)	$\tilde{C}_2$ (MeV)	$\tilde{C}_3$ (MeV)
Dalitz-Downs I (DDI)	ρ_{GI}	-43.29		
	ρ_{GII}	-39.83		
	ρ_{DG}	-68.40	25.84	
Dalitz-Downs II (DDII)	ρ_{GI}	-56.38		
	ρ_{GII}	-50.16		
	ρ_{DG}	-97.44	51.28	
Bassichis and Gal	ρ_{GI}	-63.28		
b	ρ_{GII}	-58.20		
	ρ_{DG}	-100.01		37.84
Bassichis and Gal	ρ_{GI}	-68.55		
c	ρ_{GII}	-63.05		
	ρ_{DG}	-108.35		41.00
Gibson *et al.* I (GI)	ρ_{GI}	-45.16		
	ρ_{GII}	-41.58		
	ρ_{DG}	-71.13	26.63	
Gibson *et al.* II (GII)	ρ_{GI}	-122.03		
	ρ_{GII}	-113.53		
	ρ_{DG}	-187.28		64.72
Ho-Volkov	ρ_{GI}	-94.39		
(H-V)	ρ_{GII}	-87.76		
	ρ_{DG}	-145.08		50.36
Sotona $S\Lambda 1$	ρ_{GI}	-36.81		
(reported by Zofka)	ρ_{GII}	-34.22		
	ρ_{DG}	-56.58	19.64	
Sotona $S\Lambda 4$	ρ_{GI}	-144.69		
(reported by Zofka)	ρ_{GII}	-130.84		
	ρ_{DG}	-239.15		105.49
Sotona $S\Lambda 5$	ρ_{GI}	-330.13		
(reported by Zofka)	ρ_{GII}	-293.91		
	ρ_{DG}	-569.45		267.16
Verma and Sural	ρ_{GI}	-93.96		
potential 2	ρ_{GII}	-87.62		
	ρ_{DG}	-143.36		48.69
Verma and Sural	ρ_{GI}	-180.03		
potential 3	ρ_{GII}	-166.34		
	ρ_{DG}	-281.18		102.47

compared to the ρ_{DG}. This difference can lead to a difference of about 8% in the value of $V_{\Lambda\alpha}$ at the origin if one is prepared to accept Λ-nucleon potentials with very strong repulsion at the origin like the $S\Lambda 5$. With this potential the values of the (readjusted) Λ-α potentials to be compared are

$$V_{\Lambda\alpha}^{\rho_{DG}}(0) = -91 \text{ MeV}$$

and

$$V_{\Lambda\alpha}^{\tilde{\rho}_{DG}}(0) = -84 \text{ MeV} .$$

The difference is, however, negligible for Λ-nucleon potentials with a very soft core like the Ho-Volkov one. With this potential the corresponding numbers are

$$V_{\Lambda\alpha}^{\rho_{DG}}(0) = -30.5 \text{ MeV} ,$$

and

$$V_{\Lambda\alpha}^{\tilde{\rho}_{DG}}(0) = -30.4 \text{ MeV} .$$

In view of all the results reported in this paper, it appears that in order to make progress in the determination of the Λ-α potential it would be desirable to have a better knowledge of the density distribution of the α particle and in particular of the Λ-nucleon potential.

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