Experimental evidence in support of triaxial shape of ^{150,152,154}Sm nuclei

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Theoretical $B(E2)$ values and branching ratios in respect to ^{150,152,154}Sm nuclei have been calculated recently using adiabatic time-dependent Hartree-Fock methods which are in poor agreement with experiment. The pairing-plus-quadrupole model of Kumar had already been a failure in describing samarium nuclei. Therefore the asymmetric rotor model has been employed to explain known individual $B(E2)$ values, $B(E2)$ branching ratios, and low lying energy levels. The comparison of phenomenological model results with those of microscopic model calculations has been done only to show the better fit achieved in the present work. The nonaxial parameter (γ) has been calculated by existing methods, viz., using the energy ratios $E2^{+'}/E2^{+}$, $E4^{+}/E2^{+}$, and $E2^{+'}/E6^{+}$ or $E4^{+}$ and with new approach feeding $E2^{+}$ and model dependent intrinsic quadrupole moments (Q_0). The asymmetric rotor model estimates are the only ones which stand by the simple test of Kumar ($0.5 <$ enhancement/hindrance factor $F < 2.0$). It appears that the mass number A and charge number Z play important roles in assigning the shape of the nucleus in terms of the nonaxial parameter γ.

[NUCLEAR STRUCTURE ^{150,152,154}Sm, rigid rotor model, nonaxial parameter γ, calculated low lying energy levels, $B(E2)$'s, branching ratios, compared with experiment and other models.]

INTRODUCTION

The study of samarium isotopes has been a challenging theoretical problem, since they lie in the range from near-spherical to well-deformed shapes. The ^{148}Sm nucleus is believed to be basically spherical, while ^{154}Sm is thought to be a well deformed nucleus, and ^{150,152}Sm are transitional nuclei. Various approaches[1-3] have been tried so far but none of them has been found to be fully successful in explaining the known $B(E2)$ values of interband and intraband transitions. Since the asymmetric rotor model (ARM) of Davydov and Filippov (DF) (Ref. 4) seems to be particularly useful for nuclei in the transition region between rotational and near harmonic modes of collective excitation, it has been thought worthwhile to study ^{150,152,154}Sm nuclei in the framework of the rigid rotor model. Though it is not proper to compare the phenomenological DF model on an equal footing with the microscopic models of Tamura *et al.*[1] and Kumar,[2] even then, it is permissible to emphasize the better fit achieved by the DF model over the other existing models. In previous papers of the authors[5] $B(E2)$ values for the transitions depopulating the 2^+ state of the gamma vibrational band and for the inter-rotational band in deformed even nuclei were evaluated. The results shown thereof have already contradicted Zawischa's viewpoint[6] in general and confirmed that low lying $K^\pi = 2^+$ resonances were classical gamma vibrations for samarium nuclei. The anomalous ^{152}Sm nucleus (having neutron number 90) lays in the vicinity of the experimental trend of $B(E2;2^{+'} \rightarrow 2^+/0^+)$ vs γ which was fully endorsed in the DF trend.[5] Again it is encouraging to quote Puri *et al.*[7] regarding properties of the $2^{+'}$ level in the $150 < A < 190$ region that the experimental δ value ($E2/M_1$ mixing ratio, which provides rather a sensitive test of nuclear wave functions) favored the nonaxial model calculations of Davydov and Filippov over the pairing force model of Greiner[8] or the Coriolis interaction model of Bes *et al.*[9] Furthermore, the reduced electric quadrupole transition probability (in e^2b^2 units) from the first 2^+ state changes very rapidly, from 0.141 to 0.274 between ^{148}Sm and ^{150}Sm, and from 0.274 to 0.657 between ^{150}Sm and ^{152}Sm. This indicates a rapid change in the average γ value, as can be seen from the expression for the quadrupole moment of the first 2^+ state of a triaxial nucleus (DF)

$$Q_2 = \frac{3ZR^2\beta}{(5\pi)^{1/2}} \times \frac{6\cos(3\gamma)}{7(9-8\sin^2 3\gamma)^{1/2}} .$$

TABLE I. Energy of the first ground state $E2^+$, and of the first and second excited states of the gamma band $E2^{+'}$ and $E3^+$, are listed. They were taken from Ref. 18.

Nucleus	$E2^+$ (keV)	$E2^{+'}$ (keV)	$E2^+ + E2^{+'}$ (keV)	$E3^+$ (keV)
^{148}Sm	550.1	1453.6	2003.7	1902.9
^{150}Sm	333.9	1193.81	1527.76	1504.53
^{152}Sm	121.77	1083.79	1205.56	1233.8
^{154}Sm	82.05	1522.45	1440.4	1540.0

Another point in favor of the ARM is that there exists a relation

$$E2^{+'} + E2^+ \approx E3^+$$

(see Table I) where $E2^+$ is the energy of the first ground state band and $E2^{+'}$ and $E3^+$ are the energies of the first and second excited states of the gamma band.

I. REVIEW OF METHODS FOR CALCULATING γ

The following three methods[4,10,11] have been widely used so far for evaluating the nonaxial parameter (γ) from the ratio of energies:

(a) $E2^{+'}$ and $E2^+$,

(b) $E4^+$ and $E2^+$,

(c) $E2^{+'}$ and nearest ground state level energy.

When method (a) gave an anomalous description of gamma band energies pertaining to the ^{154}Gd nucleus, as shown by Varshni *et al.*, method (b) was suggested with a remark that γ from method (a) leads to the inclusion of many non-DF nuclei.

During the last decade many workers[6,11–13] followed method (b) and commented on $B(E2)$ values of even deformed nuclei in the rare earth and actinide regions. A surprising situation has been brought to attention by Puri *et al.* while inferring the lead of the Davydov-Rostovsky model[14] over the DF model for $B(E2)$ ratios of the cascade to cross-over transition from the 2^+ vibrational band after following method (b), meant for seeking good agreement for DF nuclei in the region $150 < A < 190$. This is contrary to the true situation, since the inclusion of the coupling of rotation with β vibration (as done by Rostovsky) worsens the agreement with experiment.[5] An explanation to the above paradox can be sought upon analyzing the work of Varshni *et al.* Method (b) gives γ a few degrees larger than those values given by method (a). This enhancement in the γ at nearly 15° is equivalent to introducing the Bohr-Mottleson vibration rotation interaction correction (BMVRIC) of the form $-bI^2(I+1)^2$ in the energy value. Therefore if γ were taken from the ground state rotational band, then the energies of the remaining two levels of this band may be predicted to an accuracy well within 1%. But energies of the $2^{+'}, 3^+, 4^{+'}, \ldots,$ levels cannot then be predicted with any precision (Figs. 1 and 3). Therefore even low lying energy levels are not uniformly explained by method (b). Another objectionable outcome of method (b) is that it keeps the ^{150}Sm nucleus out of the DF range, contrary to the fact that it can be the most suitable nucleus (being transitional) for ARM. For the ^{152}Sm nucleus method (b) gives $\gamma = 22°$ and method (a) gives $\gamma = 13°$. The enhancement in γ amounting to 9° (77%) cannot be justified as being equivalent to the rotation vibration interaction term, which can be only a few degrees. Also, in the DF energy diagram the $2^{+'}$ level is brought down to cross the 4^+ level at $\gamma = 21.5°$; hence for $\gamma > 21.5°$, $E4^+$ should be greater than $E2^{+'}$. The experimen-

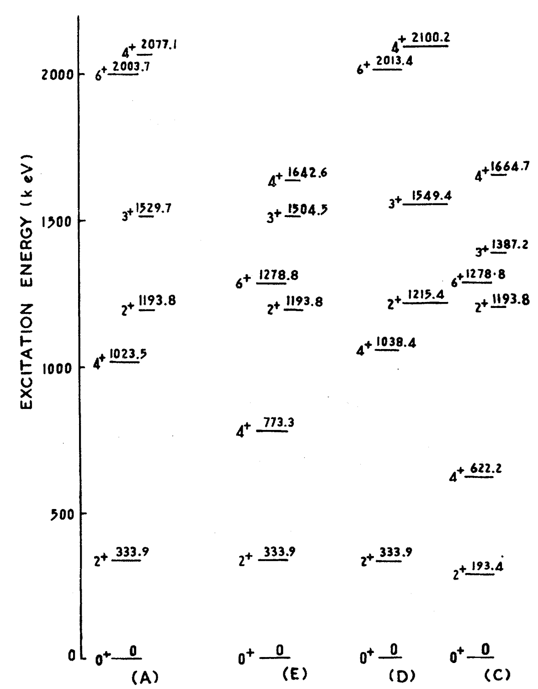

FIG. 1. Energy spectra for the ^{150}Sm nucleus. (E) is the experimental spectrum adapted from Ref. 18. (A), (D), and (C) are a calculated spectrum using γ following methods (a), (d), and (c), respectively. Method (b) is not applicable to this nucleus.

tal values of $E2^{+'}$ (=1193.8 keV) are found to be greater than those of $E4^{+}$ (=773.3 keV), which is inconsistent with the value of γ from method (b). Therefore, method (b) can only be of use if one is confined to ground state energy levels. The use of such γ for evaluating $B(E2)$ values and $B(E2)$ branching ratios, as done by some workers,[7,13] has no justification.

Method (c) of deducing the asymmetric parameter γ from the energy of the $2^{+'}$ state has already been commented on as being unreliable by Baker *et al.*[15]

II. A NEW APPROACH TO EVALUATE γ [METHOD (d)]

For a fixed value of β, violation of axial symmetry of the nucleus leads to an increase of the energy of the levels belonging to the axial nucleus in the DF model. The increase of the level energy corresponds to a decrease of the effective moment of inertia of the nucleus. For the first excited state of spin 2 the effective moment of inertia can be determined from the formula

$$E2^{+}=\frac{6\hbar^2}{2\mathscr{I}_0}\times\left[\frac{9-(81-72\sin^2 3\gamma)^{1/2}}{4\sin^2 3\gamma}\right], \qquad (1)$$

where the inertial parameter according to the general empirical rule[16] is

$$\frac{6\hbar^2}{2\mathscr{I}_0}\approx 1224A^{-7/3}\beta^{-2}\ (\text{MeV}) . \qquad (2)$$

The reduced transition probabilities $B(E2;I_i\to I_f)$ are functions of Q_0 and expressed as

$$B(E2;2^{+}\to 0^{+}) =\frac{e^2Q_0{}^2}{32\pi}\times\left[1+\frac{3-2\sin^2 3\gamma}{(9-8\sin^2 3\gamma)^{1/2}}\right]$$

and

$$B(E2;2^{+'}\to 0^{+}) =\frac{e^2Q_0{}^2}{32\pi}\times\left[1-\frac{3-2\sin^2 3\gamma}{(9-8\sin^2 3\gamma)^{1/2}}\right] . \qquad (3)$$

The above equations allow us to express Q_0 in terms of the easily observable transition probabilities $B(E2;I_i\to I_f)$, and provide us with a model dependent method of determining the intrinsic quadrupole moment Q_0. If the deformation can be characterized by a single parameter β, then Q_0 is approximately given by[17]

$$Q_0=3ZR^2\beta/(5\pi)^{1/2} . \qquad (4)$$

Using the above relations and the feeding model dependent Q_0 and $E2^{+}$ we can determine γ. It is to be noted that mass number A and charge number Z also play a role in giving the shape to the nucleus while describing the asymmetric parameter γ.

The parameters $\beta A^{2/3}$ and γ are listed in Table II for the four isotopes of samarium along with the experimental input quantities $B(E2;2^{+}\to 0^{+})$, $B(E2;2^{+'}\to 0^{+})$, and $A.E2^{+}$. According to the classical approximation to the dynamics of the triaxial core, which prefers rotation about the axis with the largest moment of inertia in order to minimize the rotational energy, we have $|\beta A^{2/3}|<4$ as a weak coupling classification in terms of core excitation states (and therefore these are vibrational nuclei); $|\beta A^{2/3}|>7$ for well deformed nuclei; and $4<|\beta A^{2/3}|<7$ more adequate for transitional nuclei.

Thus approach (d) describes samarium nuclei strictly according to the established facts. ^{148}Sm which possesses $|\beta A^{2/3}|=4.02$ is almost vibrational; ^{150}Sm which possesses $|\beta A^{2/3}|=5.58$ is a transitional nucleus with $\gamma=20.25°$; ^{152}Sm has $|\beta A^{2/3}|=8.68$ and hence is near to the well deformed structure with $\gamma=14.5°$; while for the ^{154}Sm

TABLE II. Basic experimental quantities $E2^{+}$, $B(E2;2^{+}\to 0^{+})$, and $B(E2;2^{+'}\to 0^{+})$, are given which have been taken from Refs. 1 and 18. A model dependent $e^2Q_0{}^2/16\pi$ and $\beta A^{2/3}$ have been calculated using basic experimental quantities. In columns (a), (b), (c), and (d) the values of the asymmetric parameter are placed using methods (a), (b), (c), and (d), respectively.

Nucleus	$E2^{+}$ (keV)	$B(E2;2^{+}\to 0^{+})$ (e^2b^2)	$B(E2;2^{+'}\to 0^{+})$ (e^2b^2)	$e^2Q_0{}^2/16\pi$ (e^2b^2)	Asymmetric parameter (γ) (deg) (a)	(b)	(c)	(d)	$\beta A^{2/3}$
^{150}Sm	333.95	0.274(6)	0.0088(20)	0.2828(80)	20.5		17.5	20.25	5.58
^{152}Sm	121.77	0.670(15)	0.0163(11)	0.6863(161)	13.0	22.0	12.5	14.5	8.68
^{154}Sm	82.05	0.922(40)	0.013(3)	0.935(43)	9.5	15.5	9.5	10.0	10.14

nucleus (a well deformed one) with $\gamma=10°$, $|\beta A^{2/3}|=10.14$.

As the level energies are not a very good probe of the nuclear shape, as they are rather insensitive to softness and sensitive to the inertial parameter (whose γ dependence is usually taken to correspond to irrotational flow), in addition to other effects such as Coriolis antipairing (CAP), the calculations of reduced electric quadrupole transition probabilities which are more sensitive to γ have also been carried out for some useful conclusions.

III. RESULTS AND DISCUSSIONS

A. Energy levels

Figures 2 and 3 show the energy spectra of ^{152,154}Sm nuclei. In column (E) the experimental energy levels are listed which are taken from Ref. 18. Columns (A), (D), and (B) show the theoretical ARM energy levels derived from feeding the nonaxial parameter (γ) calculated by methods (a), (d), and (b), respectively. It should be noted that the calculated values for $2^{+'}$, 3^{+}, and $4^{+'}$ in column (B) are too much lower than the experimental ones. The agreement between energy values of the gamma band in column (A) with those of (E) is best, but there the inputs are $E2^{+}$ and $E2^{+'}$. Column (D) also compares well with column (E), where the input is only $E2^{+}$. This can be qualitatively argued as the minimum finite value of γ for stable deformation, which cannot be small, since the frequency of vibration and rotation would then be comparable and separation into rotational and vibrational levels would be meaningless. For $\gamma<15°$, the nonaxial parameter deserves to be enhanced by a degree or two, equivalent to BMVRIC. This is probably the reason why column (D) results are better than those of columns (A) and (C) for ground state levels. The results of column (B) are discouraging. Again the reason that the value of γ should be lowered by a few degrees to 21.5° gets support from the DF energy diagram, which reads $E2^{+'}=E4^{+}$ at $\gamma=21.5°$, while the experimental $E2^{+'}$ ($=1.1938$ MeV) is much closer to the experimental $E4^{+}$ ($=0.366\,46$ MeV) for ^{152}Sm. This fact rejects the value of γ ($=22°$) used in column (B).

Figure 1 shows the energy spectra of the ^{150}Sm nucleus. In column (E) the experimental energy lev-

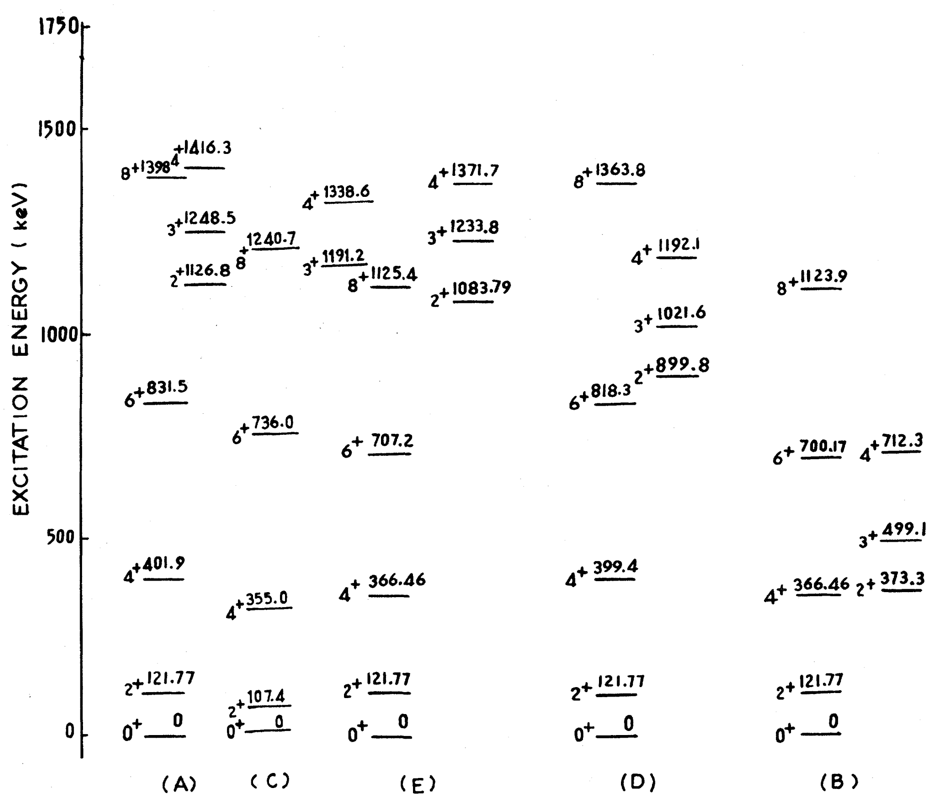

FIG. 2. Energy spectra for the ^{152}Sm nucleus. (E) is the experimental spectrum adapted from Ref. 18. (A), (C), (D), and (B) are the calculated spectrum using γ following methods (a), (c), (d), and (b), respectively.

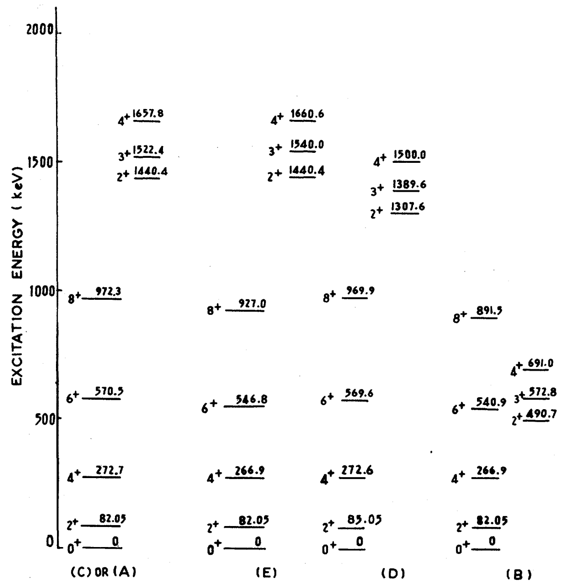

FIG. 3. Energy spectra for the ^{154}Sm nucleus. (E) is the experimental spectrum adapted from Ref. 18. (C) or (A), (D), and (B) are the calculated spectrum using γ following methods (c) or (a), (d), and (b), respectively.

els, adapted from Ref. 18, are listed. In columns (A), (D), and (C) we display the theoretical ARM energy levels derived from using the nonaxial parameter (γ) calculated from methods (a), (d), and (c), respectively. The discrepancy between calculated and observed energy values was attempted[19] to be explained by accounting for the interaction of rotation with β vibrations. This interaction can be expressed as

$$E_b({}^nI,\gamma,\beta)=\{\epsilon({}^nI,\gamma)-b[\epsilon({}^nI,\gamma)]^{1/2}\} ,$$

where b is constant, and $\epsilon({}^nI,\gamma)$ are eigenvalues in units of A $(=\hbar^2/4B\beta^2)$ for the DF model without interaction. But such a correction cannot be applied for ^{150}Sm in the results of column (C) since the discrepancies between calculated and observed values are in the opposite direction for the 3^+ and 4^+ states of the gamma band while the above equation implies that the deviations would be in the same direction for all the energy levels. In view of the fact that the effect of β vibrations has been incorporated in a better way in the model of Davydov and Chaban,[20] the addition of the cubic term in $\epsilon({}^nI,\gamma)$ to the right hand side of above equation, to improve the results, does not seem to be worthwhile.

It is pointed out here that Tamura's results[1] are also subject to this objection since discrepancies between calculated and observed values are in opposite directions for the 3^+ and 4^+ states of ^{154}Sm and ^{150}Sm and the 2^+ and 3^+ states of ^{152}Sm belonging to the gamma band. With elimination of method (c) due to its anomalous description of the energy levels of the ^{150}Sm nucleus, method (d) claims to predict a lower value of γ which is also not subject to have BMVRIC since it is 20.25 ($\gamma > 15°$).

B. Probability of electric transitions

Tables III – V contain the descriptions of experimental $B(E2)$'s of known transitions under column (E) and calculated ARM values under columns (A), (B), (C), and (D) employing γ determined from methods (a), (b), (c), and (d), respectively, for ^{150,152,154}Sm nuclei. The calculated results of

TABLE III. $B(E2)$ individual values (e^2b^2) and branching ratios for the ^{152}Sm nucleus. S_1 results were taken from the work of Tamura *et al.* (Ref. 1); S_2 results from the work of Kumar (Ref. 2); S_3 results from the work of Toyama (Ref. 21). Experimental values were taken from Ref. 1. (A), (B), and (D) results were calculated employing a nonaxial parameter following the methods of Davydov and Filippov (Ref. 4), Varshni *et al.* (Ref. 10), and the method presented in this paper. Theoretical values which differ from the experimental values (including errors) by more than a factor of 2 are underlined.

Transition	Expt.	S_1	S_2	S_3	D	B	A
$2^+\to0^+$	0.670(15)	0.673	0.648		0.652(15)	0.630	0.640
$4^+\to2^+$	1.017(14)	0.98	0.993		0.947(22)	0.917	0.926
$2^{+'}\to0^+$	0.0163(11)	0.050	0.022		0.034	0.040	0.029
$2^{+'}\to2^+$	0.0417(42)	0.053	0.051		0.091(2)	0.381	0.0737
$2^{+'}\to4^+$	0.00416(32)	0.006	0.003		0.0113(3)	0.042	0.0087
$2^{+'}\to2^+/0^+$	2.44(13)	1.06	2.33	2.56	2.64	9.48	2.5
$2^{+'}\to2^+/4^+$	11.9(13)	9.14	19.7	11.1	8.09	8.93	8.47
$3^+\to2^+/4^+$	0.95(7)	2.68	1.42	0.952	0.72	0.181	0.694
$4^{+'}\to2^+/4^+$	0.088(13)	0.34	0.16		0.053	0.0277	0.069

TABLE IV. $B(E2)$ individual values (e^2b^2) and branching ratios for the ^{150}Sm nucleus. S_1 results were taken from the work of Tamura *et al.* (Ref. 1); S_2 results from the work of Kumar (Ref. 2). Experimental values were taken from Ref. 1. (A), (C), and (D) results were calculated employing a nonaxial parameter following the methods of Davydov and Filippov (Ref. 4), Meyertervehn (Ref. 11), and the method presented in this paper. Theoretical values which differ from the experimental values (including errors) by more than a factor of 2 are underlined.

Transition	Expt.	S_1	S_2	A	D	C
$2^+\to0^+$	0.274(6)	0.275	0.233	0.256	0.264(7)	0.258
$4^+\to2^+$	0.53(6)	0.51	0.431	0.375	0.388(11)	0.377
$2^{+'}\to0^+$	0.0088(20)	0.020	0.01	0.015	0.0186(5)	0.018
$2^{+'}\to2^+$	0.0387(141)	0.024	0.125	0.051	0.108(3)	0.112
$2^{+'}\to4^+$	0.0194(100)	0.087	0.034	0.00197	0.017	0.0045
$2^{+'}\to2^+/0^+$	4.4(6)	1.18	12.6	3.33	5.83	6.22
$2^{+'}\to2^+/4^+$	2.0(7)	0.27	3.75	6.66	6.33	7.09
$3^+\to2^+/4^+$	0.29(6)	1.09	0.52	0.54	0.275	0.258
$3^+\to2^{+'}/2^+$	24(5)	4.34	10.2	16.8	14.3	14.5
$4^{+'}\to2^+/4^+$	0.050(7)	0.69	0.028	0.0023	0.0016	0.034
$4^{+'}\to3^+/2^{+'}$	3.7(13)	0.53	0.58	2.13	1.98	1.95

Tamura *et al.*,[1] Kumar,[2] and Toyama[21] are also listed for the sake of comparison. We shall impose Kumar's test[22] (i.e, $0.5<$ enhancement/hindrance factor $F<2.0$) on the entries of each column and see which of them remains successful. Tamura fails to accommodate many transitions such as $2^{+'}\to0^+$, $2^{+'}\to4^+$, $2^{+'}\to2^+/0^+$, $2^{+'}\to2^+/4^+$, $3^+\to2^+/4^+$, $3^+\to2^{+'}/2^+$, $4^{+'}\to2^+/4^+$, and $4^{+'}\to3^+/2^+$ for ^{150}Sm, $2^{+'}\to0^+$, $2^{+'}\to2^+/0^+$, $3^+\to2^+/4^+$, and $4^{+'}\to2^+/4^+$ for ^{152}Sm, and $2^{+'}\to2^+$, $2^{+'}\to4^+$, $2^{+'}\to2^+/4^+$, and $4^{+'}\to2^+/4^+$ for the ^{154}Sm nucleus, in his sixth order boson expansion description calculations of collective states. Kumar is unsuccessful in explaining $2^{+'}\to2^+$ and $2^{+'}\to2^+/0^+$ for ^{150}Sm, and $2^{+'}\to0^+$, $2^{+'}\to2^+$, $2^{+'}\to4^+$, $2^{+'}\to2^+/4^+$, and $4^{+'}\to2^+/4^+$ for the ^{154}Sm nucleus. Varshni is silent on the ^{150}Sm nucleus and fails almost everywhere in describing ^{152,154}Sm. Meyertervehn fails with $2^{+'}\to2^+$, $2^{+'}\to4^+$, and $2^{+'}\to2^+/4^+$ for ^{150}Sm. Davydov and Filippov do not accommodate $2^{+'}\to4^+$, $2^{+'}\to2^+/4^+$, and $4^{+'}\to2^+/4^+$ for ^{150}Sm and $3^+\to2^+/4^+$ and $4^{+'}\to2^+/4^+$ for the ^{154}Sm nucleus. The present work faces the least barriers and is hindered only at the $4^{+'}\to2^+/4^+$ ratio for ^{150,154}Sm nuclei.

On comparing the calculated values we find that $2^{+'}\to2^+/4^+$, $3^+\to2^+/4^+$, and $4^{+'}\to2^+/4^+$ values

TABLE V. $B(E2)$ individual values (e^2b^2) and branching ratios for the ^{154}Sm nucleus. S_1 results were taken from the work of Tamura *et al.* (Ref. 1); S_2 results from the work of Kumar (Ref. 2); S_3 results from the work of Toyama (Ref. 21). Experimental values were taken from Ref. 1. (A)/(C), (B), and (D) results were calculated employing a nonaxial parameter following the methods of Davydov and Filippov and of Meyertervehn (Refs. 4 and 11), Varshni *et al.* (Ref. 10), and the method presented in this paper. Theoretical values which differ from the experimental values (including errors) by more than a factor of 2 are underlined.

Transition	Expt.	S_1	S_2	S_3	D	B	A/C
$2^+\to0^+$	0.922(40)	0.881	0.94		0.9088(417)	0.882(10)	0.910(42)
$4^+\to2^+$	1.186(39)	1.25	1.40		1.304(40)	1.286(59)	1.306(60)
$2^{+'}\to0^+$	0.013(13)	0.021	0.033		0.026(1)	0.052(2)	0.024(1)
$2^{+'}\to2^+$	0.02	0.047	0.047		0.047(2)	0.174(1)	0.044(2)
$2^{+'}\to4^+$	0.0008	10^{-5}	0.01		0.0038(2)	0.024(1)	0.0037(2)
$2^{+'}\to2^+/0^+$	1.56	2.23	1.42	2.04	1.82	3.32	1.81
$2^{+'}\to2^+/4^+$	25.0	3916	8.0	11.1	12.34	7.09	11.2
$3^+\to2^+/4^+$	2.5	3.45	1.35	1.19	0.39	0.54	<0.39
$4^{+'}\to2^+/4^+$	0.055	0.51	0.32		0.17	0.033	0.183

following method (d) are better than those of Kumar. The $2^{+'}\to 2^+$, $2^{+'}\to 4^+$, and $4^{+'}\to 2^+/4^+$ values are better in column (D) than the values listed in column (A). Toyama describes well a few ratios for the ^{152}Sm nucleus but he is silent on ^{150}Sm. It appears from his work that it is limited to lower values of γ only and predicts almost a constant value for the $2^{+'}\to 2^+/0^+$ ratio for all well deformed nuclei.

It is, however, important to note that the experimental value of the ratio $3^+\to 2^+/4^+$ for the ^{154}Sm nucleus is reported to be 1.4 in Ref. 21 and is 0.95 according to Ref. 23, which is much lower than the value reported in Ref. 1 (=2.5). The former value reduces the hindrance factor from 6 to 3.5/2.5. It is well known that the comparison of the theoretical and experimental electromagnetic properties presents a much more severe test of a theory than does the comparison of the energy levels. When the electromagnetic transitions get weaker, the pertinent experimental data are normally supplied in the form of branching ratios. Therefore, the comparison of the theoretical branching ratios with experimental ones offers a still more stringent test of the theory. It begins to test the validity of the prediction of rather small components of the wave functions. One is puzzled to see Tamura's prediction deviating by more than a factor of 150 for the branching ratio $2^{+'}\to 2^+/4^+$ for the ^{154}Sm nucleus, while remarkable success has been achieved throughout in the present work within only a factor of 2.

The reason for the only discrepancy which remains in our work with respect to the $4^{+'}\to 2^+/4^+$ ratio for the ^{150}Sm nucleus may be the negligence of a slight gamma dependence of $E2^+$ and $B(E2;2^+\to 0^+)$ in the Davydov model, in view of uncertainties involved in the empirical rule of Grodzins.

$$E2^+\times B(E2;2^+\to 0^+)\approx(2.5\pm 1)\times 10^{-3}Z^2A^{-1}$$

[MeV(e^2b^2)] from which $\hbar^2/2\mathscr{I}_0=204/\beta^2A^{7/}$ (MeV) is derived. Again the $4^{+'}\to 2^+/4^+$ value changes abruptly at $\gamma=20°$ in ARM.

C. Branching ratios

Various theoretical predictions of $B(E2)$ branching ratios, viz., $2^{+'}\to 0^+/2^+\to 0^+$, $2^{+'}\to 2^+/2^+\to 0^+$, $2^{+'}\to 2^+/0^+$, $2^{+'}\to 4^+/2^+\to 0^+$, $2^{+'}\to 2^+/4^+$, $3^+\to 2^+/4^+$, and $4^{+'}\to 2^+/4^+$ are plotted in respect to ^{150,152,154}Sm nuclei along with the experimental values in Fig. 6. We observe that the decreasing trend in the experimental value of $2^{+'}\to 0^+/2^+\to 0^+$ versus neutron number does not coincide with that of the values of Kumar and of Tamura. Tamura's trend is in the opposite direction to that of the experimental in describing the $2^{+'}\to 2^+/0^+$ ratio. For the $2^{+'}\to 2^+/4^+$ ratio the values of Kumar and Tamura are again in contradiction to the experimental trend. For $3^+\to 2^+/4^+$,

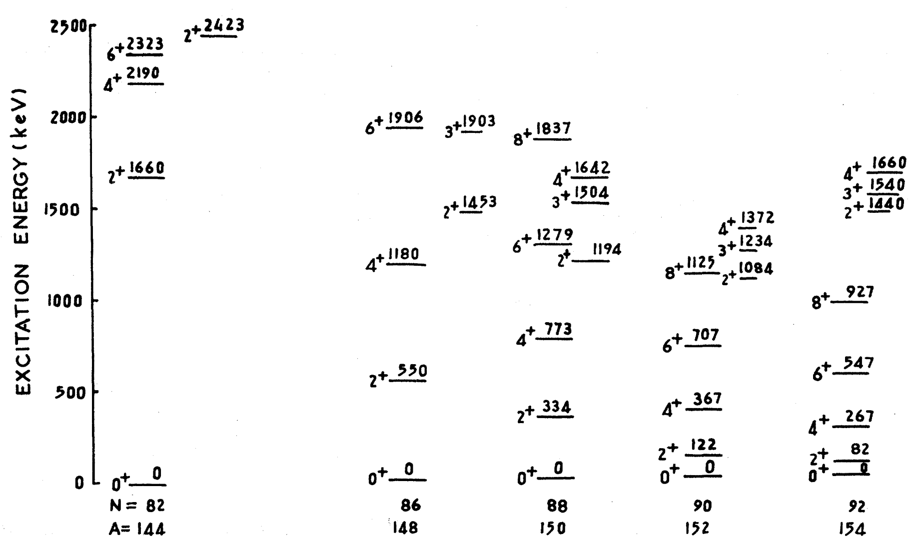

FIG. 4. The energy level diagram of the low lying ground state and gamma-vibrational levels, adapted from Ref. 18, of the even isotopes of samarium studied in this paper.

Tamura's results are slightly better, but in the $4^{+'} \to 2^{+}/4^{+}$ ratio Tamura fails miserably in both quality and quantity. Although not even one out of the three results gives a qualitative trend in $4^{+'} \to 2^{+}/4^{+}$, our results are comparatively better and nearest to the experimental one.

Table II reveals the closeness in the value of γ placed in columns (a) and (d). In deformed nuclei (^{152,154}Sm) the values of column (d) are larger than the values of column (a). This may be accounted for by the vibration rotation interaction correction and a centrifugal stretching correction. They are of much importance in DF calculations, with the analogy of molecular spectra where these corrections have the form $-bI^2(I+1)^2$ in the expression of rotational excitation energies, and become more important as the equilibrium deformation decreases. Hence γ is subject to a correction of 1° or 2° if it is calculated from the energy ratio $E2^{+'}/E2^{+}$. In order to have modified γ, Varshni *et al.* put forth

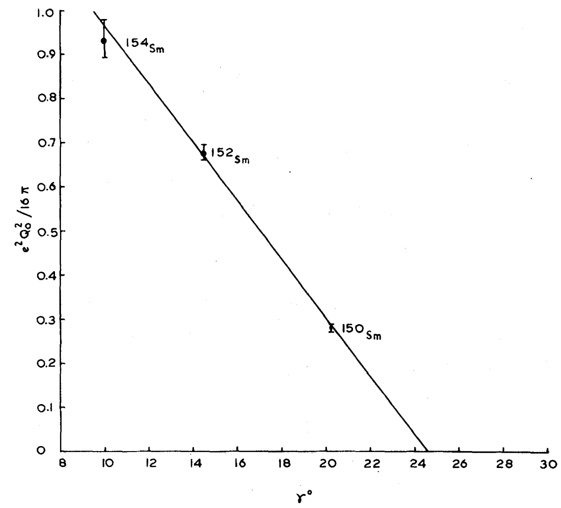

FIG. 5. Plot of model dependent $e^2Q_0^2/16\pi$ (e^2b^2) vs γ (obtained by using the new approach).

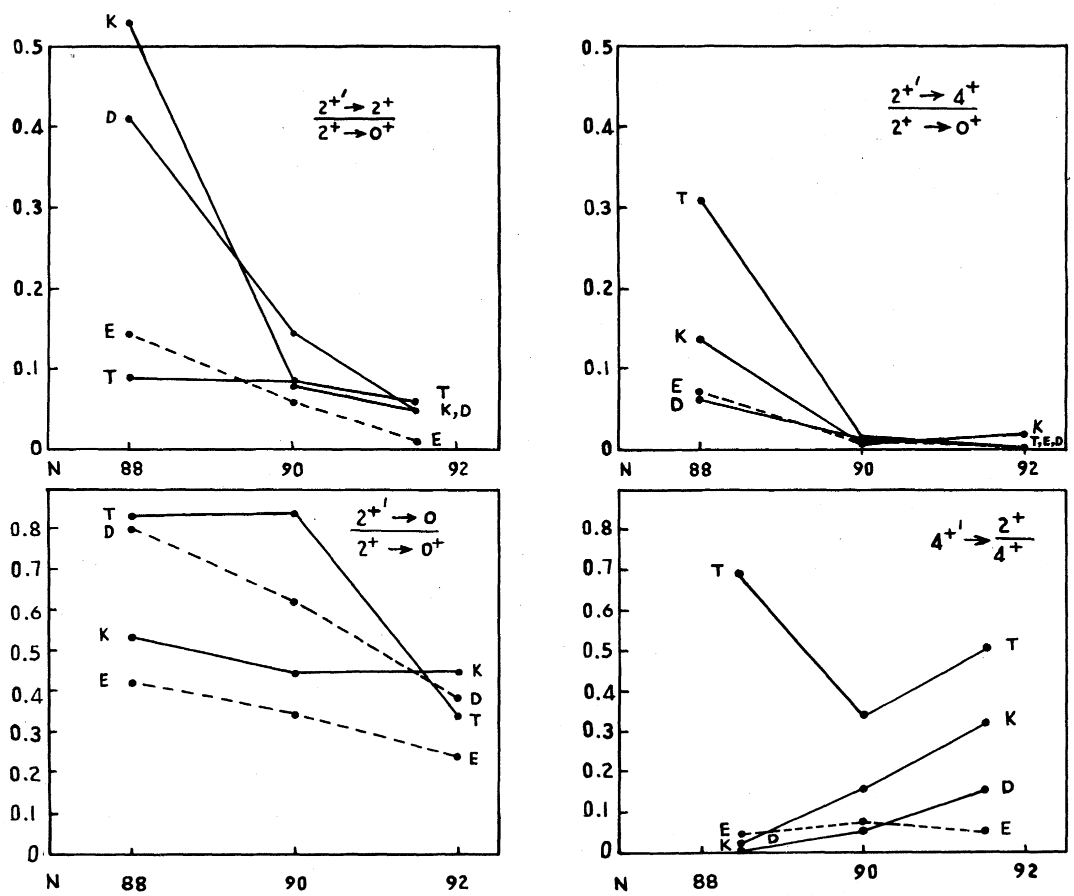

FIG. 6. $B(E2)$ branching ratios are displayed in respect to ^{150,152,154}Sm nuclei. E results are the experimental ones (Ref. 1). K results were taken from the work of Kumar (Ref. 2). T results were taken from the work of Tamura *et al.* (Ref. 1). D results were calculated using the present approach for evaluating γ.

their method of determining γ from the energy ratio $E4^+/E2^+$. This method failed misearably, as it excludes the transitional nucleus ^{150}Sm to obey the DF discipline and gives a much lower value of γ for ^{152}Sm. On the other hand, Mayertervehn probably tried to reduce the value of γ for nuclei having $\gamma > 20°$ but he could do so in the case of ^{150}Sm to such an extent that it worsens the agreement between theoretical and experimental $B(E2)$ values and also describes $2^{+'}$, 3^+, and $4^{+'}$ energy levels anomalously. In the case of the transitional nucleus (^{150}Sm) the value of γ in column (d) is slightly less than that of column (a). Although there is no convincing theoretical reason against nonaxially symmetric nuclei in the region with γ about 20° or so,[24] even for lesser values of γ the $2^{+'}$ state lies very high with respect to the 2^+ state (^{152,154}Sm nuclei in Fig. 4), and this too supports the triaxial nature of the nucleus. We observe a simple linear relationship in Q_0 and γ derived from method (d) for ^{150,152,154}Sm nuclei which gives $Q_0=0$ (i.e., $\beta=0$) at $\gamma=24.6°$ (Fig. 5). It is interesting to note that in the cases where nuclei can no longer be considered deformed in the original sense used by Bohr Mottleson (i.e., when $\gamma > 24°$), the simple linear relationship ends. In these cases the nuclear coupling scheme would no longer involve a simple one parameter coupling scheme, but would instead in-

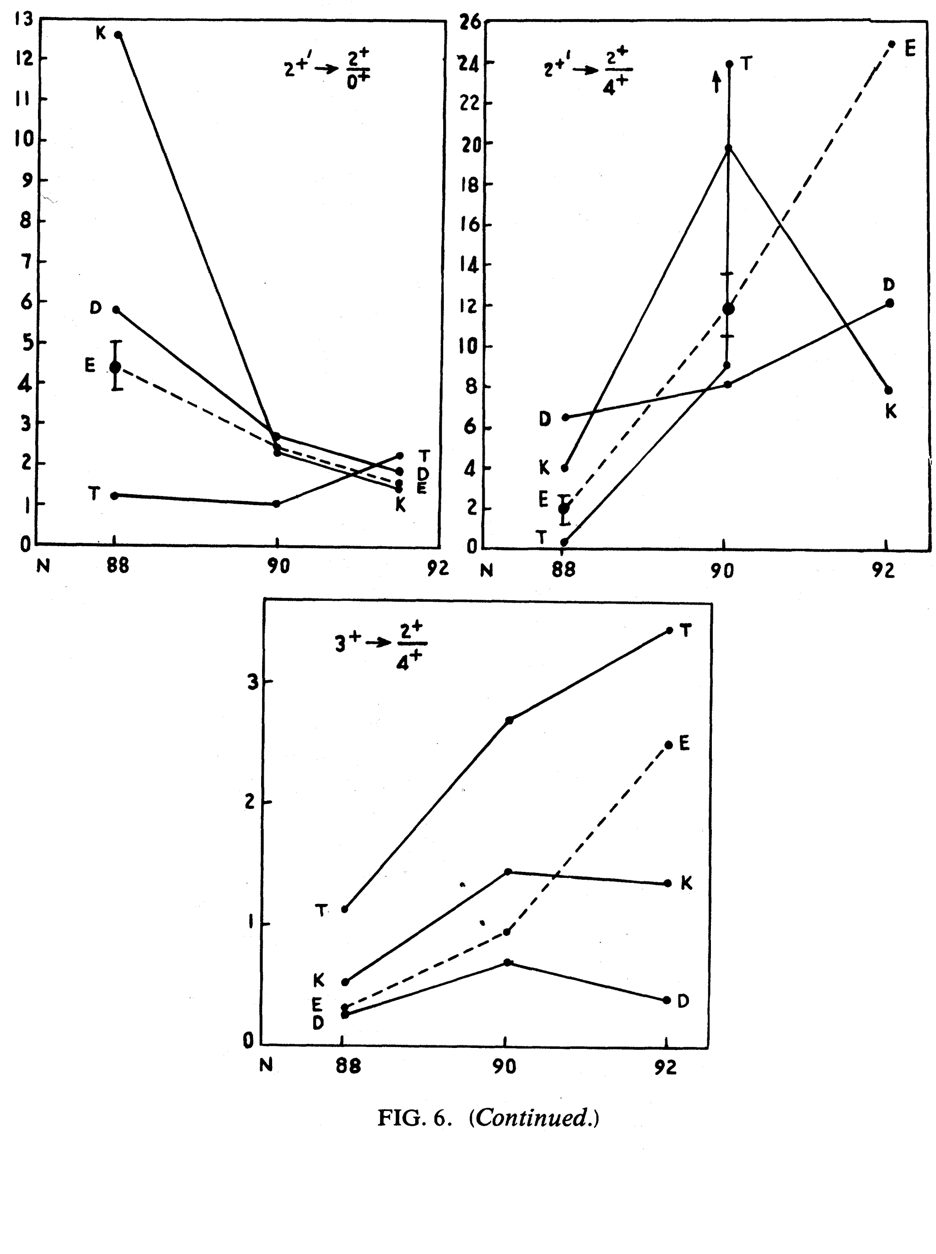

FIG. 6. (*Continued.*)

TABLE VI. Rigid rotor model estimates in units of e^2b^2 of various unknown individual transitions and branching ratios with respect to ^{150,152,154}Sm isotopes are displayed. $B(E2)$ values of gamma ray cascades have been predicted and displayed using an empirical relation and the modified form of figures drawn in Refs. 5 and 26.

Transition	^{150}Sm	Nuclei ^{152}Sm	^{154}Sm
$2^{+'}\to 4^+$			
$2^{+'}\to 2^+/0^+$			
$2^{+'}\to 4^+/2^+$			
$3^+\to 4^+$	0.115(3)	0.082(3)	0.032(1)
$3^+\to 4^+/2^+$			
$3^+\to 2^+/2^{+'}$		0.051	0.0075
$4^{+'}\to 4^+/2^+$			
$4^{+'}\to 3^+/2^{+'}$		2.165	2.23
$6^+\to 4^+$	0.461(13)	1.07(2)	1.446(66)
$4^{+'}\to 2^{+'}/2^+$	1082	65.9	52.3
$3^+\to 2^+$	0.033(1)	0.0596(14)	0.012
$3^+\to 2^{+'}$	0.473(13)	1.062(25)	1.65(7)
$4^+\to 2^{+'}$	0.0094(2)	0.0063(1)	0.0021
$4^{+'}\to 2^+$	0.0004	0.0057(1)	0.010
$4^{+'}\to 2^{+'}$	0.135(4)	0.375(1)	0.538(25)
$4^{+'}\to 3^+$	0.269(1)	0.81(2)	1.199(55)
$4^{+'}\to 4^+$	0.088(2)	0.107(2)	0.058(3)
$6^+\to 4^{+'}$	0.0136(4)	0.022	0.0072
		Empirical estimates	
$8^+\to 6^+$	0.435	1.074	1.497
$10^+\to 8^+$	0.447	1.103	1.538
$12^+\to 10^+$	0.455	1.123	1.565
$14^+\to 12^+$	0.460	1.136	1.584
$16^+\to 14^+$	0.464	1.147	1.598
$18^+\to 16^+$	0.468	1.155	1.61

volve a competition between the quadrupole coupling and the pairing correlation (i.e., when $\gamma > 24°$), and the nuclei become pseudospherical (i.e., $\beta=0$). Our calculations lend support to the triaxial nature of ^{150,152,154}Sm nuclei. The theoretical Q_2 values obtained for the ^{150,152,154}Sm isotopes are -0.918, -1.62, and -1.92 (in e b), which agree with the experimental values $-1.22(22)$, $-1.8(6)$, and $-2.14(10)$ within the experimental error and are given in Ref. 25. Also, in the range $20° < \gamma < 30°$, the present work clearly indicates that ^{150}Sm is a transitional nucleus and has rather stable triaxial shape, in contradiction to the expectation of a very soft fluctuating shape based on the collective potential calculations.

TABLE VII. The mean lives of different rotational excited levels have been predicted using an empirical relation and the modified form of figures drawn in Refs. 5 and 26.

Nuclei	I^π	Mean life (τ) in ps of I^π level
^{150}Sm	6^+	5.35(11)
^{152}Sm	12^+	1.56(3)
^{154}Sm	10^+	3.23(14)
	12^+	2.15(10)

Known $B(E2)$ values and excitation energies of samarium isotopes have been excellently explained by using the nonaxial parameters γ [method (d)], which are slightly modified from the usual values of γ [method (a)]. Such modification has been a necessity and provoked Varshni *et al.* and Meyer-tervehn to search for methods different than the usual one.

The $B(E2)$ values of gamma ray cascades and mean lives of different rotational excited levels have been predicted using an empirical relation and the modified forms of the figures drawn in Ref. 5, and are displayed in Tables VI and VII.

CONCLUDING REMARKS

Extensive calculations of low lying rotational band and gamma vibrational band energy levels, $B(E2)$ values, and branching ratios for ^{150,152,154}Sm nuclei employing the rigid rotor model with a new technique of determining γ are presented in this paper. A new empirical linear relationship has been observed in the model dependent intrinsic quadrupole moment Q_0 and the asymmetric parameter γ. This linear relationship rectifies the usual values of γ [method (a)] by enhancing it a few degrees at about $\gamma \approx 15°$ and reducing it a few degrees at $20° < \gamma < 30°$. This had been a necessity and stimulated theoreticians[10,11] to adopt other methods of evaluating γ. The γ thus obtained not only removes the problem of anomalous description of gamma band energy levels, but excellently explains the individual $B(E2)$'s and branching ratios within a factor of 2. Analyzing Tamura's results with respect to the energy spectrum of the gamma vibrational band for all the samarium isotopes and the branching ratios where almost everywhere the theory fails badly, one is forced to conclude that the physical origins of the quadrupole and octupole collective motions cannot be different from one another, as assumed in Tamura's theory. At the same time, the present work indicates that ^{150,152}Sm, the so-called transitional nuclei, have triaxial shapes which are more stable than expected from theoretical potential energy surfaces, and thus lends new support to the old Davydov model. Quantitative agreement with experiment probably indicates that the effect of β and γ fluctuations in the range $20° < \gamma < 30°$ which leads to an overall compression of the energy spectrum and effects transition probabilities, has been accounted for as a modification in the value of γ in the linear empirical relation. It is inferred that the mass number A and charge number Z also play an important role in assigning shape to the nucleus in terms of the asymmetric parameter.

On the whole, the predictions made in the present work are very useful, keeping in mind the extreme simplicity. The mean lives of some rotational excited levels which are of preliminary use for experimentalists are listed in Table VII. Our calculations assign 10^+ and 12^+ spins to known levels at 1.338 and 1.817 MeV in the case of the ^{154}Sm nucleus, which were suspected to be 4^+ in Ref. 23, and commends 12^+ spin for the 2.158 MeV observed energy level in the ^{152}Sm nucleus.

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