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Two-body current contribution to $M1$ transitions and moments in ^{38}Cl and ^{40}K

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We deduce evidence for the existence of a two-body current contribution to $M1$ transitions and moments in nuclei, based on the application of shell model theory to experimental data. A complete specification of a two-body $M1$ operator, within a shell-model configuration, can be obtained by combining data from particle-particle and particle-hole nuclei. For this purpose, new particle-hole relations are derived for $M1$ transitions. The method of analysis is applied to the particle-hole pair ^{38}Cl-^{40}K. The two-body current contributions could be effects of either configuration mixing, or mesonic currents, or both.

[NUCLEAR STRUCTURE ^{38}Cl, ^{40}K; shell model analysis of μ and $B(M1)$ to determine two-body $M1$ operator.]

I. INTRODUCTION

One of the simplest predictions of the *j-j* coupling shell model is that the ground state magnetic moments of odd-*A* nuclei with one particle or hole outside a closed shell should be given by the Schmidt moments. The prediction is a consequence of the two assumptions of simple configurations and of a one-body $M1$ operator, equal to that for free nucleons. The qualitative agreement of experimental single-particle moments with the Schmidt moments is well known. The residual deviation of the moments from the Schmidt values has been a subject of theoretical and experimental investigation for over thirty years. Both configurations mixing and mesonic currents have been studied as possible contributors to the known deviations. The subject has been extensively reviewed recently.[1]

Most of the theoretical effort has concentrated on the one-particle or one-hole moments, for which there is very precise data with which to compare.[2] The theory is then expressed in terms of a modification of the one-body magnetic moment operator from its free value by the nuclear system; this can be reexpressed as a change in the g factor. However, the theories that predict these modifications in one-body operators also predict the existence of two-body $M1$ operators, e.g., from "exchange currents" in the nucleon-nucleon interaction (i.e., mesons, etc.) or analogous model effects from configurations. There has been almost no investigation of the possible effects of the two-body $M1$ currents directly in complex nuclei at low energy. Presumably they would show up in the static magnetic moments and in $M1$ transitions in nuclei with several active nucleons. One exception is the special case of threshold radiative capture, $np \rightarrow d\gamma$, which has been investigated from the point of view of mesic current contributions by Riska and Brown.[3]

The difficulty of working with complex nuclei is clear: Transitions are not measured as accurately as ground state moments, two-body effects are presumably small compared to one-body effects, and not all needed information may be measured, for any one nuclear case, as we shall see. However, in this paper we shall show that one can find two-body effects in available nuclear $M1$ data, and even extract some information about the explicit form of the two-body $M1$ operator. We do not investigate the dynamical origin of the operator in the present work.

We work in the context of the *j-j* coupling shell model with cases for which that is a good first approximation. Suppose we take the experimental magnetic moments of closed-shell plus one nucleon to define the one-body $M1$ operators: This includes any effects induced by the nucleus on the single nucleon in an effective one-body operator. We may then calculate, within the shell model, the magnetic moments and $M1$ transitions for a relatively simple nucleus, such as two nucleons outside a closed shell, and compare to experimental data. In principle, any deviations could be ascribed to some two-body effective currents. However, we would not find this procedure very reliable. Often the total effect is of the order of magnitude of the experimental uncertainty. In addition, we shall also not find all the data we would like to have to pin down the residual two-body effect, as we shall see further, below.

Therefore, we have devised a different approach which gives us more information by combining data from different nuclei. In particular, we shall deal with nuclei in the same shell model orbits, for which we may use particle-hole relations. Consider a proton orbit a, and a neutron orbit b; we use the particle-particle nucleus (a,b) and the particle-hole nucleus (a^{-1},b) for information on two-body currents. The one-body $M1$ operators are defined from the closed shell nuclei with an a or b particle, or a hole. We consider a general two-body $M1$ operator acting on the proton and neutron in their respective orbits. (Exchange currents require a proton and neutron, in lowest order.) We derive a particle-hole (p-h) relation between the matrix elements of this operator for the (a,b) nucleus, and the (a^{-1},b) nucleus. This is a generalization of the p-h relations for spectral energies for such pairs of nuclei, originally derived by Pandya,[4] and by Goldstein and Talmi.[5,6] A similar relation for $M1$ and $E2$ transitions, comparing (a,b) and (a^{-1},b^{-1}) cases, was derived by Goode and West.[7] The techniques used here are standard tensorial operator transformations in the shell model.[8]

The p-h relation allows us to combine data for $M1$ transitions and magnetic moments from both the (a,b) and (a^{-1},b) nuclei, having subtracted the one-body effects. This allows us, in some cases, to determine all the possible matrix elements of the two-body $M1$ current operator within the shell-model space.

The best example we have found for application of this technique is the pair of nuclei ^{38}Cl and ^{40}K, which is the pair for which the energy relations (Pandya-Goldstein-Talmi) were first studied and which work to high accuracy here.[4–6,9] The orbits are $a=d_{3/2}$ (proton), $b=f_{7/2}$, neutron, and are relatively pure (see, however, Goode[10]). Both nuclei then have a shell model spectrum of four levels, with $J=2$, 3, 4, and 5, shown in Fig. 1 with experimental energies. The allowed $M1$ transitions are all measured, as well as the ground state magnetic moments, and the first excited-state moment of ^{40}K, as indicated in the figure. There are therefore nine $M1$ data with which to constrain the two-body current: We shall see that this overdetermines it.

The paper is arranged as follows: In Sec. II we study the one-body $M1$ operator, and try to constrain it from measured data. We then predict the transitions and moments in ^{38}Cl and ^{40}K, and compare with data. In Sec. III we introduce the two-body $M1$ operator, and derive the required p-h relations among the matrix elements. In Sec. IV we give numerical results for the matrix elements of the two-body operator which emerges from the analysis of ^{38}Cl and ^{40}K. Discussion follows in Sec. V.

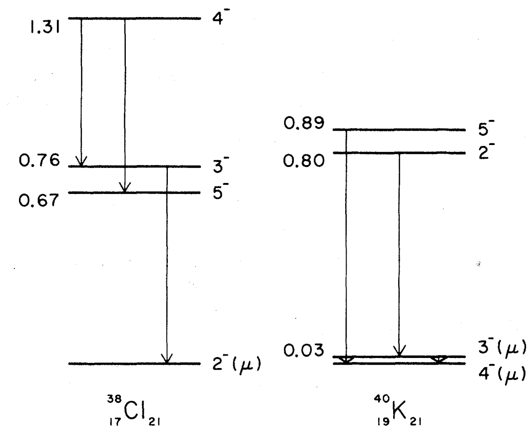

FIG. 1. Excitation energies, spins, parities, measured $M1$ transitions, and measured magnetic moments (μ) are indicated for the lowest four levels of ^{38}Cl and ^{40}K. These levels involve the shell-model orbits $d_{3/2}$ (proton) and $f_{7/2}$ (neutron).

II. ONE-BODY TRANSITION OPERATOR

We are interested in magnetic moments and $M1$ transitions in nuclei. The operator on the nuclear degrees of freedom which governs both quantities will be denoted by $\hat{O}(M1)$ or simply by $\hat{O}$. The operator is an angular momentum tensor of rank one and even parity. We consider the transitions and moments in terms of matrix elements of $\hat{O}(M1)$ between states of the j-j coupling shell model. The operator in this case may depend on the coordinates, spins, etc., of more than one nucleon: That is, it may be a many-body operator. This could come about simply because the j-j configurations are not an exact description of the nuclear states; in this case we have configuration mixing, expressed in terms of an *effective transition operator* in the j-j space. Many-body operators could also come from non-nucleon degrees of freedom, e.g., mesonic currents, in which case they represent a "real" many-body effect. In either case, we can represent the $M1$ operator as a sum of terms of definite particle rank

$$\hat{O}(M1)=\hat{O}_{\rm I}+\hat{O}_{\rm II}+\cdots, \qquad (2.1)$$

where $\hat{O}_{\rm I}$ is the one-body operator, $\hat{O}_{\rm II}$ the two-body operator, and so on. In this section we consider only $\hat{O}_{\rm I}$; $\hat{O}_{\rm II}$ will be examined in Sec. III. We do not explicitly consider higher rank in this paper.

We consider nuclei with *j-j* configurations given by (a,b) and (a^{-1},b); that is, a nucleus with two particles outside a closed shell (one proton, one neutron) and a nucleus with one hole (proton) and one particle (neutron) involving the same orbits as the two-particle nucleus. These are particle-hole (p,h) partners. The specific case of interest will be ^{38}Cl, (a,b) and ^{40}K(a^{-1},b).

For the (a,b) configuration the operator $\hat{O}_{\rm I}$ can be written (in units of the nuclear magneton) in the form

$$\hat{O}_{\rm I}=(g_a\vec{\rm J}_a+g_b\vec{\rm J}_b)\ . \tag{2.2}$$

$\vec{\rm J}_a$ and $\vec{\rm J}_b$ are the angular momentum operators for the orbits a and b and g_a and g_b are the respective g factors. The operator (2.2) also applies to the (p,h) case (a^{-1},b). This can be seen as follows: We may write the angular momentum operators in terms of the spherical unit tensor operator of rank one in the appropriate orbit

$$\vec{\rm J}=\sum_j (j(j+1)[j])^{1/2}U_{jj}{}^1\ , \tag{2.3}$$

where $U_{jj}{}^1$ is the unit tensor operator and $[j]\equiv 2j+1$. The general one-body unit tensor operator $U^{\nu}_{\rho\rho'}$ is defined such that the reduced matrix element in one-body states is given by

$$\langle a||U^{\nu}_{\rho\rho'}||b\rangle=\delta_{a\rho}\delta_{b\rho'}\ . \tag{2.4a}$$

We also introduce a unit tensor operator for holes, such that the reduced matrix element in one-hole states is

$$\langle a^{-1}||U^{\nu}_{\rho\rho'}(h)||b^{-1}\rangle=\delta_{a\rho}\delta_{b\rho'}\ . \tag{2.4b}$$

These tensor operators have the following (p,h) transformation property[11]

$$U^{\nu}_{\rho\rho'}+(-1)^{\rho-\rho'-\nu}U^{\nu}_{\rho'\rho}(h)=[\rho]^{1/2}\delta_{\rho\rho'}\delta_{\nu 0}\ . \tag{2.5}$$

Therefore, it follows that

$$U_{aa}{}^1=U_{aa}{}^1(h) \tag{2.6a}$$

and

$$\langle a||U_{aa}{}^1||a\rangle=\langle a^{-1}||U_{aa}{}^1||a^{-1}\rangle\ . \tag{2.6b}$$

It follows that matrix elements of the one-body operator (2.2) are unchanged by transforming the particle in orbit a to a hole in the same orbit. This is the well known result for magnetic moments:

$$\mu(a)=\mu(a^{-1})\ , \tag{2.6c}$$

assuming that the operator is the one-body part only, $\hat{O}_{\rm I}$.

The operator (2.2) will give matrix elements for magnetic moments (diagonal) or $M1$ transitions (off diagonal) for both the (a,b) and (a^{-1},b) nuclei by direct use of Eqs. (2.3)–(2.6) and some simple recoupling. The reduced matrix elements of $\hat{O}_{\rm I}$ can be written

$$\begin{aligned}\langle ab;J||\hat{O}_{\rm I}||ab;J'\rangle &=[JJ']^{1/2}(-1)^{b+1-a}\\ &\quad\times\{(-1)^J W(aJaJ';b\,1)(a(a+1)[a])^{1/2}g_a\\ &\qquad+(-1)^{J'}W(bJbJ';a\,1)(b(b+1)[b])g_b\}\ ,\end{aligned} \tag{2.7}$$

where W are the standard Racah coefficients. Using (2.6) we also obtain the p-h relation

$$\langle a^{-1}b;J||\hat{O}_{\rm I}||a^{-1}b;J'\rangle=\langle ab;J||\hat{O}_{\rm I}||ab;J'\rangle\ . \tag{2.8}$$

The matrix elements are completely specified by the two g factors, g_a and g_b. There are three natural choices of the g's to examine: (i) the Schmidt g factors, which are based on the free nucleon magnetic moments; (ii) the g factors which give empirical values of the magnetic moments of the neighboring odd-A nuclei; and (iii) we could find the "best" values of g_a and g_b by finding the experimental moments and transition in the (a,b) and (a^{-1},b) nuclei, e.g., by least squares fitting. Note that each of these is, in a sense, empirical, but based on different data: (i) free nucleons, (ii) odd-A nuclei, and (iii) the p-p and p-h nuclei themselves.

The two nuclei we shall study are ^{38}Cl and ^{40}K, for which $a=\pi d_{3/2}$ and $b=\nu f_{7/2}$. The Schmidt values[12] for these two orbits are given in the first column (i) of Table I. For column (ii) we use the magnetic moments of the neighboring odd-A nuclei. These are, with their configurations, for ^{38}Cl: ^{37}Cl$(\pi d_{3/2})$ and ^{37}S$(\nu f_{7/2})$; while for ^{40}K the neighbors are ^{39}K$(\pi d_{3/2}{}^{-1})$ and ^{41}Ca$(\nu f_{7/2})$. Note that the configurations of the neighbors of ^{38}Cl refer to the ^{36}S closed shell, while those of ^{40}K refer to the ^{40}Ca closed shell.

The g factors of the neighboring nuclei are obtained from the experimental magnetic moments[13] and are listed in column (ii) of Table I. The magnetic moment of ^{37}S is not measured, although the other three are available. Therefore we estimate the g factor for ^{37}S from other experimental data, using

TABLE I. The g factors g_a and g_b for the single-particle orbits $\pi d_{3/2}$ and $\nu f_{7/2}$, respectively, obtained by methods explained in the text.

Nucleus, orbit	(i) Schmidt	(ii) Odd-A neighbors	(iii) Least-square fit
^{38}Cl: g_a	0.08	0.456	0.204
^{38}Cl: g_b	−0.547	(a) −0.287 (b) −0.456 (c) −0.747	−0.615
^{40}K: g_a	0.08	0.261	0.201
^{40}K: g_b	−0.547	−0.456	−0.446

TABLE II. Reduced matrix elements for moments and $M1$ transitions of ^{38}Cl, in units of the nuclear magneton, μ_0.

J_i	J_f	Experimental (μ_0)	One body part (μ_0)	Residual (μ_0)
2	2	-5.61 ± 0.002	(i) −4.72 (iia) −3.61 (iib) −4.99 (iic) −7.38 (iii) −5.61	(i) −0.89 (iia) −2.00 (iib) −0.62 (iic) 1.77 (iii) ∼0.0
3	2	-3.47 ± 0.22	(i) −2.09 (iia) −2.49 (iib) −3.06 (iic) −4.03 (iii) −2.75	(i) −1.36 (iia) −0.98 (iib) −0.41 (iic) 0.56 (iii) −0.73
4	3	-2.01 ± 0.15	(i) −2.44 (iia) −2.88 (iib) −3.53 (iic) −4.66 (iii) −3.17	(i) 0.42 (iia) 0.86 (iib) 1.52 (iic) 2.65 (iii) 1.16
4	5	3.39 ± 0.29	(i) 2.12 (iia) 2.53 (iib) 3.09 (iic) 4.09 (iii) 2.78	(i) 1.25 (iia) 0.86 (iib) 0.29 (iic) −0.69 (iii) 0.60

some theoretical considerations, which are explained further in the Appendix. We use the following three estimates:

$$g_b(^{37}\mathrm{S})=2g_b(^{39}\mathrm{Ar})-g_b(^{41}\mathrm{Ca})\ , \tag{2.9a}$$

$$g_b(^{37}\mathrm{S})=g_b(^{41}\mathrm{Ca})\ , \tag{2.9b}$$

$$g_b(^{37}\mathrm{S})=\tfrac{7}{4}g_b(^{38}\mathrm{Cl})-\tfrac{5}{4}g_b(^{40}\mathrm{K}) +\tfrac{1}{2}g_b(^{41}\mathrm{Ca})\ . \tag{2.9c}$$

Estimate (2.9b) is based simply on the assumption that $\hat{O}\simeq\hat{O}_{\mathrm{I}}$. The other two estimates are based on theory introduced in the next section, and are derived in the Appendix.

For the third column (iii) of Table I, we fit the g factors g_a and g_b to the $M1$ data and magnetic moments of ^{38}Cl and ^{40}K separately, as discussed below. The measured information is indicated in Fig. 1: the ground state moment and three $M1$ transitions for ^{38}Cl; for ^{40}K we have, in addition, the moment of the first excited state. Thus we obtain g_a and g_b for ^{38}Cl from four data, and g_a and g_b for ^{40}K from five.

If the $M1$ operator were in fact only the one-body operator of (2.2), then we would find that g_a and g_b for ^{38}Cl equaled the g_a and g_b for ^{40}K, respectively. From Table I, we see that this equality holds exactly only for (i) the Schmidt values, for which the result is true by definition. However, for (ii) and (iii) we do not find such equality. [Note, however, $g_a(^{38}\mathrm{Cl})\simeq g_a(^{40}\mathrm{K})$ in column (iii).] The differences are not small; this is evidence that the assumption of a one-body operator for $M1$ is not valid.

In Tables II and III we compare the reduced matrix elements based on Eqs. (2.7), (2.8), and the g factors of Table I, with the experimental transitions[14] and moments[13] for ^{38}Cl and ^{40}K, respectively. For the moments we have the measured signs as well as magnitudes. However, for the transitions, only the absolute magnitudes can be extracted from the data. We have taken the signs to agree with the predictions of the Schmidt g factors (or the odd-A neighbors, which agree). We show the experimental uncertainties with the extracted reduced matrix elements in the first column: These are much smaller for the moments than for the transitions.

In the second column we give the value of Eq. (2.7) for the particular matrix element, for the three choices of g_a and g_b of Table I. For ^{38}Cl, we use the three estimates (2.9a)−(c) for g_b, given in Table I, column (ii).

In the third columns of Tables II and III we show the difference between the experimental values and those obtained from Eq. (2.7) with the various choices of g_a and g_b. We note that the residual differences are, in general, small compared with the experimental matrix elements themselves. The dif-

TABLE III. Reduced matrix elements for moments and $M1$ transitions of ^{40}K in units of the nuclear magneton, μ_0.

J_i	J_f	Experimental (μ_0)	One body part (μ_0)	Residual (μ_0)
			(i) -5.61	(i) 1.29
4	4	$-4.35\pm3\times10^{-7}$	(ii) -4.19	(ii) -0.16
			(iii) -4.25	(iii) -0.10
			(i) -4.98	(i) 1.06
3	3	$-3.94\pm9\times10^{-3}$	(ii) -4.17	(ii) 0.23
			(iii) -4.09	(iii) 0.14
			(i) 2.44	(i) 0.27
3	4	$2.71\pm9\times10^{-2}$	(ii) 2.77	(ii) -0.06
			(iii) 2.51	(iii) 0.20
			(i) 2.09	(i) 0.41
2	3	$2.52\pm2\times10^{-1}$	(ii) 2.40	(ii) 0.12
			(iii) 2.17	(iii) 0.35
			(i) -2.12	(i) 0.44
5	4	-1.69 ± 10^{-1}	(ii) -2.43	(ii) 0.74
			(iii) -2.19	(iii) 0.50

ferent values of the g factors clearly produce differences in the results in the third column: Least squares fitting tends to give the smallest remainders, but we note that ^{40}K is much better fit than is ^{38}Cl. Even the least squares fit leaves a bigger remainder than can be explained by the experimental uncertainties alone. The existence of the nonzero differences in these tables gives us more evidence that the $M1$ operator is not adequately represented by $\hat{O}_{\rm I}$. In the next section we examine the evidence for a two-body part: $\hat{O}_{\rm II}$.

III. TWO-BODY TRANSITION OPERATOR

We now examine the role of the two-body $M1$ transition operator $\hat{O}_{\rm II}$. In principle, we could determine all the two-body matrix elements of this operator (up to signs) for j-shell orbits (a,b) from the measured transitions and moments of the nucleus with configuration (a,b). However, this experimental information is never complete: Although cases for which all transitions are measured exist, usually only the ground state magnetic moment is known (with rare exceptions: Some excited-state moments are also measured). In the case of ^{38}Cl$(d_{3/2},f_{7/2})$, all three $M1$ transitions and the ground state moment are known (see Fig. 1).

If we did, in fact, have all the experimental information to determine $\hat{O}_{\rm II}$ completely, we would then want to see whether the total $M1$ operator $\hat{O}\simeq\hat{O}_{\rm I}+\hat{O}_{\rm II}$ was adequate to explain $M1$ transitions and moments in nuclei with more than two particles, that is, whether one can neglect $\hat{O}_{\rm III}$ and higher rank operators. For example, with $\hat{O}=\hat{O}_{\rm I}+\hat{O}_{\rm II}$, we would be able to predict all the transitions and moments of the p-h nucleus ^{40}K, given the complete information for the p-p nucleus ^{38}Cl. For this comparison, we would need the linear algebraic relations between the p-p and p-h matrix elements of $\hat{O}_{\rm II}$. These will be derived shortly.

With the ability to connect the p-h matrix elements with the p-p matrix elements, we can overcome the limitation of having incomplete data for the p-p case alone. Consider the case of ^{38}Cl and ^{40}K. For ^{38}Cl we have four pieces of information, as mentioned. For ^{40}K we have five: three transitions and two moments (again, see Fig. 1). For the $d_{3/2}$-$f_{7/2}$ configuration, the operator $\hat{O}_{\rm II}(M1)$ has seven independent matrix elements, corresponding to three transitions and four moments. Therefore, we have more experimental information from ^{38}Cl and ^{40}K than needed to determine $\hat{O}_{\rm II}$. This leaves us two pieces of information $(2=9-7)$ to test for operators of higher (many-body) rank. In addition, we can predict the values of the unmeasured excited state magnetic moments of ^{38}Cl and ^{40}K.

We therefore need a (p-h) relation for the matrix elements of $\hat{O}_{\rm II}$. It is useful to begin by writing the operator in multipole form:

$$\hat{O}_{\rm II}=\sum_{gh}\beta^{gh}_{aabb}[gh]^{1/2}(U^g_{aa}\times U^h_{bb})^1\ , \tag{3.1}$$

where U^ν, $\nu=g$, or h is the one-body $(a\neq b)$ unit tensor operator defined in (2.4a), the bracket implying angular momentum coupling of two tensors to rank 1, and the β^{gh}_{aabb} are *multipole coefficients.* This form of expressing $\hat{O}_{\rm II}$ is most easily obtained using the second quantized notation for unit tensors; see Ref. 8. The multipole coefficients are related to the two-body reduced matrix elements by

$$\beta^{gh}\equiv\beta^{gh}_{aabb}=\sqrt{1/3}\sum_{JJ'}\langle ab;J||\hat{O}_{\rm II}||ab;J'\rangle\begin{Bmatrix}a & b & J\\ a & b & J'\\ g & h & 1\end{Bmatrix}\ , \tag{3.2}$$

where we use the unitary (bar) 9-j symbol (which is equal to $[g\ h\ J\ J']^{1/2}$ times the usual symmetric 9-j symbol). (We drop the orbital labels a, b, on β^{gh} from here on.) This relation (3.2) may be inverted to give

$$\langle ab;J||\hat{O}_{\rm II}||ab;J'\rangle = \sqrt{3}\sum_{gh}\beta^{gh}\left\{\begin{matrix}\overline{a} & \overline{b} & \overline{J}\\ a & b & J'\\ g & h & 1\end{matrix}\right\} . \qquad (3.3)$$

Note that there are an equal number of β^{gh} and matrix elements: They contain the same information. Thus (3.1) is the most general form for $\hat{O}_{\rm II}$ within the (a,b) shell model space.

If we take the matrix elements of $\hat{O}_{\rm II}$ in Eq. (3.1) between particle-hole states, and use the p-h relationship (2.5), and standard recoupling methods, we find

$$\langle a^{-1}b;J||\hat{O}_{\rm II}||a^{-1}b;J'\rangle = \sqrt{3}\sum_{gh}(-1)^{g+1}\beta^{gh}\left\{\begin{matrix}\overline{a} & \overline{b} & \overline{J}\\ a & b & J'\\ g & h & 1\end{matrix}\right\} + \sqrt{3}[a]\beta^{01}\left\{\begin{matrix}\overline{a} & \overline{b} & \overline{J}\\ a & b & J'\\ 0 & 1 & 1\end{matrix}\right\} , \qquad (3.4)$$

where the last term comes from the right-hand side (rhs) of Eq. (2.5). The two equations (3.3) and (3.4) show that both the p-p and p-h matrix elements of $\hat{O}_{\rm II}$ may be expressed in terms of the same set of multipole coefficients β^{gh}. We will use this relation later for analysis of experimental data.

We may also combine (3.2) and (3.4) to obtain a linear p-h relation among the reduced matrix elements themselves

$$\langle a^{-1}b;K||\hat{O}_{\rm II}|a^{-1}b;K'\rangle = \sum_{JJ'}(-1)^{K+J}\langle ab;J||\hat{O}_{\rm II}||ab;J'\rangle\left\{\begin{matrix}\overline{b} & \overline{a} & \overline{K}\\ a & b & K'\\ J & J' & 1\end{matrix}\right\} + \sqrt{3}[a]\beta^{01}\left\{\begin{matrix}\overline{a} & \overline{b} & \overline{K}\\ a & b & K'\\ 0 & 1 & 1\end{matrix}\right\} , \qquad (3.5)$$

where we have used properties of the 9-j symbols. This relation can also be inverted to give the p-p matrix elements in terms of the p-h matrix elements and β^{01}.

In order to apply the p-h transformation (3.5) to transition (or moment) matrix elements, we must also include the p-h transformation of the one-body part, given in Eq. (2.8), since $\hat{O}(M1)\simeq\hat{O}_{\rm I}+\hat{O}_{\rm II}$, by assumption. We can, in fact, put the p-h transformation in a more compact form by reinterpreting the last term of Eq. (3.5) as follows. First we note that

$$\left\{\begin{matrix}\overline{a} & \overline{b} & \overline{K}\\ a & b & K'\\ 0 & 1 & 1\end{matrix}\right\} = (-1)^{a-b-K'+1}\left[\frac{KK'}{a}\right]^{1/2} W(bKbK';a1) . \qquad (3.6)$$

Substituting this in the last term of (3.5), and adding the entire expression to that for the one-body part, using (2.7) and (2.8), we obtain the p-h transformation of $\hat{O}=\hat{O}_{\rm I}+\hat{O}_{\rm II}$ in the form

$$\langle a^{-1}b;K||\hat{O}||a^{-1}b;K'\rangle = [KK']^{1/2}(-)^{b-a+1}\{(-1)^{K}W(aKaK';b1)(a(a+1)[a])^{1/2}g_a' + (-1)^{K'}W(bKbK';a1)(b(b+1)[b])^{1/2}g_b'\} + \sum_{JJ'}(-1)^{K+J}\langle ab;J||\hat{O}_{\rm II}||ab;J'\rangle\left\{\begin{matrix}\overline{b} & \overline{a} & \overline{K}\\ a & b & K'\\ J & J' & 1\end{matrix}\right\} , \qquad (3.7)$$

where we have introduced new g factors:

$$g_a' = g_a(h) , \qquad (3.8a)$$

$$g_b' = g_b + \left[\frac{3[a]}{b(b+1)[b]}\right]^{1/2}\beta^{01} . \qquad (3.8b)$$

The g' factor for the b orbit has been modified to include the last term of (3.5). This is the appropriate g factor for a single neutron in the b orbit, where the a orbit is now filled with protons. In the original one-body equation (2.2), g_b was defined for an empty a orbit. The extra term, proportional to

β^{01}, is simply the extra contribution to the magnetic moment of the filled-a-shell-plus-b-particle nucleus, due to the $\hat{O}_{\rm II}$ operator between a and b particles. We can see this directly by calculating this magnetic moment from the matrix element of $\hat{O}_{\rm II}$. First we write $\hat{O}_{\rm II}$ in the form

$$\hat{O}_{\rm II}=\sum_{gh}(-1)^{g+1}\beta^{gh}[gh]^{1/2} \times(U^{g}_{aa}(h)\times U^{h}_{bb})^{1}+(3[a])^{1/2}\beta^{01}U_{bb}{}^{1}\ , \tag{3.9}$$

where we use (3.1) and (2.5). Then we take the matrix element for the filled a shell with one b particle, and obtain

$$\langle a^{[a]},b;J=b||\hat{O}_{\rm II}||a^{[a]},b;J=b\rangle=(3[a])^{1/2}\beta^{01}\ . \tag{3.10}$$

This corresponds exactly to the second term in (3.8b) as stated [multiplied by the coefficient $(b(b+1)[b])^{1/2}$, as in Eq. (2.3)].

Similarly, we must replace the particle g factor g_a by the hole g-factor $g_a(h)$, to correct for the filling of the a shell. In general, $g_a(h)\neq g_a$, because of two-body $M1$ operators among the a particles. [This is not included in our $\hat{O}_{\rm II}$ of Eq. (3.9).]

Therefore, when we analyze the transitions and moments for the particle-hole nucleus (a^{-1},b), we shall take the g factors from the neighboring odd-A nuclei (a^{-1}) and (b), with filled a shell. Thus, the procedure used in Sec. II with regard to the one-body operator, of using the nearest neighbor g factors, is actually justified for the present case of $\hat{O}(M1)=\hat{O}_{\rm I}+\hat{O}_{\rm II}$.

IV. NUMERICAL RESULTS

We apply the preceding theoretical framework to the analysis of all the measured $M1$ data for the pp-ph pair, ^{38}Cl and ^{40}K. As discussed earlier, not all magnetic moments are measured. We therefore analyze the two nuclei simultaneously, using the p-h relations of the previous section.

As mentioned in Sec. III, we have nine measured $M1$ matrix elements for the two nuclei together, which were given as reduced matrix elements in Tables II and III. We take the difference between each experimental entry and the one-body reduced matrix element to define the experimental two-body terms:

$$\begin{aligned}\langle ab;J||\hat{O}_{\rm II}(\exp)\,|\,ab;J'\rangle &= \langle ab;J||\hat{O}(\exp)\,|\,ab;J'\rangle \\ &\quad -\langle ab;J||\hat{O}_{\rm I}||ab;J'\rangle\ ,\end{aligned} \tag{4.1a}$$

and

$$\begin{aligned}\langle a^{-1}b;J||\hat{O}_{\rm II}(\exp)||a^{-1}b;J'\rangle &= \langle a^{-1}b;J||\hat{O}(\exp)\,|\,a^{-1}b;J'\rangle \\ &\quad -\langle a^{-1}b;J||\hat{O}_{\rm I}||a^{-1}b;J'\rangle\ ,\end{aligned} \tag{4.1b}$$

where the matrix elements of $\hat{O}_{\rm I}$ are given by Eq. (2.7), with g_a and g_b taken from the nearest odd-A neighbors, as follows: ^{38}Cl: g_a from ^{37}Cl, g_b from ^{37}S; ^{40}K: g'_a from ^{39}K, g'_b from ^{41}Ca. [See Eq. (3.8) and following discussion.] The values are taken from Table I, column (ii).

Since we do not have a measured value of the magnetic moment of ^{37}S, as discussed in connection with Table I, we have made use of the three different estimates for that value, given in that table, and are labeled (a), (b), and (c).

We use the multipole form of $\hat{O}_{\rm II}$, given in Eq. (3.1), since it allows us to treat the two nuclei symmetrically. The coefficients to be determined are the β^{gh} defined in Eq. (3.2). It can be shown using the symmetries of the 9-j symbols that there are seven independent nonzero coeffficients for the specific orbits $d_{3/2}$, $f_{7/2}$ which are relevant to ^{38}Cl and ^{40}K, namely,

$$(g,h)=(0,1);(1,0);(1,2);(2,1);(2,3);(3,2);(3,4)\ .$$

We determine these seven coefficients by making a least-squares fit to the nine experimental numbers of Eq. (4.1), using Eq. (3.3) for the ^{38}Cl data and Eq. (3.4) (having removed the second term, as explained in Sec. III) for the ^{40}K data. The resulting values of the β^{gh} are displayed in Table IV for the three choices of the ^{37}S g factor.

We note that the results do depend on the choice of the ^{37}S g factor; in particular, the two β's, β^{01} and β^{10}, which are themselves intimately related to these g factors, as shown, e.g., in Eq. (3.8). The values of the β^{gh} change monotonically with the value of g_b. We note further, that only in the last column do we find $\beta^{01}>0$. This came from the determination of g_b from Eq. (2.9c), which is further explained in the Appendix. There we note that β^{01} may be independently extracted from the least-squares fit of the g_a and g_b, given in Table I, for which we found $\beta^{01}=1.09$. This [see (A9)] value agrees best with that of the third column of Table IV.

On the other hand, using Eq. (2.9a) to determine g_b, we found $\beta^{01}=-0.55$, as in Eq. (A5). This agrees in sign with the value in the first column of Table IV. Therefore, both choices of β^{01} or g_b are self-consistent (as to sign; we have not tried to force equal values). However, the choice using (2.9a) has an additional assumption with respect to that of (2.9b), which is probably not warranted, namely, that ^{39}Ar has a weak-coupling ground state: $(d_{3/2})_0^2 f_{7/2}$, $J=\frac{7}{2}$. For Eq. (2.9b) we only require the shell assignments and $\hat{O}\simeq\hat{O}_{\rm II}+\hat{O}_{\rm II}$. Therefore, we could conclude that third choice for g_b on Table I is the best justified, and therefore, the values in the third column of Table IV might be the best determined.

In Tables V and VI we list the resulting two-body reduced matrix elements for ^{38}Cl and ^{40}K, respectively. These are compared to the experimental values of Eq. (4.1), which were used to determine the β^{gh}, and therefore, the theoretical values displayed. Note that the resulting matrix elements always agree in sign, and usually in order of magnitude, with the reduced experimental values, where available. The root mean square (r.m.s.) deviation between each theoretical case and the corresponding reduced experimental value may be found (combining the four values for ^{38}Cl and five values for ^{40}K):

$$\begin{aligned}&(a)\ \text{r.m.s.}=0.161\ ,\\&(b)\ \text{r.m.s.}=0.295\ ,\\&(c)\ \text{r.m.s.}=0.569\ .\end{aligned}\tag{4.2}$$

For comparison, the quoted experimental error is $\simeq 0.164$. From these numbers, it seems that (a) is the best choice for the ^{37}S moment.

TABLE IV. Values of β^{gh}_{aabb} obtained from least square fitting with three choices of $g_b(^{37}\text{S})$, as explained in text. (Magnetons.)

	(a)	(b)	(c)
β^{01}	−1.83	−1.02	0.36
β^{10}	−0.25	−0.77	−1.66
β^{12}	0.37	0.43	0.56
β^{21}	0.82	0.35	−0.46
β^{23}	−1.85	−1.29	−0.34
β^{32}	−0.03	0.04	0.14
β^{34}	0.97	0.88	0.70

V. DISCUSSION

Comparison of Tables V and VI with Tables II and III shows that inclusion of a two-body $M1$ operator $\hat{O}_{\rm II}$ improves the fit to experiment. By use of the p-h theory, we have been able to determine all the matrix elements of $\hat{O}_{\rm II}$ within the $d_{3/2}$-$f_{7/2}$ space. Thus, we predict values for the unmeasured magnetic moments of excited states of ^{38}Cl and ^{40}K. Our least-squares procedure determines the best values for $\hat{O}_{\rm II}$, depending, however, on an uncertain estimate of the magnetic moment of ^{37}S. The multipole coefficients of $\hat{O}_{\rm II}$ are given in Table IV.

Based on internal consistency, we found that (c) probably represented the best estimate of the unknown ^{37}S g factor. However, based on the r.m.s. values (4.2), we would conclude that (a) is the best choice. The picture is consistent (with either

TABLE V. Reduced matrix elements of $\hat{O}_{\rm II}$ for ^{38}Cl (magnetons), calculated from Eq. (3.3) using values of β^{gh} in Table IV.

J_i	J_f	$\hat{O}_{\rm II}$	Reduced experimental
2	2	(a) −2.09 (b) −0.67 (c) 1.79	(a) −2.01 (b) −0.62 (c) 1.77
3	2	(a) −0.72 (b) −0.12 (c) 0.93	(a) −0.98 (b) −0.42 (c) 0.56
3	3	(a) −1.55 (b) −1.04 (c) −0.16	
4	3	(a) 0.71 (b) 1.04 (c) 1.61	(a) 0.86 (b) 1.52 (c) 2.65
4	4	(a) 1.21 (b) 0.77 (c) 0.02	
5	4	(a) −1.01 (b) −0.63 (c) 0.04	(a) −0.86 (b) −0.29 (c) 0.69
5	5	(a) −3.66 (b) −2.84 (c) −1.43	

TABLE VI. Reduced matrix elements of $\hat{O}_{\rm II}$ for ^{40}K, calculated from Eq. (3.4) using values of β^{gh} in Table IV.

J_i	J_f	$\hat{O}_{\rm II}$	Reduced experimental
		(a) 4.36	
2	2	(b) 3.34	
		(c) 1.59	
		(a) −0.12	
3	2	(b) −0.12	−0.12
		(c) −0.12	
		(a) 0.12	
3	3	(b) 0.02	0.23
		(c) −0.16	
		(a) 0.32	
4	3	(b) 0.57	0.06
		(c) 1.00	
		(a) −0.18	
4	4	(b) −0.35	−0.16
		(c) − 0.63	
		(a) 0.55	
5	4	(b) 0.63	0.74
		(c) 0.76	
		(a) 2.41	
5	5	(b) 0.63	
		(c) −2.45	

choice) with the assumption that

$$\hat{O}(M1) \simeq \hat{O}_{\rm I} + \hat{O}_{\rm II} \ .$$

There may well be room for further terms of higher particle rank; they should be smaller than either $\hat{O}_{\rm I}$ or $\hat{O}_{\rm II}$. We note the $\hat{O}_{\rm I}$ alone gives a better picture of ^{40}K than of ^{38}Cl: This is not consistent with only $\hat{O}_{\rm II}$ corrections. The higher-order terms cannot be determined completely by the present sort of empirical analysis.

We expect that $\hat{O}_{\rm II}$ must have its origin in either or both configuration mixing and mesonic current effects. The most general investigation of the effects of configuration mixing for ^{38}Cl and ^{40}K has been given by Goode.[10] Since he did not obtain substantial agreement with experiment, we may assume that not all the contribution to $\hat{O}_{\rm II}$ can be assigned to configuration mixing. We might therefore expect that some of the remaining differences are due to mesonic current effects. These are now being investigated.

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APPENDIX

We explain the three methods of estimating the unknown g factor for ^{37}S, expressed in Eq. (2.9). If the $M1$ operator were strictly a one-body operator, $\hat{O}(M1) = \hat{O}_{\rm I}$, then (2.6c) would hold; for the $d_{3/2}$ shell, this leads to the equality given in Eq. (2.9b). For the other two we assume a one plus two-body form, $\hat{O}(M1) = \hat{O}_{\rm I} + \hat{O}_{\rm II}$, with $\hat{O}_{\rm I}$ given by Eq. (2.2) and $\hat{O}_{\rm II}$ by (3.1), within the shell-model space.

First we generalize the result of Eq. (3.10), in which we showed that the two-body contribution to the magnetic moment for a single b particle ($f_{7/2}$-neutron, here) with a closed a shell ($d_{3/2}$-protons) is proportional to the coefficient β^{01}. This gives a connection to the g-factor relative to no a-shell particles, expressed in Eq. (3.8b). A similar result obtains for any (even) number of a particles with total angular momentum zero, namely:

$$g_b(n_a) = g_b(0) + n_a \Delta g \ , \tag{A1a}$$

with

$$\Delta g = \left[\frac{3}{b(b+1)[ab]} \right]^{1/2} \beta^{01} \ , \tag{A1b}$$

as can be verified by a direct calculation of the reduced matrix element

$$\langle (a^n)^0, b; J = b \,||\, \hat{O}_{\rm II} \,||\, (a^n)^0, b; J = b \rangle \ , \tag{A2}$$

with $\hat{O}_{\rm II}$ given in Eq. (3.1). The result gives that of Eq. (3.10) for $n_a = [a] \equiv 2a+1$. For the present case of $a = d_{3/2}$, we have the three cases $n=0$, ^{37}S; $n=2$, ^{39}Ar; $n=4$, ^{41}Ca; the last being the closed a-shell case. If we *assume* that $^{39}{\rm Ar}_{\rm g.s.}$ ($J = \frac{7}{2}^-$) can be treated as a $J=0$ proton-pair plus $f_{7/2}$ neutron, then we may use the linear relation (A1) among the three g factors to write

$$g_b(0) = 2g_b(2) - g_b(4) \ , \tag{A3}$$

which is equivalent to Eq. (2.9a). From the measured magnetic moments, we find

$$g_b(^{41}{\rm Ca}) = -0.456 \ , \tag{A4a}$$

$$g_b(^{39}\mathrm{Ar}) = -0.372 , \tag{A4b}$$

and therefore

$$g_b(^{37}\mathrm{S}) = -0.287 , \tag{A4c}$$

which appears in Table I as entry (a) for g_b in ^{38}Cl. The numbers of Eq. (A4) with Eq. (A1) give a predicted value of β^{01} of

$$\beta^{01} = -0.55 . \tag{A5}$$

The last estimate of $g_b(^{37}\mathrm{S})$ is also based on Eq. (A1), but with a different interpretation. In Sec. II we found the best values of g_a and g_b for ^{38}Cl and ^{40}K separately, by least squares fitting of Eqs. (2.7) and (2.8) to the experimental transitions and moments. This procedure is equivalent to the following: We assume that $\hat{O}(M1)$ has a one-body and two-body part: $\hat{O}_{\mathrm{I}}$ given by Eq. (2.2), and

$$\hat{O}'_{\mathrm{II}} = \sqrt{3}\beta^{10}(U_{aa}{}^1 \times U_{bb}{}^0)^1 + \sqrt{3}\beta^{01}(U^0_{aa} \times U_{bb}{}^1)^1 , \tag{A6}$$

that is, by Eq. (3.1) with only β^{10} and β^{01} nonzero. Using Eq. (3.6), we can reinterpret the β^{01} term of (A6) as an additional contribution to g_b, similar to that of Eq. (3.8b), which is linear in the number of a particles: This is exactly given by Eq. (A1), with $n_a = 1$ for ^{38}Cl, and $n_a = 3$ for ^{40}K. (A similar result may be obtained for g_a, depending on β^{10}, but this term is not of interest here, since $n_b = 1$ for both cases.)

Now, fitting g_b for ^{38}Cl and ^{40}K is equivalent to finding a value of $g_b(0)$ and Δg in Eq. (A1) from the $n = 1$ and 3 cases. We may extract

$$g_b(0) \equiv g_b(^{37}\mathrm{S})$$

from the resulting relation

$$g_b(^{37}\mathrm{S}) = \tfrac{3}{2} g_b(^{38}\mathrm{Cl}) - \tfrac{1}{2} g_b(^{40}\mathrm{K}) , \tag{A7a}$$

using the values from Table I, column (iii), for the right-hand side. However, Eq. (A1) also applies to ^{41}Ca, whose g factor is measured, and given in (A4a). We could just as well use the following relation:

$$g_b(^{37}\mathrm{S}) = 2g_b(^{38}\mathrm{Cl}) - 2g_b(^{40}\mathrm{K}) + g_b(^{41}\mathrm{Ca}) \tag{A7b}$$

to determine the unmeasured ^{37}S g factor. Since these two results do not agree exactly numerically, we have arbitrarily taken the average of (A7a) and (A7b), namely,

$$g_b(^{37}\mathrm{S}) = \tfrac{7}{4} g_b(^{38}\mathrm{Cl}) - \tfrac{5}{4} g_b(^{40}\mathrm{K}) + \tfrac{1}{2} g_b(^{41}\mathrm{Ca}) , \tag{A7c}$$

which is Eq. (2.9c). Using the least fit values of g_b, from Table I (iii), and (A4a) for $g_b(^{41}\mathrm{Ca})$, we find

$$g_b(^{37}\mathrm{S}) = -0.747 , \tag{A8}$$

which was used in Table I (ii)-c. The equivalent value of β^{01} is

$$\beta^{01} = 1.09 . \tag{A9}$$

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