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Separable potential for the neutron-proton system

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A separable potential is proposed for the n-p interaction in $^1S_0$ and $^3S_1$-$^3D_1$ partial waves. In the singlet S state the potential fits new phenomenological phases rather accurately in the laboratory kinetic energy range 0–600 MeV; it is also capable of reproducing singlet effective-range parameters in close agreement with experiment. In the coupled state $^3S_1$-$^3D_1$ the potential provides a correct description of the deuteron data (E_D, p_D, Q, η), while at the same time it adequately fits modern phenomenological phases up to $E_{\rm lab} \gtrsim 500$ MeV in both the S and the D wave; triplet effective-range parameters are also in agreement with experimental data. Only the mixing parameter ϵ_1 deviates from predictions of phase-shift analyses. In the construction of the potential care was taken that its off-shell behavior be reasonable. As an eminent property it thus exhibits an off-shell behavior similar to the one of the Paris potential. In particular the $^3S_1$-$^3D_1$ potential yields a deuteron S-state wave function, which in momentum space shows a zero like the Paris potential. Still the model is simple enough to be of good use in modern computer codes for few-body systems and other nuclear applications.

[NUCLEAR REACTIONS Separable potential proposed for n-p interaction; fits new on-shell data; off-shell behavior compared to Paris potential.]

I. INTRODUCTION

Apparently one of the most important goals of studying $A > 2$ nuclear systems has always been to learn more about the fundamental nucleon-nucleon (N-N) interaction. Separable representations of particular N-N force models and separable potentials themselves have served as essential tools for introducing certain features of the N-N interaction into nuclear applications mainly on few-nucleon systems.[1] The development has required one to take into account more and more aspects of the N-N force in order to arrive at conclusive results from those systems. But it has also been extremely difficult to incorporate all notions about the N-N interaction both from theory and experiment into models that guarantee a good applicability, and thus allow for either an exact solution of few-body equations or at least an acceptable approximation thereof.

Separable representations of the N-N interaction were often criticized for being unreasonable from the point of view of theory. Indeed they usually did not contain the ingredients of dynamical concepts (meson exchange), which are generally accepted and prescribe the form of an N-N potential down to internucleon distances of about 1 fm. As a consequence, even refined separable potentials relying on a heuristic ansatz with parameters determined only from experimental data were found to differ significantly from potential models derived from first principles. Such deviations mainly occurred in (half) off-shell entities like, e.g., wave functions or electromagnetic form factors of the deuteron.[2] Clearly, results from applications of these potentials to few-body problems sensitive to such aspects were not in demand and often turned out to be unreasonable.[3,4]

In fact, for separable potentials not extremely complicated, it was already difficult enough to reproduce all experimental N-N data exactly. Above all, in the coupled $^3S_1$-$^3D_1$ state a realistic description of the deuteron could hardly be achieved while maintaining an accurate fit to all scattering data.

Aiming at a practical description (i.e., not too complex an ansatz, a relatively small number of parameters, low rank, usable in few-body calculations) of the N-N interaction, we were therefore led to search for, what is most urgently needed, a separable potential for the n-p system that

(i) carefully takes into account recent *n-p* experimental data (above all from new phase-shift analyses);

(ii) concerning its off-shell behavior is modeled on some potential which is (basically) derived from first principles (the Paris potential,[5] say), and hence gives proper consideration to established theoretical notions about *N-N* dynamics.

Here we are going to present our finding for the most important *n-p* partial waves, namely, the $^1S_0$ and $^3S_1$-$^3D_1$ states.

II. REVIEW OF EXPERIMENTAL INFORMATION

While reliable *p-p* data are at hand with relatively small errors, *n-p* data bear large uncertainties and still undergo considerable alterations. Hence, when dealing with the *n-p* system, it is worthwhile to look for a consistent and trustworthy set of bound-state data and scattering phases yielded by phenomenological phase shift analyses. We will discuss the results of energy-dependent and/or energy-independent analyses given by MacGregor, Arndt, and Wright[6] (MAW-X), by Arndt, Hackman, and Roper[7] (AHR-II), by Arndt and VerWest[8] (AV), by Bugg *et al.*[9] (BASQUE), and by Bystricky, Lechanoine, and Lehar[10] (BLL). The first three show the development of analyses since the last decade (including more and more precise data); the recent analyses (AV, BASQUE, and BLL) should be based on nearly the same data but differ in the search procedure, and a comparison of these three should give an estimate of the current status of phase shift analyses. To this end let us examine in more detail the situation in the partial waves we are addressing.

A. $^1S_0$ (*n-p*)

The recent nucleon-nucleon phase-shift analyses[8,9] predict $^1S_0$ (*n-p*) phase parameters which differ significantly from the previous results.[6,7] This is true in particular with respect to energy-dependent combined analyses of *n-p* and *p-p* experimental data. The new solution of Arndt and VerWest[8] relying on a largely extended data base comes up with $^1S_0$ (*n-p*) phase shifts lying considerably lower than the energy-dependent phases ob-

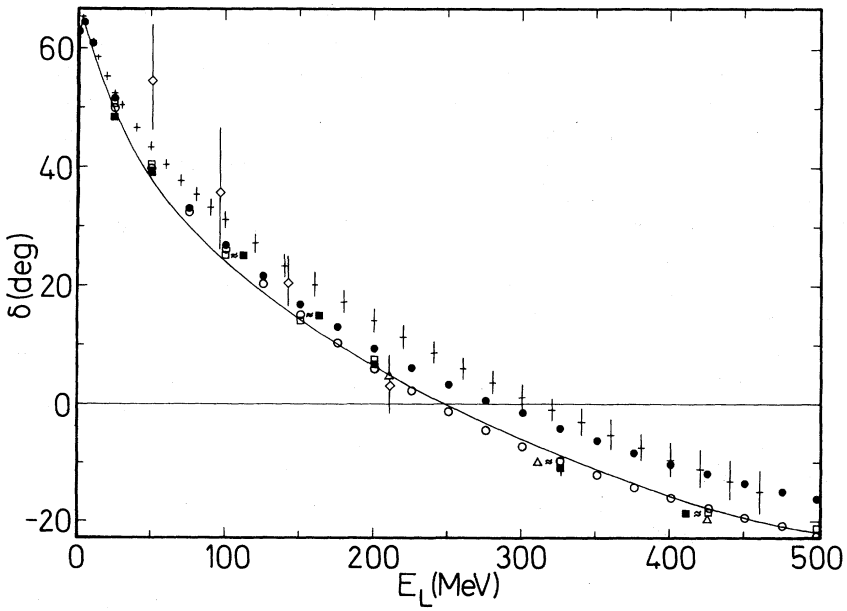

FIG. 1. $^1S_0$ (*n-p*) phenomenological phases and our separable potential prediction (solid line). Energy-dependent and energy-independent phenomenological phases of Arndt and VerWest (AV) (Ref. 8) are represented by circles (○) and squares (□), respectively. Triangles (△) give the result of an energy-independent analysis by Bugg *et al.* (BASQUE) (Ref. 9). Filled circles (●) and filled squares (■) refer to an earlier analysis of Arndt *et al.* (AHR-II) (Ref. 7) representing their energy-dependent and energy-independent solutions, respectively. Crosses (+) and diamonds (◇) show phenomenological phases with error bars as originally obtained for the $^1S_0$ (*n-p*) case in an energy-dependent (constrained solution), respectively, energy-independent analysis by MacGregor *et al.* (MAW-X) (Ref. 6).

TABLE I. Singlet effective-range parameters.

Experiment (Ref. 13)	Separable potential
$a_s=-23.748\pm0.010$ fm	$a_s=-23.76$ fm
$r_s=2.75\pm0.05$ fm	$r_s=2.69$ fm

tained in the AHR-II and MAW-X analyses. Deviations occur already at $E_{\text{lab}}\gtrsim 25$ MeV and grow with increasing energy. As a consequence, the zero of the $^1S_0$ (n-p) phase is shifted to a lower energy, namely, to $E_{\text{lab}}\simeq 240$ MeV; so it lies about 70 MeV lower than was the case in the MAW-X analysis and still 40 MeV lower than in AHR-II. Such a behavior of the $^1S_0$ (n-p) phase shift was in fact already called upon by the three latest energy-independent analyses AHR-II, BASQUE, and AV. The unpleasant gap between energy-dependent and energy-independent phase shifts, which still persisted in AHR-II, is removed now (see Fig. 1).

For the $^1S_0$ n-p state we have thus arrived at a stage where not only energy-dependent and energy-independent analyses yield similar results, but also predictions of different analyses by separate groups agree with each other. Consequently we may consider the results of the new phase-shift analysis AV to be quite reliable. Hence we are faced with phenomenological phase parameters, which indicate that the $^1S_0$ partial-wave interaction of the n-p system is considerably more repulsive than it could be assumed hitherto. The increased repulsion shows up already at moderate energies and has a profound influence on the shape of the phase-shift curve when the energy is rising.

Surprisingly enough, for the $^1S_0$ state, the n-p phase shift is thereby brought into close vicinity of the p-p phenomenological phase at energies $E_{\text{lab}}\gtrsim 25$ MeV. This means that, except for the low-energy domain,

(i) experimental data do not require a splitting of n-p phases (δ_s) and Coulomb-subtracted p-p phases[11] ($\delta_{sc}=\delta_{\text{tot}}-\sigma_{\text{Coul}}$) and

(ii) the Coulomb-distortion effect—and very probably all other indirect Coulomb effects—causing the difference $\Delta=\delta_{sc}-\delta_s$ may be considered rather unimportant; the latter issue being on concordance with theory.[12]

Whether the results of the AV analysis are consistent with low-energy effective range parameters is not quite clear. Former analyses[6,7] are not exactly in keeping with these data. Recent measurements

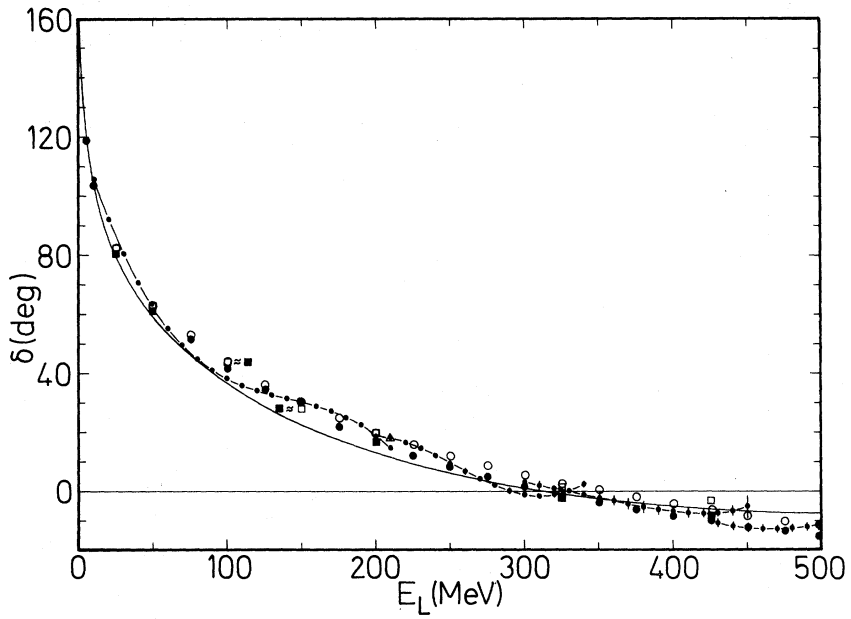

FIG. 2. $^3S_1$ phase shift for our new separable potential (solid line) compared to phenomenological data resulting from phase-shift analyses by Arndt and VerWest (AV) (Ref. 8) [energy-dependent (○) and energy-independent (□)], by Bugg *et al.* (BASQUE) (Ref. 9) (△), and by Arndt *et al.* (AHR-II) (Ref. 7) energy-dependent (●) and energy-independent (■). The dots (with error bars) linked by the dashed line represent the result of a locally energy-dependent analysis by Bystricky *et al.* (BLL) (Ref. 10).

suggest the values[13] given in Table I for singlet effective-range parameters.

B. $^3S_1$-$^3D_1$

Let us stay with phenomenological phases resulting from phase-shift analyses and thus treat the scattering problem first. Unfortunately the situation in $^3S_1$-$^3D_1$ is not in as good a shape as we have just found for $^1S_0$. Most likely because of the influence of the mixing parameter ϵ, which appears to be the most changeable quantity even in modern phase-shift analyses, the situation is not so clear cut. Taking a look at $\delta(^3S_1)$ in Fig. 2 one immediately observes that an adequate coincidence of phase-shift results from different groups is not yet reached. Likewise, predictions from energy-dependent and energy-independent phase-shift analyses do not yet agree perfectly. These unpleasant features are found also in the latest analysis, AV. In particular, corresponding energy-dependent phases at higher energies do not seem to be well supported if we compare with results of other groups. Above all the zero of the S-wave phase shift seems to lie at too high an energy as is indicated by the trend observed from AHR-II, BASQUE, and BLL, the latter analysis being a locally energy-dependent one. In the vicinity of the zero of $\delta(^3S_1)$ these analyses stand closely together; they furthermore agree with an energy-independent result by Bryan *et al.*,[14] which for $E_{\rm lab}=325$ MeV says $\delta(^3S_1)=0.5\pm1.9$ deg (not shown in Fig. 2).

The problem concerning the low-energy behavior of $^3S_1$ phases also does not seem to be perfectly settled. Recent data of triplet effective-range parameters are[13]

$$a_t(n-p)=5.424\pm0.004 \text{ fm} ,$$

$$r_t(n-p)=1.759\pm0.005 \text{ fm} .$$

While the AHR-II analysis is consistent with $a_t=5.38$ fm and $r_t=1.737$ fm, the low-energy limits of the other analyses are not clearly stated in the literature.

Next we examine $\delta(^3D_1)$. If we contrast corresponding results (see Fig. 3) of analyses just considered for $^3S_1$, we observe a deviation of AHR-II (energy dependent and energy independent) from all other groups BLL, BASQUE, and AV especially at energies above $E_{\rm lab}\approx200$ MeV. For $\delta(^3D_1)$ these latter solutions lie close together, suggesting a phase shift, which is more repulsive at high energies. But as indicated by the behavior of the locally energy-dependent analysis BLL with disparate solutions in overlapping energy regions, the dependence of $\delta(^3D_1)$ as a function of energy may generally not be so well determined as is represented by the smooth curve of some energy dependent analysis.

The situation is most unpleasant with respect to

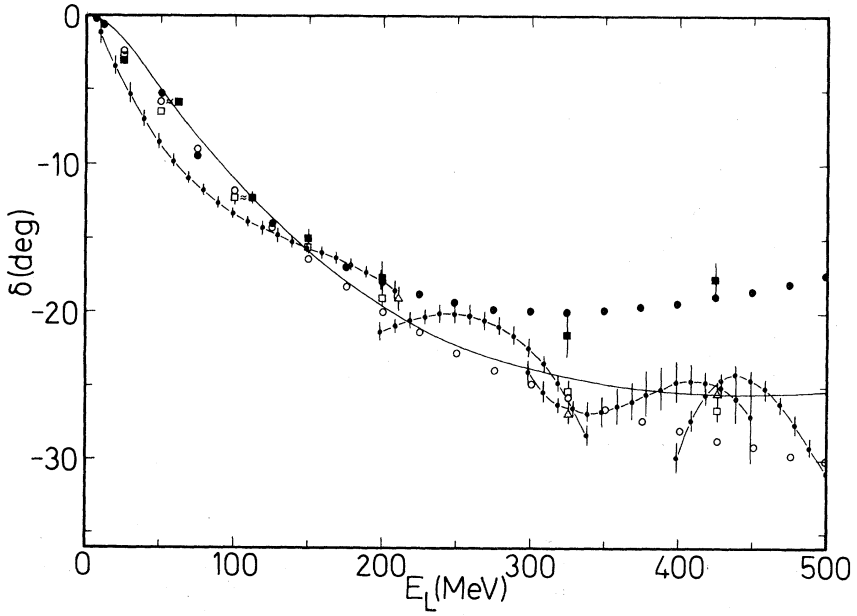

FIG. 3. $^3D_1$ phase shift of our new separable potential compared to phenomenological data. Description as in Fig. 2.

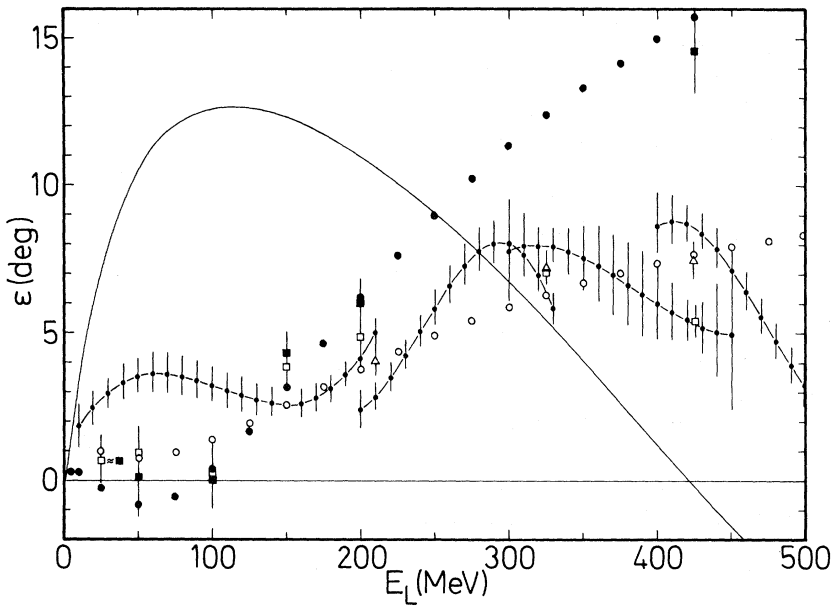

FIG. 4. Mixing parameter ϵ of our new separable potential compared to phenomenological data. Description as in Fig. 2.

the mixing parameter ϵ (see Fig. 4). Again we either find a coincidence of energy-dependent and energy-independent analyses within one group or an agreement of predictions by different groups. In the low-energy domain, AV, BLL, and AHR-II all lie apart, the latter solution even showing a mixing parameter which is negative around $E_{\text{lab}} \approx 50$ MeV. This property of the AHR-II result is the consequence of an erroneous argument[15] imposed as a constraint on the phase shift analysis. The other analyses relying on an even enlarged data base clearly rule out a negative mixing parameter and tend to raise ϵ at moderate energies. In particular they (BLL as well as AV) indicate a rapid increase of the low-energy ϵ and show a relative maximum at $E_{\text{lab}} \lesssim 50$ MeV; such characteristics of ϵ are additionally supported by two other energy-dependent analyses from Arndt *et al.*[16] and Arndt,[17] which for $E_{\text{lab}} = 50$ MeV yield $\epsilon \approx 2.78$ deg and $\epsilon = 2.97 \pm 0.85$ deg, respectively (not shown in Fig. 4).

Also at higher energies the behavior of ϵ remains ambiguous, although the three most recent analyses, BASQUE, BLL, and AV, altogether show smaller values than AHR-II. But, BLL demonstrates a multiple structure of solutions and generally the error bars are relatively large.

Now we discuss the experimental evidence on the deuteron. As can be seen in Table II, the quantities directly amenable to experiment are known to a high degree of accuracy. The weak point is the D-state probability. It has not yet become possible to constrain its value within a range of one percent or

TABLE II. Triplet effective-range parameters and deuteron properties.

	a_t (fm)	r_t (fm)	E_D (MeV)	Q_D (fm^2)	P_D (%)	η
Experiment	5.424 $\pm$0.004 (Ref. 13)	1.759 $\pm$0.005 (Ref. 13)	2.2246 $\pm$0.000 05 (Ref. 13)	0.286 $\pm$0.0015 (Ref. 29)	4–7	0.0256 $-$0.0277 (Refs. 18–21)
Separable potential	5.42	1.78	2.225	0.281	4.82	0.0274
Paris potential (Ref. 5)	5.43	1.77	2.225	0.279	5.77	0.0261

so. But it is generally believed that its value should not exceed the boundaries of 4 and 7 percent. There has been much work on the asymptotic D/S-state ratio η in the modern literature (for a compilation and critical discussion of values for η see, e.g., Refs. 18–20). The range of η given in Table II is certainly well established, the upper bound being extracted by inclusion of Coulomb corrections to the analysis of p-d scattering data.[21] We have only indirect experimental evidence of the deuteron wave function, e.g., via electromagnetic form factors measured in e-d scattering. Since in the present paper we will not treat these problems in much detail, we only take one quantity, namely the deuteron form factor $A(q^2)$, in order to check on the off-shell behavior of our potential. [For the definition of $A(q^2)$ see, e.g., Ref. 22.] As can be seen from Fig. 5, the form factor $A(q^2)$ is measured over a wide range of the momentum transfer q, wherefore it is well suited as an overall measure for the reliability of deuteron wave functions.

Having reviewed the status of experimental information in the partial-wave states that we are interested in, we now go ahead to describe our new separable potential.

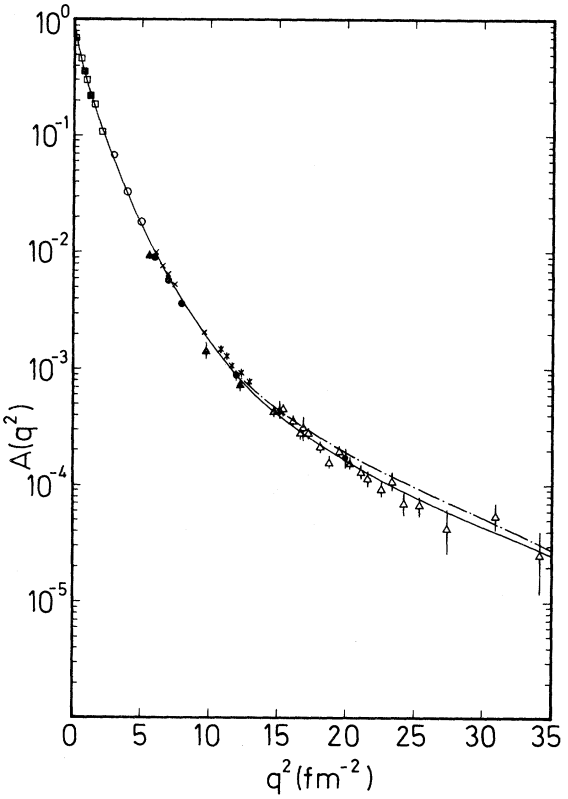

FIG. 5. Deuteron form factor A defined via the electron-deuteron cross-section formula

$$\frac{d\sigma}{d\Omega}=\left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}}\left[A(q)+B(q)\tan^2\frac{\theta}{2}\right]$$

as a function of q^2 for our new separable potential (——) and the Paris potential (—·—·—·). Experimental data are taken from Drickey *et al.* (Ref. 31) (□), Erickson (Ref. 32) (△), Buchanan *et al.* (Ref. 33) (●), Benaksas *et al.* (Ref. 34) (○), Grossetête *et al.* (Ref. 35) (■), Elias *et al.* (Ref. 36) (△), and Galster *et al.* (Ref. 37) (×).

III. SEPARABLE REPRESENTATION OF $^1S_0$ AND $^3S_1$-$^3D_1$ n-p INTERACTIONS

A. $^1S_0$

As we have seen in the discussion of the previous section, we are facing new evidence of phenomenological phases in $^1S_0$, which looks rather firmly established, but differs considerably from what was available before. Because of the increased repulsion present at higher energies, the shape just out could well lead to a change of the characteristics of the interaction. In order to take account of this new situation a description via a potential model is needed. Preferably such a potential model should be cast into separable form so that it easily lends itself to applications in Faddeev-type calculations of few-hadron systems, Brueckner-theory nuclear matter calculations, etc. Existing separable potentials for the $^1S_0$ n-p state (in particular the most frequently used[23] Mongan,[24] Graz,[25] and also other parametrizations), which are fitted to old data (MAW-X and even older ones), can no longer be considered to keep with modern phenomenological phases. Furthermore, these models provide only a poor fit to the latest values of low-energy parameters.[1] We were therefore led to construct a new separable potential for the $^1S_0$ n-p state. In order that it be applicable in existing computer codes (Faddeev calculations,...) we restricted it to be of rank 2 but prepared for more refined form factors

$$V(p',p)=g_1(p')\lambda_1 g_1(p)+g_2(p')\lambda_2 g_2(p) \tag{3.1}$$

with

$$g_1(p)=\frac{1}{p^2+\beta_{11}{}^2}+\frac{\gamma_1 p^2}{(p^2+\beta_{12}{}^2)^2}\ , \tag{3.2a}$$

$$g_2(p)=\frac{p^2}{(p^2+\beta_{21}^2)^2}+\frac{\gamma_2 p^4}{(p^2+\beta_{22}^2)^3}\ . \qquad (3.2b)$$

The parameters, whose numerical values are quoted in Table III, were determined via a least-squares fit to new AV (Ref. 8) phenomenological phases, where we imposed the constraint that also the latest low-energy parameters[13] were reproduced accurately up to experimental uncertainties. The quality of the fit may be seen from Fig. 1 and Table I.

Because of the fact that previous separable potentials mostly suffered from unreasonable off-shell properties,[2–4] we, in addition, took care of reproducing an off-shell behavior close to the one typical for local potentials, the Reid potential, say. Also the Paris potential,[5] which is basically derived from meson exchange, turns out to have a half-off-shell function quite similar to these[26]; since nowadays it is commonly considered to be a more adequate description of the N-N interaction we rather chose this model for comparison. In Fig. 6 we therefore give Noyes-Kowalski half-off-shell functions

$$f(p,k)=\frac{t(p,k;E_k)}{t(k,k;E_k)} \qquad (3.3)$$

for the original Graz potential[25]—as an example for usual separable potential behavior—, the Paris potential,[5] and our new separable potential. As can be seen from a comparison of the solid and dashed-dotted curves we succeeded in reproducing $f(p,k)$ similar to the one of the Paris potential. In particular we have a zero in the vicinity of the zero of $f(p,k)$ belonging to the Paris potential. Although this change of sign became an important property[4] it was hardly ever achieved by any separable potential.

From the on- and off-shell properties as exhibited by our separable potential, it is evident that by our choice of form factors (3.2) we achieved an excellent fit both to recent effective-range parameters as well as to modern phenomenological phases from low to rather high energies ($E_{\rm lab}>500$ MeV); at the same time we were able to keep the off-shell properties close to what nowadays can be considered reasonable.

TABLE III. Numerical values of separable potential parameters for $^1S_0$ (n-p).

$\beta_{11}=1.045\,56$ fm^{-1}	$\gamma_1=0.563\,486$ fm^0
$\beta_{12}=1.301\,48$ fm^{-1}	$\gamma_2=0.991\,115$ fm^0
$\beta_{21}=3.660\,99$ fm^{-1}	$\lambda_1=-52.7856$ MeV fm^{-1}
$\beta_{22}=2.053\,21$ fm^{-1}	$\lambda_2=48\,492.3$ MeV fm^{-1}

B. $^3S_1$-$^3D_1$

In order to provide a sufficient input for nuclear calculations we have to also treat the coupled $^3S_1$-$^3D_1$ state in the same spirit as we have just done for $^1S_0$. Only this time the task is much more complicated: On one hand it involves the deuteron bound state, wherefore the problem becomes intricate theoretically (mainly with regard to deuteron wave functions and electromagnetic form factors); on the other hand, concerning the scattering data, the experimental information is still vague, above all with respect to the mixing parameter, which causes the whole of the scattering data to remain far from being well determined.

We constructed a separable potential of rank 3, so that it should still be quite applicable in few-body calculations, which for L, $L'=0,2$ has the form (in the notation of Plessas *et al.*[1]):

$$V_{LL'}(p',p)=[g_{01}(p')g_{02}(p')g_2(p')]\Delta_L\begin{bmatrix}\lambda_{11}^{\ 0} & \lambda_{12}^{\ 0} & \lambda_{11}^{\ 02}\\ \lambda_{12}^{\ 0} & \lambda_{22}^{\ 0} & \lambda_{12}^{\ 02}\\ \lambda_{11}^{\ 02} & \lambda_{12}^{\ 02} & \lambda^2\end{bmatrix}\Delta_{L'}\begin{bmatrix}g_{01}\ (p)\\ g_{02}\ (p)\\ g_2\ (p)\end{bmatrix} \qquad (3.4a)$$

with the projection matrices

$$\Delta_0=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\quad \Delta_2=\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}, \qquad (3.4b)$$

and form factors

$$g_{01}(p)=\frac{1+\gamma_0 p^2}{(p^2+\beta_{01}^2)^2}\ , \qquad (3.4c)$$

$$g_{02}(p)=\frac{p^2}{(p^2+\beta_{02}^2)^2}\ , \qquad (3.4d)$$

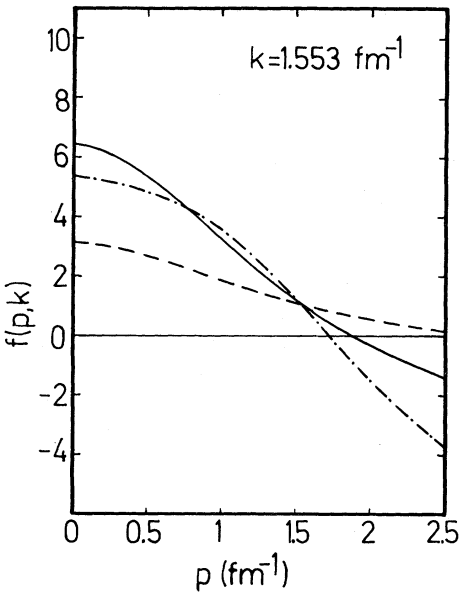

FIG. 6. Noyes-Kowalski half-off-shell function for the $^1S_0$ (n-p) state at n-p relative momentum $k=1.553$ fm^{-1} (corresponding to $E_{\text{lab}}=200$ MeV) for the separable Graz potential (Ref. 25) (— — —), the Paris potential (Ref. 5) (—·—·—·), and our new separable potential (——).

$$g_2(p)=\frac{p^2(1+\gamma_2 p^2)}{(p^2+\beta_{21}^2)(p^2+\beta_{22}^2)^2}\ . \tag{3.4e}$$

The numerical values of the parameters are quoted in Table IV. They were determined via a least-squares fit to a set of phenomenological scattering data suitably averaged over the results of BLL and AV. As constraints, we imposed that triplet effective-range parameters, the deuteron binding energy, quadrupole moment, and asymptotic D/S-state ratio were reproduced exactly, and a D-state probability of about 5 percent was reached. In addition, we took care of the off-shell behavior in order to bring it close to the one of the Paris potential. In particular, the S-state wave function of the deuteron in momentum space should meet the important requirement, like Paris, of passing through zero.

The quality of the fit to the scattering data may be seen from Figs. 2—4 and from Table II (triplet effective-range parameters). One notices an adequate description of $\delta(^3S_1)$ over the whole energy range up to $E_{\text{lab}}\gtrsim 500$ MeV. But there is an insufficiency in the shape of ϵ. Unfortunately when keeping all the above mentioned constraints we found no way of pinning down the mixing parameter at moderate energies around 100 MeV. The problem

TABLE IV. Numerical values of separable potential parameters for $^3S_1$-$^3D_1$.

$\beta_{01}=1.172\,58$ fm^{-1}	$\lambda_{11}^0=149.037$ MeV fm^{-5}
$\beta_{02}=2.643\,84$ fm^{-1}	$\lambda_{12}^0=-1036.24$ MeV fm^{-3}
$\beta_{21}=4.028\,34$ fm^{-1}	$\lambda_{22}^0=7863.27$ MeV fm^{-1}
$\beta_{22}=0.798\,223$ fm^{-1}	$\lambda_{11}^{02}=-968.452$ MeV fm^{-5}
$\gamma_0=1.117\,36$ fm^2	$\lambda_{12}^{02}=5096.75$ MeV fm^{-3}
$\gamma_2=2.530\,24$ fm^2	$\lambda^2=1433.43$ MeV fm^{-5}

is that a strong tensor force is needed at the bound-state pole for a reasonable description of the deuteron (exact quadrupole moment, D-state probability of about 5 percent), while the low-energy mixing parameter requires rather a weak tensor force. With our ansatz (3.4) it is certainly possible to fit all scattering data including an ϵ being rather small, but only at the cost of some deuteron property. Since the mixing parameter appears to be the least established experimental data (cf. Fig. 4), we rather decided to release this quantity from the weight of reproducing the prediction of a particular analysis, which anyway might not be the truth (remember the critique of experimental evidence of ϵ in the previous section). Furthermore, we have no safe indication that the influence of the mixing parameter in few-body calculations and probably other nuclear applications (like, e.g., nuclear matter) was predominant.[27] On the contrary, a recent investigation of n-d scattering by Koike *et al.*[28] using separable potentials with always the same deuteron wave function (half-off-shell behavior) but varying scattering properties (including inferior mixing parameters as well), demonstrated a strong sensitivity of the results on the S-state wave function rather than on subtle details of on-shell data (if the S-wave phase shift is kept in good order).

The description of the deuteron data is quite reasonable in our new potential (see Table II). At first sight the quadrupole moment could appear to be a bit too small (like for Paris), but one has to realize that theoretical values given in Table II represent impulse approximation results, while corrections from a relativistic treatment, meson-exchange currents, and isobar configurations add positively to these numbers.[30] Furthermore, the deuteron wave functions, notably the ones of the S state, look fairly appealing in configuration and momentum spaces (Figs. 7—10). In this respect our new potential represents a considerable improvement as compared to usual separable potential behavior[2] (like exhibited, e.g., by the former Graz

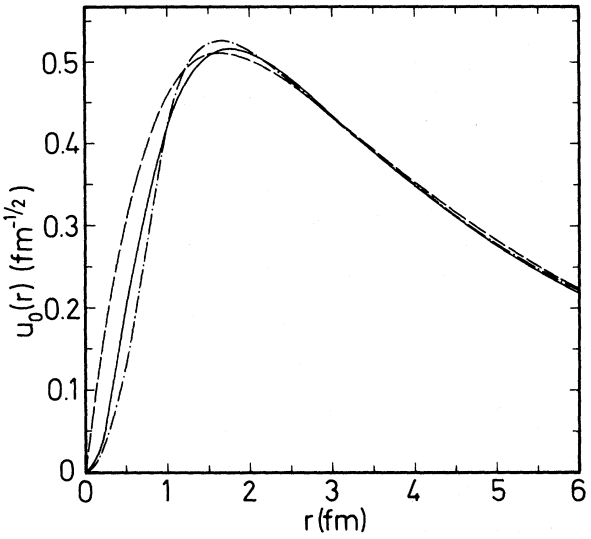

FIG. 7. *S*-state radial wave function of the deuteron in configuration space for the potentials of Fig. 6.

potential[25]). In particular, the *S*-state wave function in configuration space no longer rapidly increases at small distances, but rather shows a depression like Paris. Transformed to momentum space this property results in a zero of the *S*-state wave function at an *n-p* relative momentum of $p_0 \approx 2.4$ fm^{-1}. It turned out that this zero of $\psi_0(p)$, which occurs at not too high a relative momentum, amounts to an important feature of some *N-N* potential.[2-4,28] So it is not surprising that our new separable potential also gives a nice account of experimental data for the form factor $A(q^2)$, which we chose as an exam-

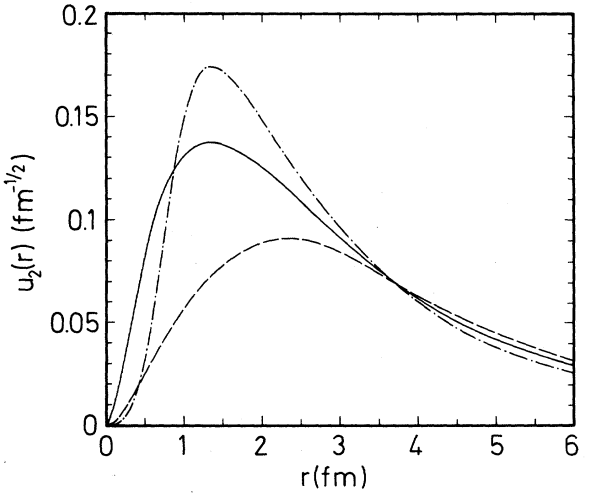

FIG. 8. *D*-state radial wave function of the deuteron in configuration space for the potentials of Fig. 6.

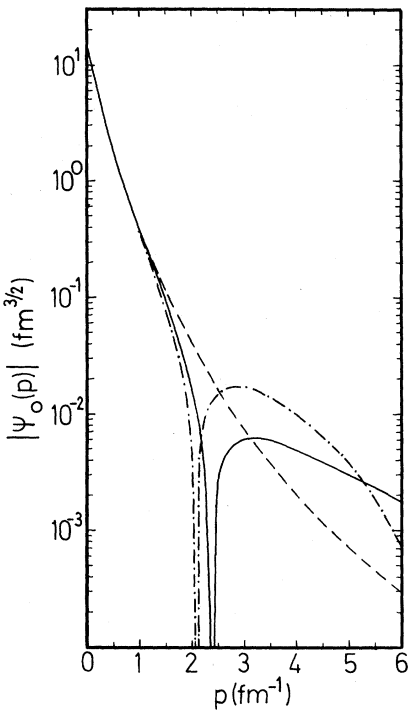

FIG. 9. *S*-state wave function of the deuteron in momentum space for the potentials of Fig. 6.

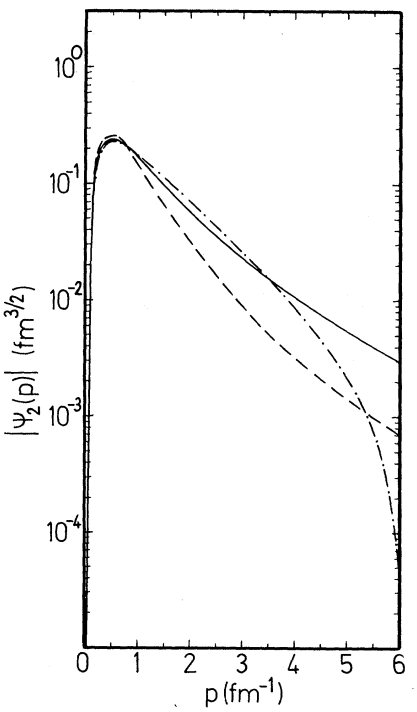

FIG. 10. *D*-state wave function of the deuteron in momentum space for the potentials of Fig. 6.

ple for comparison (Fig. 5). In fact, at higher momentum transfer, it even looks better than Paris, while at small q^2 both potentials give practically identical results. In order to show that the (half-) off-shell behavior at energies other than the bound-state pole also appears to be reasonable, we add in Fig. 11 an example on the Noyes-Kowalski half-off-shell function for the S-state defined according to Eq. (3.3). Here, too, we notice as an appealing feature that for the new potential this function passes through zero like in the case of Paris. Again we observe a striking improvement as compared, e.g., to the original Graz potential.

IV. CONCLUSION

We have presented a separable potential for the n-p interaction in $^1S_0$ and $^3S_1$-$^3D_1$ partial waves. It is capable of rather precisely reproducing all modern experimental data in these partial waves. In particular for $^1S_0$, it fits recent well established phenomenological phases up to $E_{\rm lab} \gtrsim 500$ MeV and gives accurate effective-range parameters. In the same manner also for $^3S_1$-$^3D_1$ scattering data are reproduced in accordance with recent phase-shift predictions; minor deviations only occur for the experimentally least determined quantity, namely, the mixing parameter ϵ. Deuteron properties are included very accurately.

The off-shell behavior of the separable potential was not left to arbitrariness. More readily it was modeled after the Paris potential in either partial waves. Therefore the potential also includes theoretical notions about the N-N interaction deduced from dynamical principles, which are generally accepted. As a consequence our potential no longer exhibits an unreasonable behavior of half-off-shell functions, as was often the case with earlier separable models. Above all, the deuteron wave function now appears fairly improved. In particular the zero of the S-state wave function in momentum space is present and lies close to the Paris one. Moreover, as demonstrated in a specific example for $A(q^2)$, the deuteron electromagnetic form factors are in good shape.

Still the form of the separable potential is not very complicated. For the singlet S state, it is of rank two, while for the coupled state $^3S_1$-$^3D_1$, it has (total) rank three—according to the rank of the matrix of potential strengths λ in Eq. (3.4a). The form factors, too, are rather simple rational functions, which in either partial-wave state contain only six open parameters. In addition to the potential strengths, the potential thus involves a total number of eight open parameters in $^1S_0$ and twelve open parameters in $^3S_1$-$^3D_1$. Therefore, from the point of view of practicability, our new separable potential should be well suited for few-body calculations and further nuclear problems, while it provides an overall realistic description of on- and off-shell properties of the N-N interaction.

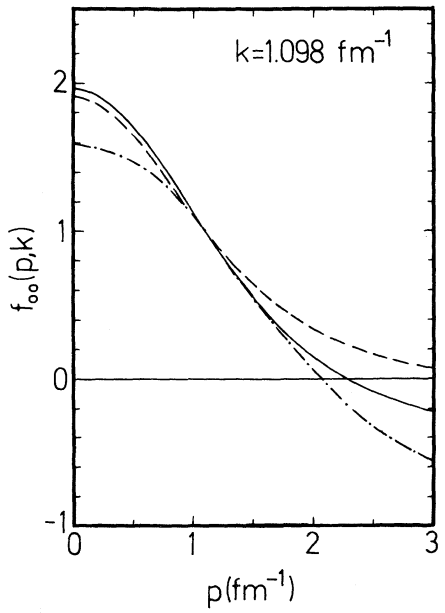

FIG. 11. Noyes-Kowalski half-off-shell function $f_{00}(p,k)$ at n-p relative momentum $k=1.098$ fm^{-1} (corresponding to $E_{\rm lab}=100$ MeV) for the potentials of Fig. 6.

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[11]We would like to mention that in all phase-shift analyses considered here $^1S_0$ (*n-p*) phase parameters were introduced in addition to $^1S_0$ (*p-p*) phases and varied separately during the combined analysis of *p-p* plus *n-p* experimental data. In a model-independent way this procedure was originally used in MAW-X, since it was then necessary to fit both data sets. The resulting differences between *n-p* and *p-p* phenomenological phases as exhibited by MAW-X and also AHR-II have sometimes given rise to speculations about violation of charge independence.

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