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Three-body approach to pionic ^{16}O

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Relativistic three-body calculations are presented for the real part of the π-^{16}O scattering length. The effects of the nucleon binding and the exclusion principle are studied in the framework of a covariant three-body model (pion, nucleon, and core) of the optical potential. The present theory is a natural generalization of the optical potential model of Celenza, Liu, and Shakin who consider only the single-triangle diagram, ignoring the nucleon rescattering. We find that the nucleon binding and the exclusion principle each have a large effect on the scattering length. However, the combined effect of both medium corrections is much smaller. Depending on the model for the πN t matrix, the single-triangle diagram alone accounts for 15% and 33% of the scattering length. With medium correction included, these numbers are 25% and 40%, respectively. Finally, we determined the s wave strength parameter B_0 in a phenomenological ρ^2 term.

[NUCLEAR REACTIONS Three-body model, nucleon binding effects, exclusion-principle corrections, pion-nucleus optical potential, pion-^{16}O scattering length.]

I. INTRODUCTION

One of the intriguing aspects of the low energy pion-nucleus interaction is the repulsion in the s wave, which is not well understood. The usual first-order optical potential models strongly underestimate this repulsion.[1] One obvious reason is the omission in the lowest-order term of higher-order contributions involving pion interactions with more than one nucleon. A second reason is that approximations such as the fixed scatterer, closure, and impulse approximation, which are often made in the evaluation of the first-order optical potential,[2] may not be adequate. This is of particular importance at low energies, since there the first-order term is the result of a rather delicate balance between attractive and repulsive πN partial-wave contributions.[3] Clearly, approximations are even harder to avoid in the evaluation of the higher-order contributions. Therefore, it is important to establish at least the first-order term in a reliable microscopic way. That being done, one may pin down interesting higher-order effects phenomenologically from the discrepancy between the first-order contribution and the experimental data.

The inadequacy of the fixed-scatterer approximation in the extremely low-energy regime has been demonstrated clearly in a very recent investigation by Bhalerao and Shakin.[4] These authors used a covariant optical potential model[5] including a full integration over the Fermi motion and a proper (relativistic) three-body subenergy choice in the elementary πN t matrix. They found that this treatment leads to a desirable increase in the s wave repulsion as compared to the more standard fixed-scatterer approaches. The optical-potential model used by Bhalerao and Shakin clearly represents a significant improvement over the fixed-scatterer theories. However, medium effects such as the nucleon binding and the exclusion principle in the intermediate scattering state are not included in it. *A priori*, there is little reason to assume that these effects are unimportant. In fact, large repulsive corrections have been reported in the literature as coming from the exclusion principle.[6] This effect is often included by means of a second-order term in the multiple-scattering series. Unfortunately, however, most evaluations of the exclusion principle effects are carried out in the framework of the fixed-scatterer or closure approximations, which leaves no room for a consistent handling of the nucleon-binding correction. This is a serious shortcoming since the nucleon binding and the exclusion principle are to be considered as two sides of the same coin in the following sense: If the nucleon binding in the intermediate scattering state is ignored, no Pauli-principle violating states enter the description. Pauli-principle corrections, e.g., through a second-order optical potential, are superfluous in such cases. If one aims at incorporating the nucleon

binding and the exclusion-principle effects consistently, one has to embark on nothing less than a three-body treatment (pion, nucleon, and core) of the optical potential.[7,8] In previous studies we have applied a three-body model in a calculation of the π-^{4}He scattering length.[8] Indeed we found a repulsive correction from the exclusion principle. Although this increased repulsion was partially compensated for by an attraction from the binding effect, the overall result of both medium corrections was shown to be repulsive.

In this work we present a study of the π-^{16}O scattering length. This quantity is known quite well from the 1s level shift and width in pionic ^{16}O. We will apply a covariant three-body theory for the optical potential.[9] In the limit of absent nucleon rescattering this theory reduces to the covariant formulation used by Bhalerao and Shakin.[4] Furthermore, we add a ρ^2 optical-potential term to our parameter-free first order optical potential to determine the higher-order effect parameter B_0 from a fit to the π-^{16}O scattering length. This way of determining B_0 avoids the difficulties discussed recently by Seki *et al.*,[10] and encountered in purely phenomenological studies, where the first-order term is also parametrized.

In Sec. II we briefly review the covariant three-body model as developed in Ref. 9. The calculational scheme used in the application to ^{16}O is presented in Sec. III. Finally, Sec. IV contains a discussion of our results.

II. COVARIANT THREE-BODY MODEL

The first-order optical potential in a physically sound definition accounts for pion scattering from a single bound nucleon, which experiences the nuclear medium through the binding interactions and the Pauli principle. It can be formulated as a three-body object if an independent-particle shell model is used for the nucleus. This was first pointed out by Revai.[11] The three-body formulation, contrary to the usual essentially two-body descriptions (impulse approximation) or one-body descriptions (fixed-scatterer approximation), allows for a consistent inclusion of the nuclear dynamics. In its most general form the three-body model is not very practical, since one would have to deal with integral equations in two continuous variables. To arrive at a practical model, we proposed in a previous investigation[8] the use of separable forms for the πN and nucleon-core (NC) interactions. In particular, for the subthreshold energy domain (e.g., pionic atoms) the assumption of separability should be reasonable for the NC interaction because here one expects a clear dominance of the bound state poles in the NC t matrix. Using separable interactions allows for a quasi-two-body treatment, which is a good starting point for numerical calculations of the optical potential. Furthermore, as is discussed in much detail in Ref. 9, the quasi-two-body formulation is very suitable for incorporating the requirements of relativistic covariance using the Blankenbecher-Sugar reduction technique.[12]

Assuming an independent-particle shell model, one can write the first-order optical potential matrix as $U^b_{ji}(\vec{k}',\vec{k};w)$, where the superscript b denotes the hole state and the subscripts i and j are the initial and final single-particle states, respectively. The πA momenta and total energy are given by $\vec{k}',\vec{k}$ and w, respectively. The optical potential in this notation is given by

$$U_{\rm opt}(\vec{k}',\vec{k};w)=\Sigma_b^{\rm occ}U^b_{bb}(\vec{k}',\vec{k};w)\ . \tag{1}$$

We also introduce a more simple auxiliary matrix $\tilde{U}^b_{ji}(\vec{k}',\vec{k};w)$ for pion bound-nucleon scattering in which both the Pauli principle and the required ground state exclusion are ignored. Then U and $\tilde{U}$ are related by the coupled integral equation

$$U^b_{ji}(\vec{k}',\vec{k};w)=\tilde{U}^b_{ji}(\vec{k}',\vec{k};w)-\Sigma_l^{\rm occ}\int\frac{d\vec{k}''}{(2\pi)^3}\frac{1}{2E_\pi(\vec{k}'')}\tilde{U}_{jl}(\vec{k}',\vec{k}'';w)G^b_l(\vec{k}'';w)U^b_{li}(\vec{k}'',\vec{k};w)\ , \tag{2}$$

where

$$G^b_l(\vec{k}'';w)=\frac{1}{2E^b_l(\vec{k}'')}\frac{1}{w^+-E_\pi(\vec{k}'')-E^b_l(\vec{k}'')} \tag{3}$$

and $E_{\pi'}$ and E^b_l are the energies of the pion and nucleus in the particle-hole states b, l, respectively. For each hole state b such a set of equations holds.

By means of Eq. (2) we can eliminate the exclusion-principle violating transitions and nuclear ground state transitions, which are implicitly present in the auxiliary matrix $\tilde{U}^b_{ji}$. This matrix $\tilde{U}^b_{ji}$ satisfies a set of coupled quasi-two-body equations if separable πN and NC interactions are assumed, as illustrated in Fig. 1. In general, $\tilde{U}^b_{ji}$ can be obtained by solving these equations. However, in the present case of subthreshold interactions, $\tilde{U}^b_{ji}$ is

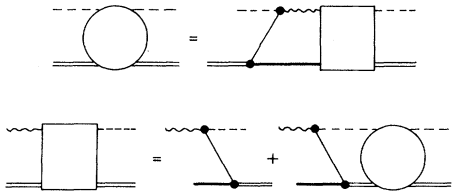

FIG. 1. Illustration of the coupled quasi-two-body equations for the auxiliary optical potential matrix $\tilde{U}^b_{ji}$. In $\tilde{U}^b_{ji}$ the nucleon interaction with the core in the intermediate system is taken into account (binding correction). Notice that $\tilde{U}^b_{ji}$, contrary to U^b_{ji}, may contain prohibited transitions to the Pauli blocked states or the nuclear ground state in the intermediate system. The dashed, single, and solid lines refer to the pion, nucleon, and core, respectively. The double and dashed lines refer to the interacting nucleon-core and interacting pion nucleon states, respectively.

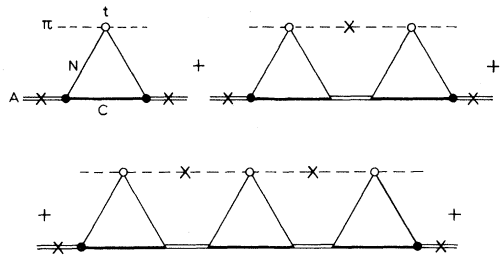

FIG. 2. At subthreshold energies we may calculate $\tilde{U}^b_{ji}$ by summing iterations of triangle diagram contributions. In the impulse approximation only the single triangle diagram is taken into account. The additional contributions where the pion is on-mass shell in the intermediate states represent binding corrections. The crosses denote which particles are taken on the mass shell.

found more directly by summing iterated triangle diagrams.[8] This is shown in Fig. 2. A crucial step in covariant treatments applying the Blankenbecler-Sugar reduction technique[12] is the mass-shell prescription of the particles and quasiparticles. We take the nucleus on the mass shell at the *external* lines and the pion at the *internal* lines (see Fig. 2). The important advantage of this choice is that it produces the correct nonrelativistic reduction.[9] As is illustrated in Fig. 3, the summation over the iterated triangle diagrams contributing to the optical potential can be carried out in two steps. The corresponding equations are

$$\chi^b_{ji}(\vec{k}',\vec{k};w)=V^{\mathrm{II},b}_{ji}(\vec{k}',\vec{k};w)+\sum_l\int\frac{d\vec{k}''}{(2\pi)^3}\frac{1}{2E_\pi(\vec{k}'')}V^{\mathrm{III},b}_{jl}(\vec{k}',\vec{k}'',w)\tilde{G}^b_l(\omega''_\pi)\chi^b_{li}(\vec{k}'',\vec{k};w) \tag{4}$$

and

$$\tilde{U}^b_{ji}(\vec{k}',\vec{k};w)=V^{\mathrm{I},b}_{ji}(\vec{k}',\vec{k};w)+\sum_l\int\frac{d\vec{k}''}{(2\pi)^3}\frac{1}{2E_\pi(\vec{k}'')}V^{\mathrm{II},b}_{jl}(\vec{k}',\vec{k}'';w)\tilde{G}^b_l(\omega''_\pi)\chi^b_{li}(\vec{k}'',\vec{k};w)\ . \tag{5}$$

Here $\tilde{G}^b_l$ is the propagator for the on-mass shell pion and interacting NC system

$$\tilde{G}^b_l(\omega_\pi)=\left\{(\omega^+_\pi-M^b_l)\left[1-(\omega^+_\pi-M^b_l)\int\frac{d\vec{Q}_{\mathrm{NC}}}{(2\pi)^3}\frac{1}{2M^b_l}\langle\vec{Q}_{\mathrm{NC}}\,|\,l\rangle\langle l\,|\,\vec{Q}_{\mathrm{NC}}\rangle\frac{1}{\omega^+_\pi-E_{C,b}(\vec{Q}_{\mathrm{NC}})-E_N(\vec{Q}_{\mathrm{NC}})}\right]2M^b_l\right\}^{-1} \tag{6}$$

and V^α with $\alpha=$I, II, III are the three triangle diagram contributions with different mass-shell prescriptions. They are of the following generic form:

$$V^{\alpha,b}_{ji}(\vec{k}',\vec{k};w)=\int\frac{d\vec{k}''}{(2\pi)^3}\frac{1}{2E_{C,b}(\vec{k}'')}\tilde{\rho}^b_{ji}(\vec{Q}'_{\mathrm{NC}},\vec{Q}_{\mathrm{NC}};\omega'_\pi,\omega_\pi)t^\alpha_{ji}(\vec{Q}_{\pi N},\vec{Q}_{\pi N};\omega''_{C,b})\ . \tag{7}$$

The quantity $\tilde{\rho}^b_{ji}$ is given by

$$\tilde{\rho}^b_{ji}(\vec{Q}'_{\mathrm{NC}},\vec{Q}_{\mathrm{NC}};\omega'_\pi,\omega_\pi)=\frac{M^b_j-E_N(\vec{Q}'_{\mathrm{NC}})-E_{C,b}(\vec{Q}'_{\mathrm{NC}})}{\omega'_\pi-E_N(\vec{Q}'_{\mathrm{NC}})-E_{C,b}(\vec{Q}'_{\mathrm{NC}})}\rho^b_{ji}(\vec{Q}'_{\mathrm{NC}},\vec{Q}_{\mathrm{NC}})\frac{M^b_i-E_N(\vec{Q}_{\mathrm{NC}})-E_{C,b}(\vec{Q}_{\mathrm{NC}})}{\omega_\pi-E_N(\vec{Q}_{\mathrm{NC}})-E_{C,b}(\vec{Q}_{\mathrm{NC}})} \tag{8}$$

and ρ^b_{ji} is the covariant density operator

$$\rho^b_{ji}(\vec{Q}'_{\mathrm{NC}},\vec{Q}_{\mathrm{NC}})=[64M^b_jM^b_iE_N(\vec{Q}'_{\mathrm{NC}})E_{C,b}(\vec{Q}'_{\mathrm{NC}})E_N(\vec{Q}_{\mathrm{NC}})E_{C,b}(\vec{Q}_{\mathrm{NC}})]^{1/2}\langle\vec{Q}_{\mathrm{NC}}\,|\,i\rangle\langle j\,|\,\vec{Q}'_{\mathrm{NC}}\rangle\ . \tag{9}$$

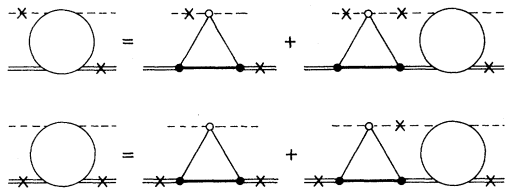

FIG. 3. Graphical representation of Eqs. (4) and (5) in which we sum over the iterations of triangle diagrams.

In Eqs. (7)–(9), $\vec{Q}_{\pi N}$ and $\vec{Q}_{\rm NC}$ are the relative two-body momenta. Furthermore, t^{α}_{ji} is the elementary πN T operator. In the absence of unambiguous prescriptions for off-mass shell models for t^{α}_{ji}, we will approximate it by an on-mass off-energy shell T operator. Such a procedure also underlies the covariant impulse approximation of Refs. 4 and 5. The subscripts ji remind us that πN partial-wave decomposition should agree with the initial and final single-particle states i and j. Adopting conventional three body notation, the quantities ω_{π} and $\omega_{C,b}$ are the invariant quasiparticle masses for the interacting NC and πN systems, respectively. When the pion is on the mass shell, ω_{π} becomes

$$\omega_{\pi}=[(w-m_{\pi})^2-2wT_{\pi}]^{1/2}\ , \tag{10a}$$

where T_{π} is the pion kinetic energy in the πA center of mass. If, on the other hand, the nucleus in the particle-hole state $\{i,b\}$ is on shell we simply get

$$\omega_{\pi}=M_i^b\ . \tag{10b}$$

In Eqs. (6)–(10) M_i^b is the invariant mass of the nucleus in the particle-hole state $\{i,b\}$. Notice that the fractions in Eq. (8) are equal to unity if ω_{π} is given by Eq. (10b). In V^I this is the case for both the initial and final states giving $\tilde{\rho}=\rho$. Therefore, V^I is just the optical potential in the covariant impulse approximation which has been discussed extensively in the work of the Brooklyn College group.[5,13] The present theory represents a natural generalization of their scheme, also taking nucleon scattering into account. In Sec. III we consider the application to the π-^{16}O scattering length.

III. CALCULATIONAL SCHEME

The general form of the theory outlined in Sec. II requires the solution of very large sets of coupled integral equations for each of the A hole states. Fortunately, this immense task only applies to the unnecessary complicated case of completely nondegenerate single-particle levels with a significant amount of NC partial-wave coupling. In our numerical calculations we have assumed degenerate $1s$ and $1p$ orbits. While the neglect of the Coulomb interaction and the spin-orbit force should not be too serious for a light nucleus such as ^{16}O, it does allow for a separation of the orbital and spin-isospin dependent part of the wave functions. With this separation one can treat the spin-isospin degrees of freedom in the coupled equations algebraically using the same methods as in Ref. 8 for the case of ^{4}He. The sum over the four intermediate single-particle states l in Eqs. (2), (4), and (5), which differ only in their spin or isospin parts, can be carried out trivially since the invariant mass M_l^b is identical for such states. Instead of matrix elements in the single-particle basis, we then consider matrix elements between the spin-isospin saturated four-nucleon states. This reduces the dimensionality of the coupled set of equations and the number of times we have to solve them by a factor of 4. This procedure can be carried through for spin-isospin symmetric nuclei if we restrict the summation over the single particle states l in Eqs. (4) and (5) to states which are occupied in the ^{16}O ground state. We ignore, therefore, virtual single-particle transitions to, e.g., the $1d$ or other excited levels. However, because of the small m_{π}/M_N ratio, one expects that coupling effects involving NC states with different orbital quantum numbers are small. This brings us to our second approximation, viz., the neglect of such couplings also between the occupied $1s$ and $1p$ orbitals. With the separation of orbital and spin-isospin degrees of freedom and the neglect of orbital couplings, the evaluation of the π-^{16}O optical potential has strong similarities to the previously studied π-^{4}He system, the major difference being that there are now four different orbital states: one for the $1s$ level,

$$\langle 1s\mid\vec{Q}_{\rm NC}\rangle=R_{1s}(\mid\vec{Q}_{\rm NC}\mid)(4\pi)^{-1/2}\ ;$$

and three $1p$ level states

$$\langle 1p,\mu\mid\vec{Q}_{\rm NC}\rangle=R_{1p}(\mid\vec{Q}_{\rm NC}\mid)Y_{1\mu}(\hat{Q}_{\rm NC})\ ,$$

differing in the magnetic quantum number μ. In the calculation of the π-^{16}O scattering length we have to consider πN S waves as well as P waves of which there are six in total. (Because of the Fermi motion, πN P waves play a role even when the pion is at rest with respect to the nucleon.) The procedure to find the optical potential is now as follows. First we have to calculate the three triangle diagram contributions $V_B^{\alpha}(TJL)$, with α=I, II, III [see Eq. (7)], for each of the four orbital states

denoted by B and the six πN partial waves, which are specified by the πN isospin T, total angular momentum J, and orbital angular momentum L. Equations (2), (4), and (5) lead to three equations for each NC orbital state B and πN partial wave TJL. In an abbreviated notation

$$\chi_B(TJL)=V_B^{\mathrm{II}}(TJL)+\frac{1}{4C_{JL}\cdot C_T}V_B^{\mathrm{III}}(TJL)\tilde{G}_B\chi_B(TJL)+\frac{1}{4C_T}\sum_{J';L'\neq L}V_B^{\mathrm{III}}(TJ'L')\tilde{G}_B\chi_B(TJL)\ , \tag{11a}$$

$$\tilde{U}_B(TJL)=V_B^{\mathrm{I}}(TJL)+\frac{1}{4C_{JL}\cdot C_T}V_B^{\mathrm{II}}(TJL)\tilde{G}_B\chi_B(TJL)+\frac{1}{4C_T}\sum_{J';L'\neq L}V_B^{\mathrm{II}}(TJ'L')\tilde{G}_B\chi_B(TJL)\ , \tag{11b}$$

$$U_B(TJL)=\tilde{U}_B(TJL)-\frac{1}{4C_{JL}\cdot C_T}\tilde{U}_B(TJL)G_BU_B(TJL)-\frac{1}{4C_T}\sum_{J';L'\neq L}\tilde{U}_B(TJ'L')G_BU_B(TJL)\ . \tag{11c}$$

The coefficients C_T and C_{JL} are readily obtained from the spin and isospin algebra. For $T=\frac{1}{2}$ and $\frac{3}{2}$, if $C_T=\frac{1}{3}$ and $\frac{2}{3}$, respectively, and $C_{JL}=1$ for πN S waves, and for P waves $\frac{1}{3}$ and $\frac{2}{3}$, if $J=\frac{1}{2}$ and $\frac{3}{2}$, respectively. The factor 4 comes from the number of nucleons in each spin-isospin saturated orbital state B. Equations (11a)–(11c) have to be solved and finally the $4\times6=24$ contributions $U_B(TJL)$ should be added,

$$U_{\mathrm{opt}}=\Sigma_B\Sigma_{TJL}U_B(TJL)\ . \tag{12}$$

This procedure is still quite involved. In particular, the numerical evaluation of the $3\times4\times6=72$ triangle diagrams $V_B^{\alpha}(TJL)$, which requires a three-dimensional integration over the recoil momentum [see Eq. (7)], is very time consuming. Therefore, we have applied one further simplification. It consists of replacing the three different $1p$ level density matrices

$$\langle\vec{Q}'_{\mathrm{NC}}\,|\,1p,\mu\rangle\langle\mu,1p\,|\,\vec{Q}_{\mathrm{NC}}\rangle$$

in Eqs. (6) and (7) by a single "spherical averaged" density matrix

$$\langle\vec{Q}'_{\mathrm{NC}}\,|\,1p\rangle\langle1p\,|\,\vec{Q}_{\mathrm{NC}}\rangle_{\mathrm{av}}=\tfrac{1}{3}\Sigma_{\mu}\langle Q'_{\mathrm{NC}}\,|\,1p,\mu\rangle\langle\mu,1p\,|\,\vec{Q}_{\mathrm{NC}}\rangle\ . \tag{13}$$

For a harmonic oscillator wave function this gives

$$\langle\vec{Q}'_{\mathrm{NC}}\,|\,1p\rangle\langle1p\,|\,\vec{Q}_{\mathrm{NC}}\rangle_{\mathrm{av}}=\frac{2}{3\pi\sqrt{\pi}}\frac{\vec{Q}'_{\mathrm{NC}}\cdot\vec{Q}_{\mathrm{NC}}}{(M_N\cdot\omega)^4}\exp\{-(\vec{Q}'^{\,4}_{\mathrm{NC}}+\vec{Q}^{\,4}_{\mathrm{NC}})/2M_N\omega\}\ . \tag{14}$$

The use of the spherical averaged density matrix together with the spherical symmetry of the nuclear Hamiltonian allows one to work with equations for the 12-nucleon state $|\,1p\rangle$, rather than with equations for each of the three 4-nucleon states $|\,1p,\mu\rangle$ separately. With this final modification, B in Eqs. (11a)–(11c) either refers to the $1s$ shell or the $1p$ shell. [Notice that for the $1p$ shell the factor 4 in Eqs. (11a)–(11c) should be replaced by a factor 12.] In this way the number of integrals and integral equations is reduced by a significant factor of 2. At this level of approximation, we have solved the resulting integrals and integral equations rigorously with standard numerical procedures.

IV. RESULTS AND DISCUSSION

Let us first summarize the ingredients that are used as an input in the calculations. From inspection of Eqs. (6) and (7) one observes that the basic nuclear structure input is given by the $1s$ and $1p$ shell wave functions $R_{1s}(\,|\vec{Q}_{\mathrm{NC}}|\,)$ and $R_{1p}(\,|\vec{Q}_{\mathrm{NC}}|\,)$ together with the averaged single-nucleon separation energies E_{1s} and E_{1p}. The masses of the core $M_{C,B}$ with a $1s$ or $1p$ hole state follow from

$$M_{C,B}=M_A-M_N-E_B\ .$$

We have assumed harmonic oscillator (HO) shell-model wave functions with different oscillator parameters a_0 and a_1 for the $1s$ and $1p$ shells. These parameters are adjusted to fit the experimental charge form factor according to the procedure described in Ref. 14. A center of mass correction has been taken into account. The values for the HO parameters and separation energies are collected in Table I. Furthermore, a model is required for the off-shell πN t matrix. We applied two well-known models in our calculations. Firstly, we considered

TABLE I. Summary of the ^{16}O input used in the calculations. These are based on harmonic oscillator shell-model wave functions. a_0 and a_1 are the respective 1s- and 1p-shell oscillator parameters. E_{1s} and E_{1p} are the respective 1s- and 1p-shell separation energies.

a_0 (fm)	1.76
a_1 (fm)	1.62
E_{1s} (MeV)	39.7
E_{1p} (MeV)	13.5

the very popular separable-potential model of Londergan, Moniz, and McVoy (LMM) (Ref. 15), which is based on inverse-scattering theory using the CERN theoretical phase shifts.[16] Secondly, we used a separable parametrization constructed by Schwarz, Zingle, and Mathelisch (SZM).[17] This model has been applied in πd calculations by several groups[18,19] and in our previous investigation of the π-^{4}He system.[8] For the P_{33} SZM give two descriptions. We have applied the model based on their Eq. (9), in which the position of the Δ resonance is fixed at the experimental value.[17] For the S_{31} and S_{11} we took the parameters obtained by Rinat *et al.*[18] in a fit to the recent πN phase shifts from Rowe *et al.*,[20] using the SZM model. Furthermore, as in our π-^{4}He studies, we kept the LMM model for the small P waves (P_{11}, P_{13}, and P_{31}) to limit the computation. These partial waves have only a minor effect on the results. (Partial wave Born cross sections in the impulse approximation at subthreshold energies for these partial waves were found to be a factor of typically 10^3-10^5 smaller than for the dominant S_{11}, S_{31}, and P_{33}.)

Results for the π-^{16}O scattering length (real part) are displayed in Table II. We have investigated four cases. Firstly, we considered the impulse approximation (ia), taking

TABLE II. The real part of the π-^{16}O scattering length in 10^{-3} fm. LMM and SZM denote the respective models for the off-shell πN t matrix. The impulse-approximation result is denoted by ia. In bd and ex, the binding correction and exclusion correction, are taken into account. For mc both of these medium corrections are included. The experimental result is taken from Koltun and Myhrer (Ref. 6).

	LMM	SZM
ia	−76	−179
bd	−59	−82
ex	−182	−328
mc	−127	−218
Experiment	−548	

$$U_B(TJL)\simeq V_B^I(TJL)\ . \tag{15}$$

Notice that the impulse approximation result is not modified by the use of the spherical-averaged density matrix [Eq. (13)]. Secondly, we performed a calculation in which only the nucleon binding is taken into account, ignoring the exclusion requirements (coming both from the Pauli principle and the required ground state exclusion). Here one uses the binding correction (bd)

$$U_B(TJL)\simeq\tilde{U}_B(TJL)\ . \tag{16}$$

Furthermore, it is of some theoretical interest to consider also the results obtained by using the exclusion correction (ex)

$$\tilde{U}_B(TJL)\simeq V_B^I(TJL) \tag{17}$$

and subsequently solving Eq. (11c). In this approximation one ignores the binding effect. Finally, the full calculation was carried through, taking both medium corrections (mc) into account. The scattering lengths were evaluated from the effective-range expansion to the s wave scattering amplitude, which we calculated by solving the relativistic Lippmann-Schwinger equation with the optical potential as the driving term.[5,8]

Qualitatively, the results are in agreement with the previous findings for the π-^{4}He case.[8] We find consistently larger values for the negative (repulsive) scattering lengths by using the more modern SZM model as compared to the older LMM model. Furthermore, we obtain a significant reduction of the s wave repulsion from the nucleon binding and a clear increase from the exclusion requirements. For the overall result in which both medium corrections are taken into account, we do find a net increase in the repulsion. However, this increase is not so large as what we found previously for the π-^{4}He system. In the present investigation we can account for 15% of the required repulsion from the impulse approximation with the LMM model, and 25% if the medium corrections are included. Using the SZM impulse approximation we get 33% and 40% with the medium corrections taken into account. This is to be contrasted with the π-^{4}He case[8] where we had 44% for the LMM model and even 75% for the SZM model with the medium corrections included.

Clearly, a large discrepancy still exists between the first-order contribution and the experimentally established s wave repulsion, even if medium effects are included. Various mechanisms have been proposed in the literature to explain this repulsion. A very recent suggestion was made by McManus and

Riska.[21] These authors found large repulsive effects from the intermediate excitation of virtual isobar-hole pairs in a "second-order" optical potential. (They referred to this mechanism as a medium correction. However, it is quite distinct from the medium corrections in the present discussion.) Notice that their second-order term, for which they applied a Fermi-gas estimate, actually corresponds in the independent-particle shell model context to the second iteration in Eq. (11c), which is predominantly a Pauli-principle correction term. An adequate evaluation of repulsive mechanisms such as virtual isobar-hole pair excitations[21] requires a level of sophistication in which dynamical correlations can no longer be ignored. However, at present there is no generally accepted, consistent scheme for a microscopic analysis of higher-order effects such as dynamical correlations, pion absorption, configurational mixing, etc., which lead to complicated intermediate np-nh states with $n>1$. (In the three-body model we only take into account the intermediate states with $n=1$.) Usually, one applies a ρ^2 parametrization to incorporate such higher-order effects.[1] The parameters appearing in it may eventually be understood on a microscopic basis. Some efforts in this direction are described in Refs. 22–24. It is quite important in such a phenomenological approach that the uncertainties in the first-order term are minimal. It has been shown recently by Seki *et al.*,[10] that the results from pionic-atom optical potentials in which the first-order term is also parametrized are not very reliable because of strong correlations between the parameters in the first-order and second-order parts. Therefore, it is of interest to determine the s wave parameters in the ρ^2 term by adding such a term to our parameter-free first-order optical potential and adjusting the ρ^2 parameters to the experimental π-^{16}O scattering length. We have applied the momentum space formulation of Liu and Shakin[13]

$$U^{h\cdot 0}(\vec{k}'\vec{k})=B_0 A^2 2M_A\left[1+\frac{m_\pi}{M_A}\right]\int\frac{d\vec{r}}{2\pi^2}\exp\{-i(\vec{k}-\vec{k}')\cdot\vec{r}\}\rho^2(\vec{r})\,, \tag{18}$$

where $\rho(\vec{r})$ is the nuclear matter density and B_0 is the complex s wave strength parameter as defined in Ref. 25. The results for the real and imaginary parts of B are listed in Table III. It is interesting to observe that the values for B_0 are in agreement with the results obtained by Liu and Shakin in a fit to the differential cross section for low energy π-^{16}O scattering.[13] For $T_\pi=40$ MeV they found

$$B_0=(0.11-i0.008)m_\pi^{-4}\,.$$

In their calculation the medium corrections were ignored. However, as we have seen, these appear to be relatively unimportant. Therefore, this agreement demonstrates that the present optical potential adjusted to pionic ^{16}O data can be extrapolated to scattering energies.

TABLE III. The real and imaginary parts of the complex πA S-wave absorption strength parameter B_0 in units of m_π^{-4}, as obtained by fitting to the π-^{16}O scattering lengths. LMM and SZM denote the respective models for the πN t matrix.

	ReB_0	ImB_0
LMM	0.13	−0.06
SZM	0.11	−0.07

Some word of caution is in order with respect to the interpretation of B_0. In view of the approximations which are still involved in our calculations, one cannot claim that B_0 derives its contributions exclusively from the higher-order effects. Nevertheless it seems unlikely that a more detailed treatment will lead to very different results. Our major simplification, the neglect of orbital couplings, can be justified as follows: (1) Effects from couplings between orbitals which are occupied in the ground state are suppressed by the Pauli principle. (2) Couplings to other orbitals are relatively unimportant in the pionic atom limit, where the dominance in the NC interaction of the bound states which are occupied in the ground state is maximal.

The main conclusion from our three-body calculations is thus that the nucleon binding and exclusion requirements each have a significant effect on the π-^{16}O scattering length. However, the overall correction from both medium effects is relatively small. A large discrepancy remains between the first-order prediction including medium corrections and the experimental value for the scattering length. This indicates the importance of more complicated mechanisms involving multiparticle-hole states in the intermediate system.

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