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Role of $\pi^0\eta'$ mixing in nuclear charge asymmetry

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We show that if all aspects of semistrong and electromagnetic particle mixing are properly taken into account, the result is that the $\pi^0\eta$ and $\pi^0\eta'$ contributions to charge asymmetry of the nucleon force do not substantially modify the charge asymmetric NN scattering lengths due to $\pi^0\eta$ mixing alone, and the $\pi^0\eta'$ effect is $\sim\frac{1}{3}$ the (already small) $\pi^0\eta$ contribution to the ^{3}He-^{3}H mass difference.

[NUCLEAR REACTIONS Charge asymmetric nuclear forces, electromagnetic mixing of pseudoscalar mesons, effects in two-nucleon and three-nucleon systems.]

I. INTRODUCTION

The study of charge asymmetry in one-boson-exchange contributions to the NN interaction is an illustration of the continuing interplay between ideas in particle physics and nuclear physics. For example, soon after the group-theoretic tadpole picture of Coleman and Glashow[1] successfully parametrized electromagnetic mass splittings, the tadpole picture of particle physics was extended to estimate the strength of the electromagnetic mixing of the exchanged mesons $\rho^0\omega$ and $\pi^0\eta$ in nuclear force models.[2,3] Later on, attempts were made to incorporate dynamical models of particle mixing into discussions of charge asymmetry.[4] Still later, it was shown that these complications were not necessary to explain the mixings.[5] Indeed, the tadpole picture and better estimates of coupling constants yield predictions[5,6] of nuclear charge asymmetry which are generally accepted in the literature.[7,8] These predictions are consistent with the nuclear experimental evidence from low energy scattering data[9,10] and the ^{3}He-^{3}H mass difference.[11] That is, the charge asymmetric NN $^1S_0$ potential due to particle mixing, $\Delta V=\Delta V_{nn}-V_{pp}$, is found to be attractive, and $\rho\omega$ mixing dominates significantly over $\pi^0\eta$ mixing.

Nowadays, however, one would like to relate electromagnetic (em) mass splittings and em mixings to a realization of the group-theoretic structure in terms of quark masses; for example, the up-down *current* quark mass difference. In the context of this paradigm, it was argued that the $\pi^0\eta'$ mixing contribution, heretofore neglected in NN charge asymmetry, is more important and of opposite sign than the $\pi^0\eta$ contribution.[12] According to Ref. 12, "meson mixing is determined from quark mass differences" and "quark mass differences now provide a theoretical framework for the use of phenomenological meson mixing as in previous calculations." In our opinion, this is turning the problem around; for charge asymmetry one can make predictions directly from the tadpole picture and the observed em hadron mass splittings. From this point of view the (deduced) quark masses are irrelevant.

Put another way, the perturbative quark model of Refs. 12 and 13 is one of two individually self-consistent but alternative current quark model realizations of chiral symmetry breaking.[14,15] It then becomes important to reexamine $\pi^0\eta'$ and $\pi^0\eta$ mixing within a picture which is as *model independent as possible;* in particular, independent of the two sets of quark mass ratios. Fortunately, the ambiguities of the singlet to octet ratios in the SU(3) tadpole picture of Refs. 5 and 6 can be minimized by exploitation[16] of the gluon interactions of quantum chromodynamics (QCD). The resulting tadpole predictions of $\pi^0\eta$ and $\rho^0\omega$ mixing agree satisfactorily with hadron em mass splittings and also $\eta_{3\pi}$ and $\omega_{2\pi}$ decays.[16,17]

In this paper we examine the role of $\pi^0\eta'$ mixing (and of $\pi^0\eta$ mixing) in nuclear charge asymmetry. We find that a consistent determination of the $\Delta I=1$ transition matrix elements $\langle\pi^0|H_{\rm em}|\eta\rangle$ and $\langle\pi^0|H_{\rm em}|\eta'\rangle$ can be made in a relatively model-independent way by relating the strength of quark annihilation diagrams to meson masses.[16,17]

In Sec. II we make a simple, but model dependent, estimate of $\langle \pi^0 | H_{em} | \eta \rangle$ and $\langle \pi^0 | H_{em} | \eta' \rangle$ and immediately proceed to solving the problem correctly in Sec. III. We display and discuss the resulting charge asymmetric NN potentials in Sec. IV, and their effects in the two-nucleon and three-nucleon systems in Sec. V. Final remarks appear in Sec. VI.

II. PRELIMINARY REMARKS

Before proceeding to the detailed derivation of the transition matrix elements $\langle \pi^0 | H_{em} | \eta \rangle$ and $\langle \pi^0 | H_{em} | \eta' \rangle$, we show (i) how to make a simple estimate of their relative strengths and (ii) that the assumptions used in this estimate lead to the relative strengths used in Ref. 12. We begin with the well-known SU(3) matrix element

$$\langle \pi^0 | H_{em} | \eta_8 \rangle = (\sqrt{3})^{-1}(\Delta m_K{}^2 - \Delta m_\pi{}^2) \approx -0.0030 \text{ GeV}^2 , \quad (1)$$

where $\Delta m_K{}^2 = m_{K^+}{}^2 - m_{K^0}{}^2$ and $\Delta m_\pi{}^2 = m_{\pi^+}{}^2 - m_{\pi^0}{}^2$. If one neglects $\eta\eta'$ mixing, then η is taken to be η_8 of the pseudoscalar octet and η' the singlet η_1. With that assumption

$$\langle \pi^0 | H_{em} | \eta \rangle = \langle \pi^0 | H_{em} | \eta_8 \rangle ,$$

and we need a further assumption about the transformation properties of H_{em} to find $\langle \pi^0 | H_{em} | \eta_1 \rangle$. One can make (i) the quark model assumption (Zweig rule) that the $s\bar{s}$ states of η and η' do not mix with the pion, and (ii) the tadpole dominance assumption that the $s\bar{s}$ part of H_{em} due to the internal photon loop is small. Then the quark model decompositions

$$| \eta_8 \rangle = (\sqrt{6})^{-1} | \bar{u}u + \bar{d}d - 2\bar{s}s \rangle$$

and

$$| \eta_1 \rangle = (\sqrt{3})^{-1} | \bar{u}u + \bar{d}d + \bar{s}s \rangle$$

lead directly to

$$\langle \pi^0 | H_{em} | \eta_1 \rangle = \sqrt{2} \langle \pi^0 | H_{em} | \eta_8 \rangle .$$

If one stops at this point and neglects both semistrong $\eta\eta'$ mixing and the internal photon loop as was done in Ref. 12, one finds that the $\sqrt{2}$ ratio in the transition strengths (in Ref. 12, $\langle \pi^0 | H_{em} | \eta \rangle = -.0038$ GeV^2 and $\langle \pi^0 | H_{em} | \eta' \rangle = -0.0054$ GeV^2) is simply a manifestation of assumptions (i) and (ii), rather than quark mass ratios.

But one cannot neglect $\eta\eta'$ mixing since one must use physical states η and η' in the meson-exchange models of the NN potential. Taking into account the phenomenological mixing angle[16] $\theta \approx -13°$ of the physical states η and η' relative to the singlet-octet basis, defined through

$$| \eta \rangle = \cos\theta \, | \eta_8 \rangle - \sin\theta \, | \eta_1 \rangle , \quad | \eta' \rangle = \sin\theta \, | \eta_8 \rangle + \cos\theta \, | \eta_1 \rangle , \quad (2)$$

the tadpole transition results are changed significantly. Combining (1), (2), and assumptions (i) and (ii) yields the conventional Zweig rule results

$$\langle \pi^0 | H_{em} | \eta \rangle = (\cos\theta - \sin\theta\sqrt{2}) \langle \pi^0 | H_{em} | \eta_8 \rangle \approx -0.0039 \text{ GeV}^2 ,$$
$$\langle \pi^0 | H_{em} | \eta' \rangle = (\sin\theta + \cos\theta\sqrt{2}) \langle \pi^0 | H_{em} | \eta_8 \rangle \approx -0.0034 \text{ GeV}^2 , \quad (3)$$

a sizable reduction of $\langle \pi^0 | H_{em} | \eta' \rangle$ from that of Ref. 12.

Looking ahead to the actual potentials $\Delta V^{\pi\eta}$ and $\Delta V^{\pi\eta'}$, we note that they are scaled by the ratios

$$\langle \pi^0 | H_{em} | \eta \rangle / (m_\eta{}^2 - m_\pi{}^2)$$

and

$$\langle \pi^0 | H_{em} | \eta' \rangle / (m_{\eta'}{}^2 - m_\pi{}^2) .$$

Since $m_\eta{}^2 \approx 16 m_\pi{}^2$ and $m_{\eta'}{}^2 \approx 50 m_\pi{}^2$, (3) suggests that the contribution of $\eta'\pi$ mixing would be suppressed by about a factor of 4 relative to $\eta\pi$ mixing. We shall see that this reduction factor decreases to about 3 when coupling constants are considered and the heretofore assumed $\sqrt{2}$ singlet to octet ratio is replaced by the model independent mixing to be presented now.

III. $\pi^0\eta'$ MIXING

The effective Hamiltonian density can be decomposed as

$$H_{em} = H_{JJ} + H_{tad} , \quad (4)$$

where

$$H_{JJ} \equiv -\tfrac{1}{2} i e^2 \int d^4x \, D^{\mu\nu}(x) T^*(J_\mu^{em}(x) J_\nu^{em}(0)) \quad (5)$$

arises from the internal photon loop and in the quark model H_{tad}, defined by

$$H_{tad} \equiv c' H(\lambda_3) \equiv \tfrac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d) , \quad (6)$$

is the small, $O(\alpha)$, $\Delta I = 1$ tadpole density. While both H_{JJ} and H_{tad} transform like $\Delta I = 1$, present opinion is that $H_{\rm tad}$ and the current quark mass difference $m_u - m_d$ in (6) are not of electromagnetic origin. (Nevertheless we follow conventional notation and denote the sum of H_{JJ} and $H_{\rm tad}$ as $H_{\rm em}$.) However, we will *not* need to know $m_u - m_d$ in order to compute $\langle \pi^0 | H_{\rm em} | \eta, \eta' \rangle$. We accomplish this by exploring the diagonal matrix elements of $H_{\rm em}$ which are related to meson electromagnetic mass splittings. If we make the definition

$$(H_{\rm em})_{\Delta P} \equiv \langle P^+ | H_{\rm em} | P^+ \rangle - \langle P^0 | H_{\rm em} | P^0 \rangle \qquad (7)$$

for a pseudoscalar meson P, then

$$(H_{\rm em})_{\Delta P} = (H_{\rm tad})_{\Delta P} + (H_{JJ})_{\Delta P} = \Delta m_P{}^2 \; . \qquad (8)$$

The model dependence of the derivation leading to (3) lies in assumptions (i) and (ii), which imply a singlet to octet ratio of $\sqrt{2}$ in the transformation properties of $H_{\rm em}$, which is valid for $H_{\rm tad}$ but *not* for H_{JJ}. To avoid this assumption, we note that off-diagonal matrix elements of $H_{\rm em}$ are easily found in the basis $\eta_{\rm NS}$, containing only nonstrange quarks in the combination $(\bar{u}u + \bar{d}d)/\sqrt{2}$, and η_S, with quark content $s\bar{s}$. The physical η, η' states are then

$$\begin{aligned} |\eta\rangle &= \cos\phi\, |\eta_{\rm NS}\rangle - \sin\phi\, |\eta_S\rangle \; , \\ |\eta'\rangle &= \sin\phi\, |\eta_{\rm NS}\rangle + \cos\phi\, |\eta_S\rangle \; , \end{aligned} \qquad (9)$$

where the mixing angle[16] $\phi = \theta + \tan^{-1}\sqrt{2} \approx 42°$ corresponds to $\theta \approx -13°$ in (2). Then, for example, (4) and (9) imply

$$\begin{aligned} \langle \pi^0 | H_{\rm em} | \eta' \rangle = {} & \sin\phi \langle \pi^0 | H_{JJ} | \eta_{\rm NS} \rangle \\ & + \cos\phi \langle \pi^0 | H_{JJ} | \eta_S \rangle \\ & + \sin\phi \langle \pi^0 | H_{\rm tad} | \eta_{\rm NS} \rangle \; , \end{aligned} \qquad (10)$$

since $H_{\rm tad}$ does not couple $|\pi^0\rangle$ and $|\eta_S\rangle$.

Now we need to estimate pseudoscalar matrix elements H_{JJ}. At the quark level, H_{JJ} receives contributions from single-photon exchange in the "scattering graphs" (of strength a_P) of Fig. 1 and

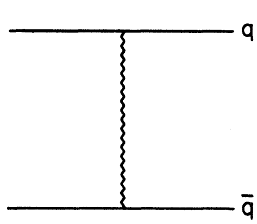

FIG. 1. Electromagnetic quark scattering diagram (strength a_P) contributing to H_{JJ}. The photon is denoted by a wavy line.

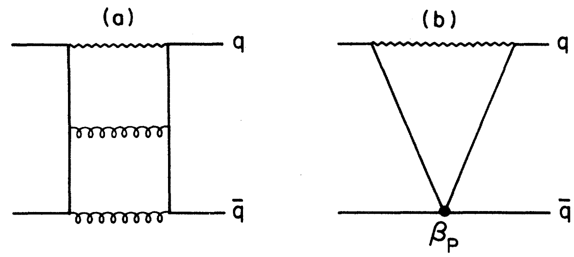

FIG. 2. (a) Electromagnetic quark annihilation diagram (strength γ_P). Only the minimal number of gluons (helical lines) consistent with C parity are shown. (b) The gluon box of (a) is approximated by the "point" of strength β_P, determined by semistrong annihilation diagrams similar to (a) (but lacking the photon).

single-photon exchange in the quark annihilation graphs (of strength γ_P) of Fig. 2. Hadronic matrix elements of H_{JJ} can be found using the SU(3) structure constants tabulated, for example, in Ref. 15. One can alternatively but equivalently couple the photon to the quark charges $(u,d,s) = (+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ and use the quark content of hadronic states, such as $|\pi^+\rangle = |\bar{d}u\rangle$ and $|K^+\rangle = |\bar{s}u\rangle$, to show that, for the charged hadronic states,

$$\begin{aligned} \langle \pi^+ | H_{JJ} | \pi^+ \rangle &= \langle K^+ | H_{JJ} | K^+ \rangle \\ &= \tfrac{2}{3}(-)(-\tfrac{1}{3}) a_P = \tfrac{2}{9} a_P \; , \end{aligned} \qquad (11)$$

where we note that the $I = 0$ quark annihilation diagram does not contribute to π^+ states. The other matrix elements of H_{JJ} are similarly[16]

$$\begin{aligned} \langle \pi^0 | H_{JJ} | \pi^0 \rangle &= -\tfrac{5}{18} a_P + \tfrac{1}{2}\gamma_P \; , \\ \langle K^0 | H_{JJ} | K^0 \rangle &= -\tfrac{1}{9} a_P \; , \\ \langle \pi^0 | H_{JJ} | \eta_{\rm NS} \rangle &= -\tfrac{1}{6} a_P + \tfrac{1}{6}\gamma_P \; , \\ \langle \pi^0 | H_{JJ} | \eta_S \rangle &= -\tfrac{1}{6}\sqrt{2}\gamma_P \; . \end{aligned} \qquad (12)$$

We now eliminate a_P in favor of meson mass differences by using the SU(3) properties of the tadpole

$$\begin{aligned} \langle \pi^0 | H_{\rm tad} | \eta_{\rm NS} \rangle &= (H_{\rm tad})_{\Delta K} \\ &= \Delta m_K{}^2 - (H_{JJ})_{\Delta K} \; , \end{aligned} \qquad (13)$$

where the second equality comes from (8). The tadpole does not contribute to pion mass splitting, so that

$$(H_{JJ})_{\Delta\pi} = \Delta m_\pi{}^2 = \tfrac{1}{2} a_P - \tfrac{1}{2}\gamma_P \; . \qquad (14)$$

Using Eqs. (11)–(14) to eliminate the unknown strength a_P from (10), we may express the required matrix element as

$$\langle \pi^0 | H_{\rm em} | \eta' \rangle = \sin\phi(\Delta m_K{}^2 - \Delta m_\pi{}^2) - \sqrt{1/6}\gamma_P \cos\theta \, , \quad (15)$$

where

$$\gamma_P = 3(H_{JJ})_{\Delta K} - 2(H_{JJ})_{\Delta\pi} = 3(H_{JJ})_{\Delta K} - 2\Delta m_\pi{}^2 \, , \quad (16)$$

has been employed to eliminate $(H_{JJ})_{\Delta K}$ in (13), and we have used the identities

$$\cos\theta = \sqrt{1/3}(\cos\phi + \sqrt{2}\sin\phi) \, ,$$

$\sin\theta = \sqrt{1/3}(\sin\phi - \sqrt{2}\cos\phi)$. The same analysis[16] for $\pi^0\eta$ mixing yields from (11)–(14) and (16),

$$\langle \pi^0 | H_{\rm em} | \eta \rangle = \cos\phi(\Delta m_K{}^2 - \Delta m_\pi{}^2) - \sqrt{1/6}\gamma_P |\sin\theta| \, . \quad (17)$$

Since $(\Delta m_K{}^2 - \Delta m_\pi{}^2)$ gives a negative contribution to (16) and (17), the quark annihilation graphs of Fig. 2 enhance both $\langle \pi^0 | H_{\rm em} | \eta \rangle$ and $\langle \pi^0 | H_{\rm em} | \eta' \rangle$ because γ_P also enters (15) and (17) with an associated minus sign.

We can estimate this strength γ_P of these em graphs by relating it to the semistrong quark annihilation diagram analogous to Fig. 2(a) (but without the exchanged photon) of strength[16]

$$\beta_P = \frac{(m_{\eta'}{}^2 - m_\pi{}^2)(m_\eta{}^2 - m_\pi{}^2)}{4(m_K{}^2 - m_\pi{}^2)} \approx 0.28 \text{ GeV}^2 \, , \quad (18)$$

which successfully explains semistrong $\eta\eta'$ mixing and leads to the mixing angles $\phi \approx 42°$ and $\theta \approx -13°$ used in (3). Given the scale of β_P, one can estimate γ_P by replacing one gluon by the photon and ignoring the off-shell momentum dependence of the gluon exchange box and replacing it by the "point" of strength β_P as depicted in Fig. 2(b). The result is[17]

$$\gamma_P = -\frac{i\alpha\beta_P}{\pi^3} \int \frac{d^4k(k^2 + m_{\rm con}{}^2)}{k^2(k^2 - m_{\rm con}{}^2)} \approx \frac{\alpha}{\pi}\beta \ln\frac{\Lambda^2}{m_{\rm con}{}^2} \approx 0.003 \text{ GeV}^2 \, , \quad (19)$$

where we have taken the QCD renormalization point mass of $\Lambda \approx 3$ GeV and an average nonstrange constituent quark mass $m_{\rm con} \approx 0.34$ GeV. The logarithm factor in (19) is insensitive to the ratio $\Lambda / m_{\rm con} \sim 10$.

The final result of inserting (19) into (15) and (17) then becomes

$$\langle \pi^0 | H_{\rm em} | \eta \rangle = (-0.0039 - 0.0003) \text{ GeV}^2 = -0.0042 \text{ GeV}^2 \, ,$$

$$\langle \pi^0 | H_{\rm em} | \eta' \rangle = (-0.0035 - 0.0012) \text{ GeV}^2 = -0.0047 \text{ GeV}^2 \, . \quad (20)$$

These results improve and correct the estimate of Ref. 5 and are less model dependent than the simple estimates of Eq. (3). The theoretical value $\langle \pi^0 | H_{\rm em} | \eta \rangle \approx -0.0042$ GeV^2 compares well with the "experimental" value[15–17] obtained from the decay $\eta \to 3\pi^0$,

$$| \langle \pi^0 | H_{\rm em} | \eta \rangle |_{\rm exp} \approx f_\pi{}^2 | M_{\eta 3\pi^0} | = 0.0048 \pm 0.0006 \text{ GeV}^2 \, . \quad (21)$$

In like manner, the analogous treatment of the vector meson transition yields[16,17]

$$\langle \rho^0 | H_{\rm em} | \omega \rangle_{\rm theory} \approx -0.0034 \pm 0.0003 \text{ GeV}^2 \, , \quad (22)$$

again in good agreement with the matrix element,

$$\langle \rho^0 | H_{\rm em} | \omega \rangle_{\rm exp} \approx -0.0034 \pm 0.0004 \text{ GeV}^2 \, , \quad (23)$$

extracted from $\rho \to 2\pi$ decay and the observed ρ^0-ω interference phase in $e^+e^- \to \pi^+\pi^-$ (see Refs. 8 and 18 for a critical remark on this extraction). We view these good agreements as lending strong support to the gluon-induced mixing scheme, the importance of annihilation diagrams, and the reliability of our theoretical estimate of $\langle \pi^0 | H_{\rm em} | \eta' \rangle$.

IV. SCATTERING AMPLITUDES AND POTENTIALS

We estimate the charge asymmetric contribution to nuclear forces due to mixing of pseudoscalar mesons in single particle exchange diagrams. The amplitude corresponding to the diagrams of Fig. 3 can be written as[5]

$$T_{NN}^{\pi\eta} = -g_\pi g_\eta \langle \pi^0 | H_{\rm em} | \eta \rangle \times \frac{\bar{u}(p_1')\tau^3\gamma_5 u(p_1)\bar{u}(p_2')\gamma_5 u(p_2)}{(m_\pi{}^2 - t)(m_\eta - t)} + (1 \leftrightarrow 2) \, , \quad (24)$$

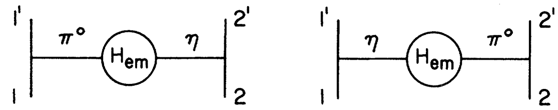

FIG. 3. Particle mixing diagrams contributing to charge asymmetry of the nuclear force. Here η stands for either η or η'.

where $t \equiv (p_1' - p_1)^2 = (p_2' - p_2)^2$, $\tau^3 | p \rangle = | p \rangle$, $S = 1 - iT$ fixes the sign of T, and η stands for either η or η'. The nonrelativistic reduction of (24) in the center-of-mass frame of the two nucleons is described in detail and the potential in all partial waves given in Ref. 18. The measurable effects of $\pi^0\eta$ and $\pi^0\eta'$ mixing appear to be strongest in the $^1S_0$ state. We therefore specialize to the $^1S_0$ coordinate space potential[5]

$$\Delta V^{\pi\eta}(r) = -\frac{g_\eta g_\pi}{4\pi} \frac{\langle \pi^0 | H_{\rm em} | \eta \rangle}{m_\eta{}^2 - m_\pi{}^2} \times \left[\frac{m_\pi{}^2}{M^2} \frac{e^{-m_\pi r}}{r} - \frac{m_\eta{}^2}{M^2} \frac{e^{-m_\eta r}}{r} \right], \quad (25)$$

where $\Delta V \equiv V_{nn} - V_{pp}$, M is the average nucleon mass, η stands for either η or η', and $g_\pi \equiv g_{\pi NN}$, $g_\eta \equiv g_{\eta NN}$, etc.

The relative importance of $\Delta V^{\pi\eta}$ and $\Delta V^{\pi\eta'}$ depends on the relative size of the coupling constants g_η and $g_{\eta'}$, and on the transition matrix elements of $H_{\rm em}$ in (20). The ratio of the couplings of η and η' to the nucleon can be read off from (9) and the Zweig rule, that $| \eta_S \rangle$, the $s\bar{s}$ state of η and η', does not couple to the nucleon. We find the very simple result, by including $\eta\eta'$ mixing (9) and employing $g_{\eta_S} = 0$:

$$g_{\eta'} / g_\eta = \tan\phi \approx 0.90 \; . \quad (26)$$

This ratio (26) is very *insensitive* to the semistrong mixing angle ϕ which is well understood[19] to be near 45°, i.e., $\theta \approx -11°$. Indeed, a more detailed diagonalization procedure incorporating quark-gluon annihilation graphs[16] leads to $\phi \approx 42°$ or $\theta \approx -13°$, which we have employed in (26). Note that the relative strength of g_η and $g_{\eta'}$ is fixed by the phenomenological $\eta\eta'$ mixing; *uncertainties* in d/f ratios at the SU(3) level and assumptions (similar to those of Sec. II) at *the quark level simply drop out of the ratio (26)*. This ratio (26) is smaller than the ratio $g_{\eta'} / g_\eta = \sqrt{2}$ assumed in Ref. 12 which ignores $\eta\eta'$ mixing [i.e., $\theta = 0$ in (2)]. We would like to emphasize that semistrong mixing cannot be neglected, and (26) is preferable.

Combining the transition matrix elements (20) with the coupling constant ratio (26) we deduce that

$$g_{\eta'} \langle \pi^0 | H_{\rm em} | \eta' \rangle / g_\eta \langle \pi^0 | H_{\rm em} | \eta \rangle \approx 1.0 \quad (27)$$

in (25) so that the overall strength of the $\Delta V^{\pi\eta'}$ is effectively scaled down relative to $\Delta V^{\pi\eta}$ by the ratio of the η to η' masses in the propagator denominators of (25):

$$(m_\eta{}^2 - m_\pi{}^2)/(m_{\eta'}{}^2 - m^2) \approx 0.31 \; . \quad (28)$$

Thus we might expect that the charge asymmetric effects of $\pi^0\eta'$ mixing would be reduced by about a factor of 3 from the already small effects of $\pi^0\eta$ mixing.[5,10,11]

Before calculating charge asymmetric effects, however, we must establish the absolute magnitudes of the coupling constants g_π and g_η [the ratio (26) then fixes $g_{\eta'}$]. The πNN coupling constant g_π is extracted from the nucleon pole in the process $\pi N \to \pi N$. Its value is very well determined to be[20]

$$g_\pi{}^2 / 4\pi \approx 14.3 \; . \quad (29)$$

Analogous extractions of the ηNN coupling constant g_η from the nucleon pole in the amplitude $\pi N \to \eta N$ differ by orders of magnitude: from the value[21] $g_\eta{}^2/4\pi \lesssim 0.15$ down to[22] $0.5 > g_\eta{}^2/4\pi \approx 0.0025$. Similarly, the discrepancy functions of a forward dispersion relation analysis of NN scattering[23] imply a small upper limit for this coupling constant; $g_\eta{}^2/4\pi \lesssim 1.0$.

A theoretical relationship between g_η and g_π can easily be derived with the SU(3) structure constants expressed in the strange-non-strange basis of (9). Again exploiting $g_{\eta_S} = 0$, we obtain the simple formula

$$g_\eta = \cos\phi \, g_{\eta_{\rm NS}} = \cos\phi \, g_\pi (3f - d) \; , \quad (30)$$

where $f + d = 1$. A "chiral perturbation theory" calculation predicts[24] a ratio $d/f = 2.37$ which implies $g_\eta{}^2/4\pi \approx 0.28$. Also, fits to the Cabibbo theory of semileptonic decays find an axial current d/f ratio of 1.86 ± 0.08. Moreover, the remaining coupling constants[25] in the pseudoscalar octet yield d/f ratios ranging around 2 which corresponds to the value, including $\eta\eta'$ mixing,

$$g_\eta{}^2 / 4\pi \approx 0.9 \; , \quad (31)$$

a little higher than the value used in Ref. 5, where θ was assumed to be zero, or $g_\eta \approx g_{\eta 8}$. The experimental determinations mentioned above imply $2 \lesssim d/f \lesssim 3$. We will use (31) for numerical calcula-

TABLE I. Shifts in low-energy scattering parameters as ΔV of (25) is added to the charge symmetric Reid and de Tourreil-Rouben-Sprung potentials, the contribution ΔE to the ^{3}He-^{3}H mass difference, and the volume integral J of ΔV. Similar results are reproduced from Ref. 6 for orientation.

	Reid		dTRS			
	Δa (fm)	Δr (fm)	Δa (fm)	Δr (fm)	ΔE (keV)	J (MeV fm^3)
Maximum $\pi^0\eta$	+0.07	−0.011	+0.25	−0.015	24	−0.025
Maximum $\pi^0\eta'$	−0.10	−0.001	+0.01	−0.004	8	−0.008
Total	−0.03	−0.012	+0.26	−0.019	32	−0.033
$\rho^0\omega$ contribution	+0.74	−0.014	+1.02	−0.021	56	−6.15

tions and remark that, although g_η is uncertain and g_η is sensitive to the d/f ratio, (31) probably provides an upper limit on the importance of pseudoscalar mixing in nuclear charge asymmetry.

Some NN force models have suggested values for g_η as large as $g_\eta{}^2/4\pi \approx 4$. This value requires a substantial breaking of SU(3) symmetry in a direction opposite to that suggested by particle physics. These large values of g_η are obtained *only* in one-boson-exchange models of the NN interaction. In these models the coupling constants for mesons more massive than the pion are taken as free parameters in a least squares fit to the phase shifts. Coupling constants obtained from these simple pole models should be considered neither realistic nor theoretical, for they must absorb both the neglect of the important contribution of the 2π continuum and all uncertainties about the true nature of the short range part of the NN interaction. Thus we return to (31) as a more objective upper bound of g_η.

V. CHARGE ASYMMETRIC EFFECTS IN TWO-BODY AND THREE-BODY SYSTEMS

We might expect from (25) that the charge asymmetric effects of $\pi^0\eta'$ mixing would be reduced by about a factor of 3 from the already small effects of $\pi^0\eta$ mixing.[5,10,11] The numerical results in two- and three-nucleon systems are presented in Table I. It is traditional to add a model for $\Delta V = V_{nn} - V_{pp}$ to a model for the charge symmetric interaction, and calculate the charge in the two-body scattering length Δa and effective range Δr. We chose the Reid soft-core potential[26] which, despite its name, has a large repulsion at small r, and the de Tourreil-Rouben-Sprung (dTRS) potential[27] which has a "super-soft core" and a meson-theoretic outer region. The values of Δa and Δr were obtained with the variable-phase method. The $^1S_0$ state comprises 90% of the $I=1$ component of the trinucleon wave function, so the contribution

$$\Delta E = \langle {}^3\text{He} \mid V_{pp} \mid {}^3\text{He} \rangle - \langle {}^3\text{H} \mid V_{nn} \mid {}^3\text{H} \rangle$$
$$= -\langle {}^3\text{He} \mid \Delta V \mid {}^3\text{He} \rangle$$

to the trinucleon binding energy difference can be estimated by considering only the $^1S_0$ part of ΔV. The values of ΔE were obtained with the nearly model-independent perturbative method based on the hyperspherical formula which is discussed in Ref. 11. We include comparative effects of $\rho^0\omega$ mixing[6] in Table I for orientation (a tabulation of effects of other mechanisms of charge asymmetry can be found in Ref. 10). The effects of pseudoscalar mixing in Table I are *maximum values*, because g_η of (31) is rather larger than the most likely final determination of g_η. Therefore the results of Table I can be scaled *down* for the reader's favorite value of g_η.

In Table I it is clear that the relative strengths of the two potentials are not reflected equally in the two-body and three-body systems. The contribution ΔE scales with the factor of 3 in the relative strengths, but the Δa is strongly dependent on the model of the charge symmetric interaction. If this model has a super-soft core, Δa from $\pi\eta'$ mixing is nearly zero; but if the model has a strong repulsive core, Δa from $\pi\eta$ mixing changes sign and the total Δa from pseudoscalar mixing is nearly zero. To understand this we must consider the radial dependence of the potentials in (25). They are not monotonic but are weakly repulsive at long range due to the Yukawa function weighted by $m_\pi{}^2/M^2$, and strongly attractive at short range because of the Yukawa weighted by the larger $m_\eta{}^2/M^2$ or $m_{\eta'}{}^2/M^2$. Indeed the reduction factor of about 3, as one replaces η by η', effects only the long range (pionlike) part of the potential because $m_\eta{}^2/(m_\eta - m_\pi{}^2) \approx 1$ for both η and η'. Evidently the results of Table I should be understood by realizing that integrals

TABLE II. The same quantities calculated with a single Yukawa potential (32) with unit strength $V_0=1$ MeV fm and masses corresponding to m_π, m_η, and $m_{\eta'}$. The results are scaled by the factor m^2/M^2 where M is the nucleon mass.

	Reid		dTRS			
	$\frac{m^2}{M^2}\Delta a$ (fm)	$\frac{m^2}{M^2}\Delta r$ (fm)	$\frac{m^2}{M^2}\Delta a$ (fm)	$\frac{m^2}{M^2}\Delta r$ (fm)	$\frac{m^2}{M^2}\Delta E$ (keV)	$\frac{m^2}{M^2}J$ (MeV fm^3)
π^0	0.0574	−0.0004	0.0593	−0.0004	2.783	0.555
η	0.0728	−0.0017	0.0909	−0.0017	5.043	0.555
η'	0.0374	−0.0036	0.0707	−0.0036	5.065	0.555

over $\Delta V^{\pi\eta}$ and $\Delta V^{\pi\eta'}$ have different sensitivities to the cancellation in (25), as first noted in Ref. 12.

We illustrate this sensitivity by first calculating the volume integral

$$J=4\pi\int Vr^2dr$$

of a single Yukawa potential

$$V=V_0e^{-mr}/r\ , \tag{32}$$

with unit strength $V_0=1$ MeV fm. The integral is $J=4\pi V_0/m^2$, so that m^2J is constant. The tabulation of J in Table I reflects the overall factor of $\sim\frac{1}{3}$ in (25) because the r^2 weighting factor in J preserves the cancellation between the long range and short range parts of ΔV as η is changed to η'.

In Table II we display Δa, Δr, ΔE, and J for potentials of type (32) weighted by the factor m^2/M^2, where $m=(m_\pi, m_\eta, m_{\eta'})$. The cancellation in (25) is clearly unaltered as η is changed to η' for J and ΔE, but not for Δa. We can see that for the scattering length calculation, the cancellation in (25) is quite different as η is changed to η', even to the extent of a change of sign of Δa. Presumably this is related to the perturbative formula

$$\frac{\Delta a}{a}=Ma\int\Delta V(r)\,|\,u_0(r)\,|^2dr\ , \tag{33}$$

which has a weighting factor of r, because the zero energy wave function $u_0(r)$ of the Reid potential is approximately proportional to $\sqrt{r}$ for r greater than a core radius.[6]

Finally, we remark that the nearly complete cancellation in Δa, with respect to the Reid potential, shown in Table I is not found in Ref. 12 for two reasons. The first is their neglect of $\eta\eta'$ mixing which, by a trivial extension of the discussion in Sec. II, yields their ratio

$$g_{\eta'}\langle\pi^0\,|\,H_{\rm em}\,|\,\eta'\rangle/g_\eta\langle\pi^0\,|\,H_{\rm em}\,|\,\eta\rangle=(\sqrt{2})(\sqrt{2})=2$$

rather than the correct ratio from (28) of unity. The second reason is their use of large pseudoscalar nucleon coupling constants which exacerbate the incomplete cancellation in Δa caused by their neglect of semistrong mixing.

VI. SUMMARY

In this work we have shown, taking into account *all* aspects of pseudoscalar mixing, that pseudoscalar $\Delta I=1$ transitions $\pi^0\eta$ and $\pi^0\eta'$ play a rather small role for nuclear charge asymmetry in the two-body and three-body systems. In our previous work on this subject,[5,6,10,18] we took into account semistrong $\eta\eta'$ mixing in the transition matrix element $\langle\pi^0\,|\,H_{\rm em}\,|\,\eta\rangle$, contrary to a claim made in Ref. 28. Recent efforts in this field[12,28] have found an enhancement of the effect of pseudoscalar mixing by considering rather large η_8NN and η_1NN coupling constants. In addition, Ref. 12 neglected $\eta\eta'$ mixing in both the $\Delta I=1$ transitions and the coupling constants, and Ref. 28 used the transition matrix elements of Ref. 12 and attempted to incorporate $\eta\eta'$ mixing in an inconsistent and incomplete manner. Here we show that if semistrong $\eta\eta'$ mixing is completely accounted for, and even more importantly, if the *electromagnetic* quark annihilation graph of Ref. 16 is incorporated in the calculation of $\langle\pi^0\,|\,H_{\rm em}\,|\,\eta\rangle$ and $\langle\pi^0\,|\,H_{\rm em}\,|\,\eta'\rangle$, then the combined contribution of $\pi^0\eta$ mixing and $\pi^0\eta'$ mixing is not substantially altered from the already small contribution from $\pi^0\eta$ mixing alone. The sensitivity of their effect on NN scattering lengths to the choice of a charge symmetric potential model is substantial, making it difficult to draw final conclusions. The scattering length difference, however, is the least well known of charge asymmetric effects.[10] On the other hand, the $\pi^0\eta'$ mixing contribution to the charge asymmetry in the $A=3$ bound state is $\sim\frac{1}{3}$ the already small contribution from $\pi^0\eta$ mixing, and the charge asymmetry in the $A=3$ bound state is well established.[11]

Thus we reaffirm the accepted[7,29] conclusions of

Refs. 5, 6, and 10 that the effect on scattering lengths of the $\Delta I = 1$ $\rho^0\omega$ transition is rather greater than that of the (combined) $\pi^0\eta$ and $\pi^0\eta'$ transitions. Furthermore, in the three-body system, the $\rho^0\omega$ transition dominates over the pseudoscalar transitions, but the relative sign of the combined $\pi^0\eta$, $\pi^0\eta'$, and $\rho^0\omega$ transitions is positive, providing closer agreement between theory and experiment.

After completion of this work, we received a report from the authors of Ref. 12. They have now[30] extended their analysis to include semistrong mixing so that the combined contribution from $\pi^0\eta$ and $\pi^0\eta'$ mixing to the NN scattering lengths of the Reid soft-core potential is small and negative, in qualitative agreement with Table I. They did not, however, display the effects on their results of the wider range of phenomenological charge symmetric potentials considered in Refs. 6 and 8. We continue to disagree with their choice of ηNN coupling constants, as their smallest value is 3 times our suggested upper limit (31). Finally, we are effectively extending their analysis to include $\langle \pi | H_{JJ} | \eta, \eta' \rangle$ contributions which were neglected in Ref. 30. We stress that our final expressions (15) and (17) are rigorously complete in the context of the quark model, which leads to 20% corrections to the overall $\pi^0\eta$ and $\pi^0\eta'$ strengths relative to Ref. 30.

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