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Lifetime of interference effects in collisional time-dependent Hartree-Fock dynamics

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A kinetic equation in operator form for the one body density matrix is derived by means of a convenient truncation of the quantal Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy. Using projection techniques in a procedure similar to the derivation of the master equation, the evolution of the diagonal and the nondiagonal terms of the density matrix can be disentangled. A quantal, non-Hermitian master equation for the diagonal part is extracted and it is seen that corrections to the collision frequency or to the relaxation time arise as a consequence of the creation and destruction of incoherent events.

NUCLEAR REACTIONS Reduced one body density matrix, BBGKY hierarchy, quantal kinetic equation, coupling between probabilistic and interference matrix element, collision frequency and relaxation time, dynamical quantal corrections.

I. INTRODUCTION

The kinetics of nuclear matter has become a field of intensive research, especially in view of the observation of the transport processes associated with a wide set of nuclear phenomena. From the well-known hydrodynamical modes of nuclei[1] to the stochastic, Markovian events that induce a drift-plus-diffusion behavior of the fragment energy or mass distribution after a damped heavy ion collision,[2–5] one finds a broad range of situations that involve an average motion of the nuclear fluid. Among the various attempts to give a microscopic foundation to the observed time evolution of the macroscopic variables registered in a given experiment, the time dependent Hartree-Fock (TDHF) model[6,7] has received considerable attention.[8–15]

The most severe criticisms of the TDHF method point towards the independent-particle character of the theory. It is well known that this model provides a nonlinear, self-consistent equation of motion of either the single-particle (sp) wave function or density matrix out of which a Slater determinant for the N-body system is constructed. This feature prevents any evaluation of effects linked to particle correlations and it has been remarked that the TDHF model underestimates the widths of the experimental distributions of macroscopic observables.[16,17] The recognition of this limitation pushed several researchers to search for corrections to the mean-field approach that could include some relevant many body correlations disregarded in the independent particle model. Thus, the question examined earlier by Martin and Schwinger[18] and Kadanoff and Baym[19] has been revived in light of nuclear dynamics, and recent attempts to derive collisional corrections to the TDHF model constitute a fascinating chapter of current nuclear literature.[20–23]

Within our scope, the contributions to this field can be classifed in three groups. One of them adheres to the Green's functions philosophy.[18,19] In this framework, a hierarchy of reduced Green's functions for the many body Schrödinger equation is derived, and a convenient truncation procedure leads to a collisional TDHF equation[20,21] similar to the quantal Boltzmann equation for the kinetics of dilute gases.[24–27] Another approach relies on the assumption of the existence of a relaxation time for the one-body dynamics and incorporates an inhomogeneous term in the TDHF equation, responsible for the shift towards equilibrium.[22] The third category[23] stems from the master equation methods introduced in heavy-ion dynamics by Nörenberg and collaborators.[2–4,28,29] In this approach, one projects the many-body Liouville-von Neumann

equation of motion (hereafter, we consider $\hbar=1$),

$$i\dot{\rho}_N = L_N \rho_N \tag{1}$$

with the Liouvillian

$$L_N = [H_N, \ ] \tag{2}$$

and the Hamiltonian

$$H_N = H_0 + H_1 = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i<j=1}^{N} V_{ij} \ . \tag{3}$$

The projection techniques incorporated by Nakajima[30] and Zwanzig[31] are used to extract an uncorrelated many body state ρ_0 (a Slater determinant of sp densities) and its counterpart $\rho_c = \rho - \rho_0$. An average of the reduced equation of motion for ρ_0 over $N-1$ particles gives rise to a collisional TDHF equation similar to those obtained by the above-mentioned authors.

In this work we present a derivation, rather standard in kinetic theory, based on the quantal Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy for reduced distribution functions.[32] This formalism is practically identical to the ones employed in Refs. 20 and 23. However, our interest here is to discuss in depth the peculiar kind of process that one includes or disregards when going from the completely reversible, exact Liouvillian dynamics to a kinetic type of description, valid for either weakly coupled or dilute systems. Furthermore, we extract a kinetic, i.e., a collisional TDHF equation for the full one body density matrix, rather than for its diagonal part.[20,23] In this perspective, employing the projection techniques, we examine the coupling between interference and probabilistic terms of the one body density and establish the characteristic times for the evolution of three kinds of processes. These can be identified as the free flow of the probability, the creation of phase shifts or the onset of incoherence brought in by the collisions that tend to annihilate the probabilities, and the destruction of the initial phase correlations among sp amplitudes. This procedure is similar to that employed by Prigogine, Balescu, and collaborators[33–35] to examine the passage from the reversible (i.e., Liouvillian) to the irreversible (i.e., causal or kinetic) motion.

In Sec. II, we present the derivation of the quantal kinetic equation stemming from the equations of motion for reduced distribution functions and the classification of correlation patterns by Balescu (see, for example, Ref. 32). Section III is devoted to setting the spectral representation of the irreversible equation of motion in the weakly interacting or dilute gas approximation. This amounts to obtaining the matrix expression of the collision term and its linearized version. The kinetics of the nondiagonal one body density is examined in Sec. IV, with emphasis on the consequences of its coupling to the occupation probabilities of sp orbitals. An investigation of the relative weight of the different contributions to the collisional derivative is carried out in Sec. V. It is seen that the effect of transitions from coherent to incoherent situations, when averaged over a large portion of the sp equilibrium spectrum, is responsible for an increase of the collision rate, close to equilibration. The summary and final discussion complete Sec. VI.

II. THE KINETIC EQUATION FOR NUCLEAR MATTER

In the following, we take as a starting point the equation of the BBGKY hierarchy, already a classic in statistical mechanics (see Ref. 32, and references therein). In matrix notation, it reads,

$$i\dot{\rho}_S = L_S \rho_S + \mathrm{Tr}_{S+1} \sum_{i=1}^{S} L_1(i,S+1)\rho_{S+1} \ , \tag{4}$$

where $\rho_S \equiv \rho_S(1,2,\ldots,S)$ denotes the reduced S-body density operator,

$$\rho_S \equiv \frac{N!}{(N-S)!} \mathrm{Tr}_{S+1,\ldots,N} \rho_N \ , \tag{5}$$

L_S is a Liouvillian for S particles with one and two body terms,

$$L_S = \sum_{i=1}^{S} L_0(i) + \sum_{i>j=1}^{S} L_1(ij) \ , \tag{6}$$

and Tr_j indicates the trace over the variables of particle j. We recall that the set of coupled equations (4) that range from $S=1$ to $S=N$, namely the complete hierarchy, is exact and equivalent to the Liouville equation (1) for the N-body system. In particular, the first two members of the hierarchy are

$$i\dot{\rho}_1(1) = L_0(1)\rho_1(1) + \mathrm{Tr}_2 L_1(1,2)\rho_2(1,2) \ , \tag{7a}$$

$$i\dot{\rho}_2(1,2) = [L_0(1)+L_0(2)+L_1(1,2)]\rho_2(1,2) + \mathrm{Tr}_3[L_1(1,3)+L_1(2,3)]\rho_3(1,2,3) \ . \tag{7b}$$

A useful framework in which to disentangle these equations and profit from a view over the microscopic structure of the clusters is provided by the decomposition in vacuum-and-correlation patterns.[32] The former will be defined as

$$\rho_S^{(0)}=\Pi_S(1/2/\cdots/S)=\prod_{i=1}\rho_1(i)\ , \tag{8}$$

i.e., a vacuum of S particles is a physical state of coexistence in statistical independence. The correlation patterns of S particles are associated to the difference $\rho_S-\Pi_S(1/\cdots/S)$ and, in turn, possess a detailed structure according to the various possibilities of clustering S particles. As an example, we write the first few reduced densities.

$$\begin{aligned}\rho_2(1,2)&=P_2(1/2)\Pi_2(1/2)+P_2(12)\Pi_2(12)\\&=\rho_2^{(0)}+\rho_2^{(c)}\ ,\end{aligned} \tag{9a}$$

$$\begin{aligned}\rho_3(1,2,3)&=P_3(1/2/3)\Pi_3(1/2/3)+P_3(1/23)\Pi_3(1/23)+P_3(2/13)\Pi_3(2/13)\\&\quad+P_3(3/12)\Pi_3(3/12)+P_3(123)\Pi_3(123)\\&=\rho_3^{(0)}+\rho_3^{(c)}\ .\end{aligned} \tag{9b}$$

The symbol P denotes antisymmetrization operations between the arguments and is assumed to conserve the correlations on its right. It should be remarked that the bar between arguments is an indication of factorization of left-and-right clusters.[32]

A conspicuous analysis of the correlation patterns and their equations of motion extracted from the hierarchy can be found in Ref. 32. Far from entering into this subject in full, we will just work out in detail the vacuum and correlation components of Eq. (7b) and discuss the approximations needed to achieve a sensible truncation of the hierarchy at this level.

It is a straightforward exercise to obtain the equations for $\rho_2^{(0)}$ and $\rho_2^{(c)}$. Indeed, using the definition (8) and Eq. (7a), one easily gets[32]

$$\begin{aligned}i\dot{\rho}_2^{(0)}&=[L_0(1)+L_0(2)]\rho_2^{(0)}\\&\quad+\mathrm{Tr}_3P(1/2)\{L_1(1,3)[P_2(1/3)\Pi_3(1/2/3)+P_2(13)\Pi_3(2/13)]\\&\qquad+L_1(2,3)[P_2(2/3)\Pi_3(1/2/3)+P_2(23)\Pi_3(1/23)]\}\ .\end{aligned} \tag{10}$$

Subtracting (10) from (7b) one finds[32]

$$\begin{aligned}i\dot{\rho}_2^{(c)}&=[L_0(1)+L_0(2)+L_1(1,2)]\rho_2^{(c)}\\&\quad+\mathrm{Tr}_3\{[L_1(1,3)P_3(1/23)+L_1(2,3)P_3(1/23)-P_2(1/2)L_1(2,3)P_2(23)]\Pi_3(1/23)\\&\qquad+[L_1(1,3)P_3(2/13)+L_1(2,3)P_3(2/13)-P_2(1/2)L_1(1,3)P_2(13)]\Pi_3(2/13)\\&\qquad+[L_1(1,3)+L_1(2,3)][P_3(3/12)\Pi_3(3/12)+P_3(123)\Pi_3(123)]\}\\&\quad+L_1(1,2)P(1/2)\Pi_2(1/2)\\&\quad+\mathrm{Tr}_3\{[L_1(1,3)+L_1(2,3)]P_3(1/2/3)-P_2(1/2)[L_1(1,3)P_2(1/3)+L_1(2,3)P_2(2/3)]\}\Pi_3(1/2/3)\ .\end{aligned} \tag{11}$$

It is noticeable that the time evolution of $\rho_2^{(c)}$ is governed by three contributions: (i) a homogeneous flow, driven by the two particle Liouvillian $L_2(1,2)$ [cf. Eq. (6)]; (ii) the propagation of three-body correlations, and (iii) a source that concentrates the results of creating dynamical correlations upon the two and three body vacua. We can write, symbolically,

$$i\dot{\rho}_2^{(c)}=L_2(1,2)\rho_2^{(c)}+\mathscr{P}_3(1,2,3)\rho_3^{(c)}+C_2(1,2)\rho_2^{(0)}+C_3(1,2,3)\rho_3^{(0)}\ , \tag{12}$$

where $\mathscr{P}_3$, C_2, and C_3 are linear functionals of the interaction L_1.

The general solution to this equation can be set as follows,

$$\rho_2^{(c)}(t)=U_2(1,2,t)\rho_2^{(c)}(0)-i\int_0^t d\tau\, U_2(1,2,\tau)\mathscr{P}_3(1,2,3)\rho_3^{(c)}(t-\tau)$$
$$-i\int_0^t d\tau\, U_2(1,2,\tau)[C_2(1,2)\rho_2^{(0)}(t-\tau)+C_3(1,2)\rho_3^{(0)}(t-\tau)]\ , \tag{13}$$

with the full two-body propagator,

$$U_2(1,2,t)=\exp\{-iL_2(1,2)t\}\ . \tag{14}$$

The kinetic equation in its usual form (a closed nonlinear equation for the evolution of ρ_1) is obtained upon replacement of (13) in (7a) when suitable approximations for the three-body correlation patterns are performed. Since one could feel inclined to truncate the hierarchy at this level, it is necessary to examine the conditions for the validity of such an attempt.

A set of rather common simplifications at this stage consists of the following: First, we assume that either the initial two-body correlations $\rho_2^{(c)}(0)$ vanish or the propagator $U_2(1,2,t)$ decays with a short lifetime, as compared to macroscopic, i.e., observation times. This would imply neglection of the first term in Eq. (13) for the times under consideration. The latter is a reasonable assumption when one deals with short range particle-particle potentials, since the propagator kernel is expected to decay in an interval comparable to the duration of the interaction. In addition, taking into account the structure of $\rho_3^{(c)}(t-\tau)$ indicated in Eq. (9b), we express the second term in (13) as

$$-i\int_0^t d\tau\, U_2(1,2;\tau)\mathscr{P}_3(1,2,3)\rho_3^{(c)}(t-\tau)$$
$$=-i\int_0^t d\tau\, U_2(1,2;\tau)\mathscr{P}_3(1,2,3)[\rho_1(1)\rho_2^{(c)}(2,3)+\rho_1(2)\rho_2^{(c)}(1,3)+\rho_1(3)\rho^{(c)}(1,2)+\Pi_3(123)]_{(t-\tau)}$$
$$\approx -i\int_0^t d\tau\, U_2(1,2;\tau)\mathscr{P}_3(1,2,3)[\rho_1(1)U_2(2,3;t-\tau)\rho_2^{(c)}(2,3;t=0)+\rho_1(2)U_2(1,3;t-\tau)\rho_2^{(c)}(1,3;t=0)$$
$$+\rho_1(3)U_2(1,3;t-\tau)\rho_2^{(c)}(1,3;t=0)+\Pi_3(123;t-\tau)]+0(L_1{}^2)\ . \tag{15}$$

The arguments invoked to disregard the initial correlations propagated up to time t allow us to discard as well the integral in (15). This is true in view of the fact that $U_2(\tau)$ acts as a short-range filter; thus, for the small values of τ left for integration, $U_2(t-\tau)\,\rho_2^{(c)}(0)$ approximately behaves as $U_2(t)\rho_2^{(c)}(0)$, a negligible factor accompanying a finite integral. We are left with a two-body correlation density of the following appearance,

$$\rho_2^{(c)}(t)\sim -i\int_0^t d\tau\, U_2(1,2;\tau)[C_2(1,2)\rho_2^{(0)}(t-\tau)+C_3(1,2,3)\rho_3^{(0)}(t-\tau)]+0(L_1{}^2)\ . \tag{16}$$

For the macroscopic times under consideration, $\rho_2^{(c)}$ is entirely originated in a "living" source; indeed, the processes it embodies are those identifiable as dynamical correlations, created upon an earlier vacuum at time $t-\tau$ and propagated up to time t. This is explicitly displayed in (16) where we are showing the lowest-order contribution as a function of L_1 contained in C_2 and C_3. The type of analysis performed in the preceding paragraphs can be easily generalized and one can convince oneself that in the higher order processes included in $0(L_1{}^2)$, the effect of propagation of the initial correlations is negligible. The result given in (16) is, in fact, a series in L_1 that reflects its repeated action upon the two and three body vacua.

It can be shown[33–35] that Eq. (16) is a particular version of a more general equation, that arises whenever there exist two definite, different time scales in the many body system, namely a microscopic one governing individual scattering events in the medium and a macroscopic one related to the thermodynamic state (i.e., density) as well as to some average of the two body interaction (i.e., the transition probability or cross section). If such a separation exists, one speaks of an asymptotic or kinetic regime, in which any dynamical correlations can be expressed as

$$\rho^{(c)}=\underline{c}\,\rho^{(0)}\ , \tag{17}$$

where $\rho^{(c)}$ and $\rho^{(0)}$ are vectors in Liouville space and

$\underline{c}$ represents the so-called creation superoperator.[32–35] Now, in our simplified approach leading to Eq. (16), we have prepared ourselves for a perturbation analysis. As we introduce $\rho_2^{(c)}$ in (9a) and this one in (7a), we find, up to the lowest order in $L_1{}^2$, the kinetic equation for weakly coupled quantum systems,

$$\dot{\rho}_1(1) = -iL_0(1)\rho_1(1) - i\,\mathrm{Tr}_2 L_1(1,2) \times \rho_2^{(0)}(1,2) + W(\rho_1) \,. \tag{18}$$

Equation (7) contains the three characteristic modes of evolution in a kinetic process: (i) free flow, (ii) mean-field or Vlassov flow, and (iii) a collision term that here reads

$$W(\rho_1) = -\mathrm{Tr}_2 L_1(1,2) \int_0^\infty d\tau\, U_2(1,2;\tau)[C_2(1,2)\rho_2^{(0)}(1,2;t-\tau) + C_3(1,2,3)\rho_3^{(0)}(1,2,3;t-\tau)] \,. \tag{19}$$

For our subsequent discussion it will be useful to decompose $W(\rho_1)$ as

$$W(\rho_1) = -\int_0^\infty d\tau\, \mathrm{Tr}_2[\psi_2(\tau)U_0(1,2;-t)\Pi_2(1/2)] - \int_0^\infty d\tau\, \mathrm{Tr}_{2,3}[\psi_3(\tau)U_0(1,2,3;-\tau)\Pi_3(1/2/3)] \tag{20}$$

with the two and three body irreducible collision operators,

$$\psi_2(\tau) = L_1(1,2)U_2(1,2;\tau)L_1(1,2)P_2(1/2) \,, \tag{21a}$$

$$\psi_3(\tau) = L_1(1,2)U_2(1,2;\tau)[L_1(1,3)P_3(1/2/3) - P_2(1/2)L_1(1,3)P_2(1/3) + L_1(2,3)P_3(1/2/3) - P_2(1/2)L_1(2,3)P_2(2/3)] \,, \tag{21b}$$

while U_0 stands for the unperturbed propagator generated by L_0.

As already pointed out by several authors,[18,19,32] the appearance of three body processes in the collision kernel is a pure quantal effect that holds even in the case of a dilute gas (Boltzmann system).[24–27] It must be traced to the existence of exchange correlations among indistinguishable particles that allow for nonvanishing three particle diagrams even in the lowest, i.e., $L_1{}^2$ interaction order; these processes are depicted in Fig. 1, making use of the graphical language proposed by Prigogine[37] and Balescu.[32] We note in passing that if one introduces the mean-field Liouvillian

$$\mathscr{L}(1) = L_0(1) + \mathrm{Tr}_2 L_1^A(1,2)\rho_1(2) \tag{22}$$

it corresponds precisely to the Hartree-Fock Hamiltonian[6–15]

$$\mathscr{L}(1) = [\mathscr{H}(1)\,,\,] = [H_0(1) + \mathrm{Tr}_2 V^A(1,2)\rho_1(2)\,,\,] \,. \tag{23}$$

Equation (18) is thus of the type that has become popular in theoretical nuclear physics as a "collisional TDHF" equation and different versions are available, according to the choice of the intermediate propagator $U_2(1,2;\tau)$ in the collision kernel.[20,23]

III. SPECTRAL REPRESENTATION OF THE KINETIC EQUATION

The next task is to write Eq. (18) in a manageable form that allows the examination of the time scales of interest. We have found it useful to work in the

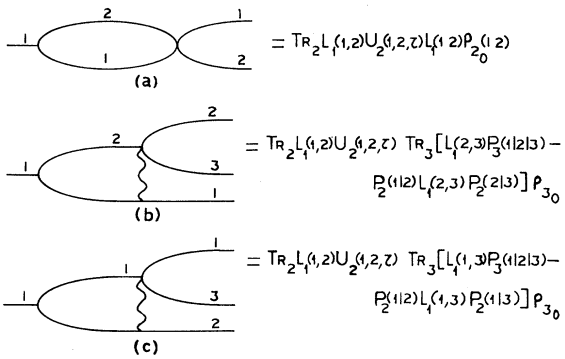

FIG. 1. Typical diagrams of a kinetic equation, according to Prigogine and Balescu. (a), (b), and (c) show the contributions to the two- and three-body collision operators. (a) The two body collision diagram is the only one existing for classical diluted or weakly interacting gases. (b) and (c) The three-body collision diagrams for quantal gases, where the wiggly line indicates that antisymmetrization of sp orbitals denoted as 1, 2, and 3 has been carried out.

Liouville representation[36] making extensive use of the following ingredients:

(a) Vectors in Liouville space are operators in Hilbert space; a basis of the former is the set $\{|\alpha\rangle\langle\alpha'|\}$, where $|\alpha\rangle$, $|\alpha'\rangle$, are elements of a basis of the latter.

(b) Operators in Liouville space are superoperators in Hilbert space; those appearing in the current developments are factorizable superoperators[35] and their combinations. In particular, the Liouvillian is

$$L = H \otimes I - I \otimes H \ , \tag{24}$$

where I is the identity in Hilbert space and $\otimes$ is the factorizability symbol that indicates the position of the vector acted upon by L. Its spectral representation is

$$L = \sum L^{\gamma\delta\gamma'\delta'}_{\alpha\beta\alpha'\beta'} |\alpha\beta\rangle\langle\alpha'\beta'| \otimes |\gamma\delta\rangle\langle\gamma'\delta'| \ , \tag{25a}$$

$$L^{\gamma\delta\gamma'\delta'}_{\alpha\beta\alpha'\beta'} = H_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}\delta_{\delta\delta'} - H_{\gamma\delta\gamma'\delta'}\delta_{\alpha\alpha'}\delta_{\beta\beta'} \ , \tag{25b}$$

where we assume that Greek indices stand for a set of sp labels. We note as well that if $|\alpha\rangle$ is a time-independent energy eigenbasis, the unperturbed sp propagator is

$$U_0(1;t) = \sum_{\alpha\beta} e^{-iw_{\alpha\beta}t} |\alpha\rangle\langle\alpha| \otimes |\beta\rangle\langle\beta| \tag{26}$$

with $w_{\alpha\beta} = \epsilon_\alpha - \epsilon_\beta$.

The remaining elements to be used hereafter are

$$P_2(1/2) = \tfrac{1}{2}\sum_{\alpha\beta} |\alpha\beta - \beta\alpha\rangle\langle\alpha\beta| \ , \tag{27}$$

$$P_3(1/2/3) = \frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma} |\alpha\beta\gamma - \alpha\gamma\beta + \beta\gamma\alpha - \beta\alpha\gamma + \gamma\alpha\beta - \gamma\beta\alpha\rangle\langle\alpha\beta\gamma| \ , \tag{28}$$

$$\Pi_2(1/2) = \sum_{\alpha\beta\alpha'\beta'} \rho_{\alpha\alpha'}\rho_{\beta\beta'} |\alpha\beta\rangle\langle\alpha'\beta'| \ , \tag{29}$$

$$\Pi_3(1/2/3) = \sum_{\alpha\beta\gamma\alpha'\beta'\gamma'} \rho_{\alpha\alpha'}\rho_{\beta\beta'}\rho_{\gamma\gamma'} |\alpha\beta\gamma\rangle\langle\alpha'\beta'\gamma'| \ . \tag{30}$$

The calculation of the collision kernel in Eq. (20), using (21), is straightforward, although lengthy; indications about details and methodology are given in Appendix A. The final result, after some suitable assumptions regarding the variation of the matrix elements of the interaction [see Appendix A from Eq. (A6) to (A13)] can be cast into the appealing form

$$\dot{\rho}_{\alpha\alpha'} \sim -i\sum_{\beta\beta'} \mathscr{L}^{\beta\alpha'}_{\alpha\beta}\rho_{\beta\beta'} + \sum_{\substack{\beta\gamma\delta \\ \beta'\gamma'\delta'}} W^{\alpha'\beta'\gamma'\delta'}_{\alpha\beta\gamma\delta} \{\rho_{\gamma\gamma'}\rho_{\delta\delta'}(1-\rho_{\alpha\alpha'})(\delta_{\beta\beta'} - \rho_{\beta\beta'}) - \rho_{\alpha\alpha'}\rho_{\beta\beta'}(\delta_{\gamma\gamma'} - \rho_{\gamma\gamma'})(\delta_{\delta\delta'} - \rho_{\delta\delta'})\} \ , \tag{31}$$

where the symbol $W^{\alpha'\beta'\gamma'\delta'}$ encloses two interactions and an energy form factor,

$$W^{\alpha'\beta'\gamma'\delta'}_{\alpha\beta\gamma\delta} = F(w,\Gamma) V^A_{\alpha\beta'\gamma\delta} V^A_{\gamma'\delta'\alpha'\beta} \ . \tag{32}$$

Equation (31) is the spectral representation of the kinetic equation (18) under the simplifying hypothesis of smooth behavior of the interaction matrix elements with respect to the sp labels. The general form of the collision kernel is given in Eqs. (A6)–(A8), where we recognize the usual "gain-minus-loss" pattern of irreversible evolution. This is evident as one neglects the off-diagonal matrix elements of ρ_1 in (31), in order to get the quantal Boltzmann equation[24–27]

$$\dot{\rho}_\alpha = -i\sum_\beta \mathscr{L}^{\beta\beta}_{\alpha\alpha}\rho_\beta + \sum_{\beta\gamma\delta} W_{\alpha\beta\gamma\delta}\{\rho_\gamma\rho_\delta(1-\rho_\alpha)(1-\rho_\beta) - \rho_\alpha\rho_\beta(1-\rho_\gamma)(1-\rho_\delta)\} \ . \tag{33}$$

Here $W_{\alpha\beta\gamma\delta}$ is just the transition probability for the sp scattering event $(\alpha\beta)\rightarrow(\gamma\delta)$ and the Pauli principle is enforced by the initial (i.e., ρ_γ, ρ_δ) and the final $(1-\rho_\alpha, 1-\rho_\delta)$ sp occupation probabilities. In this frame, Eq. (31) is an obvious generalization of this intuitive description of balance, although obtained from a complete reasoning that leads to it

with a proper selection of the transition matrix elements.

IV. THE KINETICS OF THE NONDIAGONAL ONE BODY DENSITY

The passage from Eq. (31) to Eq. (33) is founded on the hypothesis that off-diagonal matrix elements $\rho_{\alpha\alpha'}$ are small in the vicinity of equilibrium. Indeed, it can be proven on very general statistical arguments that the final stage of kinetic evolution is invariant under collisions,[32] i.e., $W(\rho_{1\,\mathrm{eq}})=0$ and

$$\lim_{t\to\infty}\rho_{\alpha\alpha'}(t)=\frac{1}{1+e^{-(\epsilon_\alpha-\epsilon_F)/T}}\delta_{\alpha\alpha'}\ . \tag{34}$$

However, one is usually interested in dynamical properties involving expectation values of observables. This indicates the convenience of knowing to a better extent the range of validity of the diagonal kinetic equation (33). It is thus important to examine the intrinsic time scales for the decay of both diagonal and off-diagonal components of ρ_1 towards the asymptote (34). Towards this aim, we perform upon Eq. (31) a projection analysis similar to that leading to the general master equation.[30,38] This is rather easily done after linearization of the collision term; the advantages of this procedure to achieve a reasonable estimate of the time scales of interest will become evident as we proceed. We have already observed that the reversible flow in the general equation (18) is driven by the self-consistent Liouvillian whose spectral Liouville representation is

$$\mathscr{L}=\sum_{\alpha\beta\alpha'\beta'}(\mathscr{H}_{\alpha\beta}\delta_{\beta'\alpha'}-\mathscr{H}_{\beta'\alpha'}\delta_{\alpha\beta})\times|\alpha\rangle\langle\beta|\otimes|\beta'\rangle\langle\alpha'|\ . \tag{35}$$

We now try to write the collision term similarly to the flow term in (31), namely,

$$W(\rho_1)=\sum_{\alpha\alpha'\beta\beta'}|\alpha\rangle\langle\alpha'|\,\mathscr{K}^{\beta'\alpha'}_{\alpha\beta}(\rho_1)\rho_{\beta\beta'}=\mathscr{K}(\rho_1)\rho_1\ , \tag{36}$$

with $\mathscr{K}$ a one-body superoperator of the form (35),

$$\mathscr{K}(\rho_1)=\sum_{\alpha\alpha'\beta\beta'}\mathscr{K}^{\beta'\alpha'}_{\alpha\beta}|\alpha\rangle\langle\beta|\otimes|\beta'\rangle\langle\alpha'|\ . \tag{37}$$

Of course, the matrix elements $\mathscr{K}^{\alpha'\beta'}_{\alpha\beta}$ depend upon ρ_1,

$$\mathscr{K}^{\beta'\alpha'}_{\alpha\beta}\equiv\mathscr{K}^{\beta'\alpha'}_{\alpha\beta}(\rho_{\alpha\alpha'},\rho_{\gamma\gamma'},\rho_{\delta\delta'})\ . \tag{38}$$

It is evident that very near equilibration, when Eq. (33) is valid, we have

$$\mathscr{K}^{\beta'\alpha'}_{\alpha\beta}=\mathscr{K}_{\alpha\beta}\delta_{\alpha\alpha'}\delta_{\beta\beta'}$$

with $\mathscr{K}_{\alpha\beta}$ a function of diagonal matrix elements of ρ_1. In general, we expect that approximation (36) gives us some insight regarding the mutual influence of the probabilistic and interference parts of ρ_1 during the motion. Especially, we would like to extract some characteristic time scale for the vanishing of these sp interference processes.

In view of (36), the general kinetic equation (18) takes the simple form

$$\dot\rho_1=(-i\mathscr{L}+\mathscr{K})\rho_1\ , \tag{39}$$

where we see that the linearized kernel is related to the width of the sp states, or, as usually stated in kinetic theory, the maximum finite eigenvalue of its inverse $\mathscr{K}^{-1}$ gives the relaxation time of the sp density. Now, since diagonalization of the inverse kernel $\mathscr{K}^{-1}$ is usually a rather complicated numerical task, the standard estimate in kinetic and transport theory consists of taking the numerical value of the element $\mathscr{K}^{\alpha\alpha}_{\alpha\alpha}(\rho_{1\,\mathrm{eq}})$ and making the assignment

$$\tau_{r\alpha}{}^{-1}\approx\mathscr{K}^{\alpha\alpha}_{\alpha\alpha}(\rho_{1\,\mathrm{eq}})\ . \tag{40}$$

This evaluation gives a relaxation time for the sp state α, out of which mean free paths and transport coefficients can be extracted as function of the sp observables listed in α. Typical calculations of the mean free path for a single nucleon impinging upon nuclear matter have been performed, as a function of the nuclear energy, for different temperatures of the environment.[39,40] Another remark at this point is that the eigenvalues of $\mathscr{K}$ are precisely the sp widths Γ_α introduced in Appendix A to smooth away the singularity of a distribution, except for an unimportant factor of units.

It is convenient to split the sp Hilbert space in two orthogonal subspaces, such that

$$\rho_1=\rho_D+\rho_N\ . \tag{41}$$

In the present approximation scheme, we have $\mathscr{K}=\mathscr{K}[\rho_1(t)]$. The diagonal and nondiagonal projections of the linearized kinetic equation provide the coupled system,

$$\dot\rho_D=(-i\mathscr{L}+\mathscr{K})_{DD}\rho_D+(-i\mathscr{L}+\mathscr{K})_{DN}\rho_N\ , \tag{42a}$$

$$\dot{\rho}_N=(-i\mathscr{L}+\mathscr{K})_{ND}\rho_D+(-i\mathscr{L}+\mathscr{K})_{NN}\rho_N \ . \tag{42b}$$

This set of equations gives rise to a generalized, non-Hermitian masterlike equation[38] for ρ_D; following the usual procedure, we integrate Eq. (42b), getting

$$\rho_N(t)=U_N(t)\rho_N(0) -i\int_0^t d\tau\, U_N(\tau)\{(\mathscr{L}+i\mathscr{K})_{ND}\rho_D\}(t-\tau) \ , \tag{43}$$

with the intrinsic propagator

$$U_N(\tau)=T\exp\{-i\int_0^\tau d\tau'(\mathscr{L}+i\mathscr{K})(\tau')\} \ , \tag{44}$$

T being the time-ordering symbol.

Equation (43) is to be regarded as the description of a class of sp correlations, namely those associated with interference between sp states. Their existence stems from two independent sources: The first term in (43) describes the propagation and decay of the initial phase correlations, while the second one corresponds to those dynamical phase shifts produced during the time evolution of the whole system. In the language of modern kinetic theory,[32–35] we would say that these dynamic correlations are "created" upon the "vacuum" ρ_D [cf. Eq. (17)], according to a non-Hermitian memory kernel. Replacement of Eq. (43) into (42a) yields the non-Hermitian master equation,

$$\dot{\rho}_D(t)=(-i\mathscr{L}+\mathscr{K})_D(t)\rho_D(t) -\int_0^t d\tau(-i\mathscr{L}+\mathscr{K})_{DN}(t)U_N(\tau)(-i\mathscr{L}+\mathscr{K})_{ND}(t-\tau)\rho_D(t-\tau)+(-i\mathscr{L}+\mathscr{K})_{DN}(t)U_N(t)\rho_N(0) \ . \tag{45}$$

The first observation, as one glances at this formula, is that the first term gives the linearized version of the close-to-equilibrium, full-diagonal equation (33). The extra terms are corrections induced by the coupling to the off-diagonal one-body density. We are now in a condition to establish typical frequencies and decay times for these added contributions and compare them with those of the motion near equilibration.

A second remark is convenient at this point. Even though we are assuming that the current regime is not extremely close to equilibrium, neither is it too far from it; the core of the derivation of the one-body kinetic equations resides in our belief of the validity of a long-time description of the evolution. We feel entitled to write

$$\rho_1(t)=\rho_{1\,\mathrm{eq}}+\delta\rho(t) \tag{46}$$

with $\rho_{1\,\mathrm{eq}}$ a thermal Fermi distribution given by Eq. (34). As quoted above, it is invariant under particle collisions, and its stationarity property

$$\mathscr{L}(\rho_{1\,\mathrm{eq}})\rho_{1\,\mathrm{eq}}=0 \tag{47}$$

defines the sp mean field eigenenergies in (34). One can then realize that Eq. (45) is in fact an evolution law for $\rho_1(t)$; let us analyze, for example, the linearized kernel in the first term, $\mathscr{K}_{DD}$. A look at Eq. (34) will convince the reader that it is essentially a second-degree polynomial in ρ_1. We have, in abridged symbolical notation

$$\begin{aligned}\mathscr{K}_{DD}(\rho_1)&=\mathscr{K}_{DD}(\rho_{\mathrm{eq}}+\delta\rho_1)\\ &=A\rho_1+B{\rho_1}^2\\ &=\mathscr{A}(\rho_{\mathrm{eq}})+\mathscr{B}(\rho_{\mathrm{eq}})\delta\rho_1\\ &\quad+\mathscr{C}(\rho_{\mathrm{eq}})\delta{\rho_1}^2 \ .\end{aligned} \tag{48}$$

Then, the evolution term $\mathscr{K}_{DD}\rho_D$ can be written

$$\begin{aligned}\mathscr{K}_{DD}(\rho_1)\rho_D&=A(\rho_{\mathrm{eq}})\rho_{\mathrm{eq}}\\ &\quad+[\mathscr{A}(\rho_{\mathrm{eq}})+\mathscr{B}(\rho_{\mathrm{eq}})\rho_{\mathrm{eq}}]\delta\rho_D\\ &\quad+0(\delta{\rho_D}^2) \ .\end{aligned} \tag{49}$$

Now, since

$$\begin{aligned}\mathscr{K}(\rho_{\mathrm{eq}})&=\mathscr{K}_{DD}(\rho_{\mathrm{eq}})\rho_{\mathrm{eq}}\\ &=\mathscr{A}(\rho_{\mathrm{eq}})\rho_{\mathrm{eq}}=0 \ ,\end{aligned} \tag{50}$$

we have

$$\mathscr{K}_{DD}\rho_D=-\frac{1}{\tau_D}\delta\rho_D \ , \tag{51}$$

defining in this way the time operator whose highest noninfinite eigenvalue gives the relaxation scale of the diagonal matrix elements, τ_r.

In this spirit, we hope that Eq. (45) corresponds to a damped motion of the type

$$\delta\dot{\rho}_D \sim \left[-iw_D - \frac{1}{\tau_D} - \frac{1}{\tau'_D} \right] \delta\rho_D + D\rho_N(0) \ . \tag{52}$$

Here we are including in w_D every oscillating contribution, actually the one arising from the mean field $\mathscr{L}$ plus those cross products in the integral kernel. The second relaxation time appearing in (52) can be traced to the real part of the memory kernel,

$$\frac{i}{\tau'_D} \sim \int_0^\infty d\tau \{ -\mathscr{L}_{DN}(t) U_N(\tau) \mathscr{L}_{ND}(t-\tau) + \mathscr{K}_{DN}(t) U_N(\tau) \mathscr{K}_{ND}(t-\tau) \} \ , \tag{53}$$

and D expresses the destruction of the initial phase correlations. It must be kept in mind that every coefficient in this equation of motion is indeed a time-dependent, nonlinear one-body superoperator.

V. STUDY OF THE TYPICAL TIME SCALES

The objective of this section is to establish the characteristic time for the destruction of $\rho_{ND}(0)$ and the magnitude of the correction τ'_D. According to Eqs. (43) and (44), the decay time for the initial correlations is given by the off-diagonal linearized kernel $\mathscr{K}_{NN}$. It is thus necessary to analyze this as well as the remaining blocks into which we have split the superoperator $\mathscr{K}$, whose construction is carried out in Appendix B [see Eqs. (33)]. In Eq. (39) we have the full diagonal part $\mathscr{K}_{DD}$, where we recognize the transition probabilities at the site of the weighting factors of the polynomial in ρ_D. Since $\rho_\alpha \leq 1$, the leading contribution to $\mathscr{K}^{\nu\alpha}_{\alpha\nu}$ corresponds to linear terms with respect to these numbers. The order of magnitude, for $\rho_D \sim \rho_{\rm eq}$, would be

$$\mathscr{K}^{\nu\alpha}_{\alpha\nu} \sim \overline{nF(w_{\alpha\gamma\nu\delta}, \Gamma_{\alpha\gamma\nu\delta}) \mid V^A_{\alpha\gamma\nu\delta} \mid^2} \ , \tag{54}$$

where n is the number of particles per unit volume and the bar denotes a typical average value taken over the summation indices γ,δ. It is not so simple to estimate the remaining matrix elements; one could, however, reason as follows. It can be assumed that $\rho_{\gamma\gamma'}$ is peaked at $\gamma=\gamma'$ and spreads around a narrow range σ for $\gamma\neq\gamma'$. Then one could expect that a sensible estimate for a summation of the type $\sum_{\gamma\gamma'} f(\gamma\gamma')$ would be $\Delta\sum_\gamma f(\gamma)$, where $\Delta = (\sigma$ times the average level density) measures the number of sp states over which f_1 can spread, keeping measurable values. This means that one could expect

$$\mathscr{K}_{ND}, \mathscr{K}_{DN}, \mathscr{K}_{NN} \sim \Delta \mathscr{K}_{DD} \ . \tag{55}$$

Expression (55) tells us that the decay time of the initial correlations, actually the largest bounded, non-negative eigenvalue of $\tau_N = -\mathscr{K}_{NN}{}^{-1}$ is proportional to $\tau_D/\Delta < \tau_D$. This means, for most times of interest concerning the observation of the motion, these initial phase shifts can be forgotten.

The situation is a bit different as we consider the second relaxation time τ'_D given by (53). In this expression we recognize that the propagator $U_N(\tau)$ whose lifetime we have just analyzed leads the time evolution of the integral kernel. The shortness of this decay time (provided $\Delta \gg 1$) in comparison to t justifies the replacement of the upper integration limit of Eq. (45) by infinity. The ratio τ_D/τ'_D can be estimated as follows: Since the integrand is nonzero for an interval τ not larger than the lowest eignevalue of τ_N, we can represent the integral as

$$\frac{1}{\tau'_D} \sim \tau_N \{ -\mathscr{L}_{DN}(t) \mathscr{L}_{ND}(t) + \mathscr{K}_{DN}(t) \mathscr{K}_{ND}(t) \} \ . \tag{56}$$

In the stationary regime the Hartree-Fock Hamiltonian becomes diagonal, thus

$$\mathscr{L}_{DN}(\text{eq}) = \mathscr{L}_{ND}(\text{eq}) = 0 \ .$$

We neglect these reversible contributions to (56) and obtain

$$\frac{1}{\tau'_D} \sim -\tau_N \mathscr{K}_{DN}(t) \mathscr{K}_{ND}(t) \ ; \tag{57}$$

thus,

$$\frac{\tau_D}{\tau'_D} \sim \frac{\mathscr{K}_{DN} \mathscr{K}_{ND}}{\mathscr{K}_{NN} \mathscr{K}_{DD}} \sim \Delta \ . \tag{58}$$

We recall that the number Δ counts the sp states γ', with respect to a given γ, for which $\rho_{\gamma\gamma'}$ is measurable and comparable to ρ_γ. Thus, Δ must be a time dependent quantity reaching the value zero as $t \to \infty$. This result indicates that when one writes the linearized kinetic equations for very-close-to-equilibrium situations,

$$i\dot{\rho}_D(t)=(-i\mathcal{L}_{DD}+\mathcal{K}_{DD}+\mathcal{K}'_{DD})(\mathrm{eq})\rho_D(t)$$
$$=\left[-i\mathcal{L}_{DD}-\frac{1}{\tau_D}-\frac{1}{\tau'_D}\right](\mathrm{eq})\rho_D(t)\ ,$$

the extra contribution $\mathcal{K}'_{DD}$ can be important for short times but negligible in the very-close-to-equilibrium regime. Since relaxation times and the associated transport coefficients are usually calculated introducing ρ_{eq} in the integrand for the loss term of the collisional derivative,[39,40] we conclude that interference events do not intervene in these calculations. However, they could be non-negligible in earlier stages of the evolution.

The meaning of this extra frequency is worth being analyzed. It measures the number of processes of a selected kind, per unit time, that provoke variations in the sp level occupation probability. These processes correspond to the following sequence: A transition takes place, carried along by the collision kernel $\mathcal{K}_{ND}$, whose effect is to break the coherence to some extent. This kernel, acting upon a sp probability at time $t-\tau$, just induces some phase shift between the two amplitudes, leaving the system endowed with a correlation ρ_N of the dynamical type. This correlation lasts a short time represented by τ_N and is destroyed by a second transition that restores the coherence at a later time t. This image generalizes that of sp correlations originated in two-body interactions, like those created and destroyed by the kernel $\mathcal{K}_{DD}$. However, it is important to signal that this effect conserves the basic scheme, consisting in the creation and subsequent annihilation of correlations, whatever their kind, as the motor of kinetic evolution. In the present case, we see that off-diagonal matrix elements of ρ_1 can be responsible for an effect that modifies the rate of collisions to an extent that, although unimportant in the vicinity of equilibrium, could be perhaps significant in the earliest motion.

VI. DISCUSSION

In this work we have derived a very general kinetic equation that fits into the scheme of collisional TDHF laws of motion for the reduced one body density operator. This derivation has been performed by means of a standard procedure in nonequilibrium statistical mechanics. Starting from the quantal BBGKY hierarchy and truncating it at the lowest interaction order (i.e., $L_1{}^2$) one ends up with the desired equation of evolution, where one can recognize that processes such as propagation of initial correlations, collisions involving more than two particles, and dynamical correlations among more than three particles have been disregarded. This kinetic equation is more general than its ancestors since it is an operational equation for the full one body matrix, rather than for the diagonal one. This generality allows a further disentangling treatment; with a somehow obvious projection, one writes a coupled system of equations for the diagonal and off-diagonal components of ρ_1. If the linearization of the collision kernel has been carried in advance, the resulting system is a first order one, and can be rewritten as a generalized, non-Hermitian master equation for ρ_D.

This new equation of motion is driven by nonlinear (i.e., depending on ρ_1) generators and exhibits the three characteristic terms: (i) free flow of ρ_D; (ii) propagation and destruction of dynamical correlations on phase shifts created upon an element of the ρ_D, and (iii) propagation of the initial incoherent events $\rho_N(0)$. The analysis of the time dependence of these terms rests on the hypothesis that at a given time t, $\rho_{\gamma\gamma'}(t)$ is concentrated near $\gamma=\gamma'$ and spreads around the diagonal matrix elements, Δ being a measure of the number of states $\gamma\neq\gamma'$ over which the magnitude of ρ_1 remains observable. One can realize that Δ must be a function of time; indeed, it can be larger for $t=0$, according to the preparation of the system, and tends to zero for the $t\to\infty$. This fact gives rise to the following observations: On one hand, one finds that the decay time for the initial correlations is shorter than the relaxation time of the diagonal density matrix by a factor Δ^{-1}. Since Δ is large near the origin of the t-axis, one can rely on the fact that these initial phase shifts are unobservable for any macroscopic time of interest. On the other hand, the extra collisional frequency or its inverse, the extra relaxation time due to a dynamical process of creation-and-annihilation of correlation, is not contemplated in the usual diagonal kinetic equation.[20–23] It introduces a pure quantal effect that originates in the possibility of the existence of correlations of a particular kind that do not possess a classical analog.

The numerical resolution of the equations presented here poses a gigantic task, even more delicate than the solution of the TDHF equations, with or without collisions. Even though it is highly unlikely that detailed calculations of the extra collisional frequency can be performed for the time being, it would be desirable if, in the light of the arguments here presented, estimates of its magnitude were given in parallel to calculations of relaxation times and transport coefficients of the nuclear fluid.

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APPENDIX A. CONSTRUCTION OF THE COLLISION MATRIX

We illustrate the kind of calculations carried out in this work displaying the contribution to Eq. (20) that arises from the first term in (21b). We assume in addition that the two-body propagator $U_2(1,2;\tau)$ in the collision kernels (21) can be reasonably approximated by the free-particle one,

$$U_0(1,2;\tau)=U_0(1;\tau)U_0(2;\tau)\ ,$$

generated by the mean-field Liouvillian in equilibrium. This means the eigenfrequencies $w_{\alpha\beta}$ in (26) are differences of eigenenergies of the static Hartree-Fock Hamiltonian once equilibration has been attained, i.e., as $\tau\to\infty$. Using Eqs. (25) to (27) we find, for example,

$$\begin{aligned}
&L_1(1,2)U_0(1,2;\tau)L_1(1,3)P_3(1/2/3)U_0(1,2,3;-\tau)\Pi_3(1/2/3)\\
&\quad=\sum_{\text{all labels}} L^{\mu\nu\sigma\tau}_{1\,\alpha\beta\gamma\delta}L^{MNST}_{1\,ABCD}\rho_{aa'}\rho_{bb'}\rho_{cc'}\exp[-i\tau(w_{mn}+w_{pq}-w_{m'n'}-w_{p'q'}-w_{rs})]\\
&\qquad\times|\alpha(1)\beta(2)\rangle\langle\gamma(1)\delta(2)|\,m(1)p(2)\rangle\langle m(1)p(2)|\,A(1)B(3)\rangle\\
&\qquad\times\langle C(1)D(3)|\,m'(1)p'(2)r'(3)\rangle^{A}\langle m'(1)p'(2)r'(3)|\,a(1)b(2)c(3)\rangle\\
&\qquad\times\langle a'(1)b'(2)c'(3)|\,n'(1)q'(2)s'(3)\rangle\langle n'(1)q'(2)s'(3)|\,M(1)N(3)\rangle\\
&\qquad\times\langle S(1)T(3)|\,n(1)q(2)\rangle\langle n(1)q(2)|\,\mu(1)\nu(2)\rangle\langle\sigma(1)\tau(2)|\ .
\end{aligned}\tag{A1}$$

We are showing the sp number arbitrarily assigned to each sp label in order to facilitate the bra-ket contraction. The superscript A indicates the antisymmetrization endorsed by $P_3(1/2/3)$. The time integral over the right half of the real axis is

$$\int_0^\infty e^{-iw\tau}d\tau=\Pi\delta_-(w)=\Pi\delta(w)-iP\left[\frac{1}{w}\right],\tag{A2}$$

with P the principal part. Thus this procedure allows us to write for the three body collision term in (20),

$$\begin{aligned}
&\int_0^\infty d\tau\,\Psi_3(\tau)U_0(1,2,3;-\tau)\Pi_3(1/2/3)\\
&\quad=\sum_{\text{all labels}} L^{\mu\nu\sigma\tau}_{1\,\alpha\beta\gamma\delta}L^{MNST}_{1\,ABCD}\rho_{aa'}\rho_{bb'}\rho_{cc'}\,|\,\alpha\beta B\rangle\langle\sigma\tau T\,|\\
&\qquad\times\{\Pi\delta_-(w_{\gamma\mu}+w_{\delta b}-w_{aa'}-w_{cc'})\delta_{a'M}\delta_{c'N}\delta_{b'\nu}\delta_{s\mu}\\
&\qquad\quad\times[\delta_{\gamma A}\delta_{\delta a}\langle CD|\,cb\rangle^{A}+\delta_{\gamma A}\delta_{\delta c}\langle CD|\,ba\rangle^{A}+\delta_{\gamma b}\delta_{\delta A}\langle CD|\,ac\rangle^{A}]\\
&\qquad\quad+\Pi\delta_-(w_{\gamma a}+w_{\delta\nu}-w_{bb'}-w_{cc'})\delta_{a'\mu}\delta_{c'N}\delta_{b'M}\delta_{S\nu}\\
&\qquad\quad\times[\delta_{\delta A}\delta_{\gamma b}\langle CD|\,ca\rangle^{A}+\delta_{\delta A}\delta_{\gamma c}\langle CD|\,ab\rangle^{A}+\delta_{\gamma A}\delta_{\delta a}\langle CD|\,bc\rangle^{A}]\}\ .
\end{aligned}\tag{A3}$$

We notice that the second term inside the outermost brackets is identical to the first under the following redefinition of labels,

$$\gamma\leftrightarrow\delta,\ \ \mu\leftrightarrow\nu,\ \ a'\leftrightarrow b',\text{and } a\leftrightarrow b\ . \tag{A4}$$

Then, as we average over the coordinates of particles 2 and 3, we get the three body collision term

$$\begin{aligned}&\int_0^\infty d\tau\ \mathrm{Tr}_{2,3}\Psi_3 U_0(1,2,3,-\tau)\Pi_3(1/2/3)\\&\quad=\sum_{\text{all labels}}|\alpha\rangle\langle\alpha'|\rho_{aa'}\rho_{bb'}\rho_{cc'}\Pi\delta_-(w_{\delta b}+w_{\gamma\mu}-w_{aa'}-w_{cc'})\\&\qquad\times(\delta_{\gamma A}\delta_{\delta a}\langle CD\,|\,cb\rangle^A+\delta_{\gamma A}\delta_{\delta c}\langle CD\,|\,ba\rangle^A+\delta_{\gamma b}\delta_{\delta A}\langle CD\,|\,ac\rangle^A)\\&\qquad\times(L1^{\mu b'\alpha'\beta}_{\alpha\beta\gamma\delta}+L1^{b'\mu\alpha'\beta}_{\alpha\beta\delta\gamma})L1^{a'c'\mu B}_{ABCD}\ .\end{aligned} \tag{A5}$$

Some further tedious algebra with attention to cancellations and convenient renaming of labels from time to time rewards one with the sight of the final expression,

$$W(\rho_1)=\sum_{\alpha\alpha'}|\alpha\rangle\langle\alpha'|\,(G_{\alpha\alpha'}-L_{\alpha\alpha'}) \tag{A6}$$

with the gain-and-loss balance weights,

$$\begin{aligned}L_{\alpha\alpha'}=\sum_{\substack{abcd\\a'b'c'd'}}\{&\rho_{aa'}\rho_{bb'}\delta_{cc'}\delta_{dd'}(V_{\alpha b'dc}V'^A_{d'c'ab}\delta_{a'\alpha'}+V_{dc'\alpha'b}V'^A_{a'b'dc}\delta_{\alpha a})\\&-\rho_{aa'}\rho_{bb'}\rho_{cc'}\delta_{dd'}V^A_{\alpha b'dc}V'^A_{d'c'ab}\delta_{a'\alpha'}-\rho_{aa'}\rho_{bb'}\rho_{dd'}\delta_{cc'}V'^A_{a'b'dc}V^A_{d'c'\alpha'b}\delta_{\alpha a}\}\ ,\end{aligned} \tag{A7}$$

$$\begin{aligned}G_{\alpha\alpha'}=\sum_{\substack{abcd\\a'b'c'd'}}\{&\rho_{aa'}\rho_{bb'}\delta_{cc'}\delta_{d\alpha}\delta_{d'\alpha'}(V'_{b'a'\alpha'c}V^A_{\alpha c'ba}-V_{a'b'\alpha'c}V'^A_{\alpha c'ba})\\&-\rho_{aa'}\rho_{bb'}\rho_{cc'}\delta_{d\alpha}\delta_{d'\alpha'}(V'_{b'a'\alpha'c}V^A_{\alpha c'ba}-V_{a'b'\alpha'c}V'^A_{\alpha c'ba})\\&-\rho_{aa'}\rho_{bb'}\rho_{dd'}\delta_{cc'}(V'_{b'a'cd}V^A_{\alpha c'ba}\delta_{d'\alpha'}+V_{\alpha'cb'a'}V'^A_{bac'd'}\delta_{\alpha d})\}\ .\end{aligned} \tag{A8}$$

For simplicity, hereafter we absorb the δ_- factors in the interaction matrix, elements, i.e.,

$$V'_{abcd}=V_{abcd}\Pi\delta_-(w_{ac}-w_{bd})\ . \tag{A9}$$

It should be noticed that, while the real part of the δ_- distribution is in charge of selecting energy-conserving scattering events of strength V_{abcd}, its imaginary part usually gives a vanishing contribution when the summation over sp labels is converted into a sum over the transferred energy, among other observables. This in fact happens since the term under the sum or integral symbol is, in general, an even function of the transferred energy.

Now, although expressions (A7) and (A8) are exact, they are of limited utility in their general fashion. A particularly interesting situation arises under the following assumptions. First, let us broaden the singular kernels, replacing them by Lorentzian factors, namely,

$$\Pi\delta(\epsilon_a+\epsilon_b-\epsilon_c-\epsilon_d)\approx\frac{\Gamma_{abcd}}{w_{abcd}^2+\Gamma_{abcd}^2}\ , \tag{A10}$$

where

$$w_{abcd}=\epsilon_a+\epsilon_b-\epsilon_c-\epsilon_d\ , \tag{A11}$$

and

$$\Gamma_{abcd}=\Gamma_a+\Gamma_b+\Gamma_c+\Gamma_d\ , \tag{A12}$$

Γ_a being a sp width. Actually the Lorentzian factor appears as the real part of the integrated time exponential in (A1) when the sp levels are described via a complex frequency that reflects their finite lifetime and can be introduced into the theory through some convenient representation for the two body propagator $U_2(1,2;\tau)$ other than the present one (see, for example, Ref. 23). The broadened ker-

nel (A10) filters the sp energies contributing to the summations in (A7) and (A8) to within some narrow range. Thus, we further suppose that for each selection of labels in these expressions, all terms adding up contribute the same Lorentzian factor that comes out as a form factor $F(w,\Gamma)$, with w the energy transferred in the transition under consideration. In these conditions it is reasonable to complete the picture with the assumption that the interaction matrix elements vary smoothly, as functions of the sp labels, within the given energy range. This means, with some convenient reaccommodation of labels, we can write

$$W(\rho_1)\sim \sum_{\substack{\alpha\beta\gamma\delta\\ a'\beta'\gamma'\delta'}} |\alpha\rangle\langle\alpha'| F(w,\Gamma)V^A_{\alpha\beta'\gamma\delta}V^A_{\gamma'\delta'\alpha'\beta} \times\{\rho_{\gamma\gamma'}\rho_{\delta\delta'}(1-\rho_{\alpha\alpha'})(\delta_{\beta\beta'}-\rho_{\beta\beta'}) -\rho_{\alpha\alpha'}\rho_{\beta\beta'}(\delta_{\gamma\gamma'}-\rho_{\gamma\gamma'})(\delta_{\delta\delta'}-\rho_{\delta\delta'})\} . \tag{A13}$$

APPENDIX B. LINEARIZATION OF THE COLLISION MATRIX

The kinetic equation (31) is an evolution law for the coefficient of $|\alpha\rangle\langle\alpha'|$ in the operator equation of motion (18). Our purpose is to write Eq. (18) as

$$\dot\rho_1=\{-i\mathscr{L}(\rho_1)+\mathscr{K}(\rho_1)\}\rho_1 , \tag{B1}$$

so that the coefficients of its spectral representation build up the coupled system,

$$\dot\rho_{\alpha\alpha'}=\sum_{\nu\nu'}\{-i\mathscr{L}^{\nu'\alpha'}_{\alpha\nu}+\mathscr{K}^{\nu'\alpha'}_{\alpha\nu}\}\rho_{\nu\nu'} . \tag{B2}$$

We must write the kernel $\mathscr{K}$ in such a way that (B2) represents Eq. (31). This can be done on inspection of Eqs. (A13) or (31); one readily finds

$$\begin{aligned}\mathscr{K}^{\nu'\alpha'}_{\alpha\nu}=&\delta_{\nu\beta}\delta_{\nu'\beta'}\sum_{\gamma\delta\gamma'\delta'}W^{\alpha'\nu'\gamma'\delta'}_{\alpha\nu\beta\delta}(-\tfrac{1}{3}\rho_{\gamma\gamma'}\rho_{\delta\delta'}-\rho_{\alpha\alpha'}\delta_{\gamma\gamma'}\delta_{\delta\delta'}+\tfrac{1}{2}\rho_{\alpha\alpha'}\rho_{\gamma\gamma'}\delta_{\delta\delta'}+\tfrac{1}{2}\rho_{\alpha\alpha'}\rho_{\delta\delta'}\delta_{\gamma\gamma'})\\ &+\delta_{\nu\gamma}\delta_{\nu'\gamma'}\sum_{\beta\delta\beta'\delta'}W^{\alpha'\beta'\nu'\delta'}_{\alpha\beta\nu\delta}(\tfrac{1}{2}\rho_{\delta\delta'}\delta_{\beta\beta'}-\tfrac{1}{2}\rho_{\delta\delta'}\rho_{\alpha\alpha'}\delta_{\beta\beta'}-\tfrac{1}{3}\rho_{\delta\delta'}\rho_{\beta\beta'}+\tfrac{1}{2}\rho_{\alpha\alpha'}\rho_{\beta\beta'}\delta_{\delta\delta'})\\ &+\delta_{\nu\delta}\delta_{\nu'\delta'}\sum_{\beta\gamma\beta'\gamma'}W^{\alpha'\beta'\gamma'\nu'}_{\alpha\beta\gamma\nu}(\tfrac{1}{2}\rho_{\gamma\gamma'}\delta_{\beta\beta'}-\tfrac{1}{2}\rho_{\gamma\gamma'}\rho_{\alpha\alpha'}\delta_{\beta\beta'}-\tfrac{1}{3}\rho_{\gamma\gamma'}\rho_{\beta\beta'}+\tfrac{1}{2}\rho_{\alpha\alpha'}\rho_{\beta\beta'}\delta_{\gamma\gamma'}) .\end{aligned} \tag{B3}$$

It is convenient to mark the significance of the block superoperators appearing in Eqs. (42), namely the diagonal-diagonal, diagonal-nondiagonal, etc., ones;

$$\mathscr{K}_{DD}=\sum_{\nu}\mathscr{K}^{\nu\alpha}_{\alpha\nu}|\alpha\rangle\langle\nu|\otimes|\nu\rangle\langle\alpha| , \tag{B4}$$

$$\mathscr{K}_{DN}=\sum_{\nu\neq\nu'}\mathscr{K}^{\nu'\alpha}_{\alpha\nu}|\alpha\rangle\langle\nu|\otimes|\nu'\rangle\langle\alpha| , \tag{B5}$$

$$\mathscr{K}_{ND}=\sum_{\nu}\mathscr{K}^{\nu\alpha'}_{\alpha\nu}|\alpha\rangle\langle\nu|\otimes|\nu\rangle\langle\alpha'| , \tag{B6}$$

$$\mathscr{K}_{NN}=\sum_{\nu\neq\nu'}\mathscr{K}^{\nu'\alpha'}_{\alpha\nu}|\alpha\rangle\langle\nu|\otimes|\nu'\rangle\alpha'| . \tag{B7}$$

A trivial verification of (B3) is attained as one calculates the matrix elements of $\mathscr{K}_{DD}$,

$$\begin{aligned}\mathscr{K}^{\nu\alpha}_{\alpha\nu}=&\delta_{\nu\beta}\sum_{\gamma\delta}W_{\alpha\nu\gamma\delta}(-\tfrac{1}{3}\rho_\gamma\rho_\delta-\rho_\alpha+\tfrac{1}{2}\rho_\alpha\rho_\gamma+\tfrac{1}{2}\rho_\alpha\rho_\delta)\\ &+\delta_{\nu\gamma}\sum_{\beta\delta}W_{\alpha\beta\nu\delta}(\tfrac{1}{2}\rho_\delta-\tfrac{1}{2}\rho_\delta\rho_\alpha-\tfrac{1}{3}\rho_\delta\rho_\beta+\tfrac{1}{2}\rho_\alpha\rho_\beta)\\ &+\delta_{\nu\delta}\sum_{\beta\gamma}W_{\alpha\beta\gamma\nu}(\tfrac{1}{2}\rho_\gamma-\tfrac{1}{2}\rho_\gamma\rho_\alpha-\tfrac{1}{3}\rho_\gamma\rho_\beta+\tfrac{1}{2}\rho_\alpha\rho_\beta) .\end{aligned} \tag{B8}$$

It is now straightforward to verify that $\sum_\nu \mathscr{K}^{\nu\alpha}_{\alpha\nu}\rho_\nu$ gives the diagonal Boltzmann equation written in formula (33).

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