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Antisymmetrized deuteron stripping

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A three-body model of the antisymmetrized deuteron-nucleus system is evaluated approximately in terms of an effective neutron-proton interaction that can be used in the standard distorted-wave Born approximation matrix element for stripping. Although there is a significant reduction of interior contributions to stripping at low bombarding energy, this effect becomes less important at higher energy; in particular, it has little influence on the reaction ^{24}Mg(p,d) at 94 MeV.

[NUCLEAR REACTIONS Scattering theory, antisymmetrized three-body model for deuteron stripping. Application to ^{24}Mg(p,d) at 94 MeV.]

I. INTRODUCTION

The dynamics of the deuteron plus nucleus system has been studied repeatedly since the first direct reaction theories of stripping were introduced thirty years ago. Recent analyses[1-4] developed detailed evaluations of plausible ansatz solutions of the three-body model: neutron plus proton plus nucleus. What is at last clear from this work is that although "three-body effects" are large, as indicated by the large breakup part of the model three-body wave functions, such effects are less significant than a correct treatment of internal coordinates of the target nucleus.

Internal coordinates of the nucleus affect the construction of three-body models in two ways. First, they produce imaginary parts in the potentials in the three-body model: These imaginary potentials cause major modifications of the solutions[5] and they can have complicated energy dependences.[6] Second, antisymmetrization with respect to nuclear coordinates modifies both the derivation of a three-body wave function and its application for the calculation of matrix elements. In this paper we consider in detail the consequences of antisymmetry in the application of a three-body model for deuteron stripping.

Particle identity is traditionally[7] inserted in the stripping matrix element by applying an antisymmetrizer to the DWBA ansatz wave function, to give

$$\mathcal{A}\Phi\chi_d^{(+)}(\tfrac{1}{2}(\vec{r}_p+\vec{r}_n))\phi_d(|\vec{r}_p-\vec{r}_n|),$$

where Φ is the internal wave function of the target nucleus. New effects arise because the proton, for example, that emerges asymptotically can originate either from the incident deuteron (direct term) or from various orbitals of the struck nucleus (exchange terms). Standard classifications[7] of the exchange terms distinguish "knockon exchange," in which the incident projectile interacts with the ejected proton, from "heavy particle stripping" (HPS) and "optical potential exchange" terms, in which the projectile interacts with the remainder of the struck nucleus. Early interest in HPS was motivated[7] by back-angle features of experimental angular distributions. However, the effects in question were subsequently satisfactorily understood in terms of DWBA calculations of the direct term. In fact, in a careful DWBA context the near orthogonality of bound and continuum single-particle (s.p.) orbitals causes all the exchange terms in the traditional (d,p) matrix element to be very small, with the result that they have not received systematic investigation. (By contrast, the knockon term in inelastic scattering associates an interaction with every active s.p. wave function; as a result it is not reduced by orthogonality and it is frequently as

large as the direct term.)

In an improved analysis of antisymmetry in a three-body model, the DWBA ansatz is replaced by the expression $\mathcal{A}\Phi\psi(\vec{r}_p,\vec{r}_n)$, and the occupied orbitals of Φ are allowed to partially block the possible dynamical development of the relative wave function $\psi(\vec{r}_p,\vec{r}_n)$. If this relative wave function is required to be orthogonal to the occupied s.p. orbitals of Φ, it is found to reduce to a solution of a three-body equation of Bethe-Goldstone form,[8-10] with projection operators that enforce the orthogonality property. The (d,p) matrix element based on $\psi(\vec{r}_p,\vec{r}_n)$ computed in this manner consistently includes all the exchange effects discussed previously, in the form of an improved calculation of the direct term. It is this procedure we consider in the present article. We seek a sufficiently accurate Pauli modified wave function $\psi(\vec{r}_p,\vec{r}_n)$.

Previous analyses of blocking[8,9] use explicit ansatz expressions for the three-body wave function. In this work such explicit assumptions are avoided. We instead combine the Pauli projection operators into a simplified effective neutron-proton interaction V_{np}^{eff} that can be used in place of V_{np} in stripping calculations.

The operator V_{np}^{eff} can be regarded as the analog for deuteron stripping of the medium-dependent t matrix that appears in theories of other direct reactions. (Some complications of this interpretation, associated with nonlocalities of V_{np}^{eff}, are discussed at the end of Sec. II.) This use of a t matrix is not entirely new. It is not generally recognized that the standard DWBA theory of deuteron stripping already has the structure of a distorted wave impulse approximation, in terms of the *free t matrix,* in which V_{np} multiplies the eigenfunction $\phi_d(r)$. This elementary DWIA structure has been one of the reasons for the success of the standard theory.

Medium effects were considered previously in a rather formal article by Döhnert[11] and in an explicit analysis of ^{16}O(p,d) by Preedom,[12] based on Green's semiempirical bound-state G matrix.[13] Although the bound-state G matrix contains substantial Pauli effects,[12] we will see that the characteristic momentum relations in a stripping reaction tend to suppress such effects. Antisymmetrization effects in deuteron stripping and elastic scattering have also been considered by Thompson,[14] using a generalization of his method for calculating nonorthogonality terms in rearrangement collisions.[15]

The Pauli effects in V_{np}^{eff} are only large at low bombarding energies, when they tend to weaken the neutron-proton interaction in the nuclear interior. Other effects that suppress stripping contributions from the nuclear interior are the Perey effect,[16] breakup effects calculated in the adiabatic approximation,[17,18] and the phase averaging implied by the careful use of distorted waves.[19] Although Pauli suppression of the interior is redundant with these other effects, it can be important at low energy if the entrance and exit channel momenta are substantially mismatched, so that phase averaging is not effective.

II. THEORY

Analyses of antisymmetry in the neutron plus proton plus nucleus system[8-10] all arrive at a reduced three-body equation of Bethe-Goldstone form

$$(E-K-U_n-U_p)\psi(\vec{r}_p,\vec{r}_n)=QV_{np}\psi(\vec{r}_p,\vec{r}_n)\ , \tag{1}$$

for the wave function $\psi(\vec{r}_p,\vec{r}_n)$ of relative motion. Here K is the kinetic energy operator for the two nucleons, and U_n and U_p are single-particle potentials for interaction of the nucleons with the target nucleus. Although U_n and U_p are treated as local optical potentials, the derivation of Eq. (1) normally interprets them as Hartree-Fock potentials, which already incorporate important effects of antisymmetry. In addition we require the reduced wave function $\psi(\vec{r}_p,\vec{r}_n)$ to be orthogonal to the occupied single-particle eigenstates of the Hamiltonian

$$h_n+h_p\equiv K+U_n+U_p\ . \tag{2}$$

This requirement comes into question on the right hand side of Eq. (1), because V_{np} can to some extent cause transitions into the occupied single particle states. This problem is dealt with by the operator Q, which projects $V_{np}\psi$ on to unoccupied product states of h_n+h_p. Evidently the left hand side of Eq. (1) preserves the projection property of Q; the boundary conditions for ψ also usually conform to this property, therefore Eq. (1) leads to a ψ that is entirely orthogonal to occupied single-particle (s.p.) states.

Let us rewrite Eq. (1) in the form

$$(E-H_0)\psi=(Q-1)V_{np}\psi\ , \tag{3}$$

with

$$H_0\equiv h_n+h_p+V_{np}\ . \tag{4}$$

Here $(Q-1)$ is a projector on to product states that have at least one *occupied* s.p. orbital. The right hand side of Eq. (3) is a correction to the three-body model defined by H_0. Previous analyses have been

based on simple approximations of this correction term, using factored expressions for ψ. We now develop an improved analysis of the correction.

We consider an integral equation equivalent of Eq. (3),

$$\psi=\psi_0+(E^+-H_0)^{-1}(Q-1)V_{np}\psi\ , \tag{5}$$

with ψ_0 the exact solution of the uncorrected three-body problem

$$(E-H_0)\psi_0(\vec{r}_p,\vec{r}_n)=0\ . \tag{6}$$

Spurious effects appear in ψ_0, because V_{np} causes coupling to occupied single-particle states. The second term of Eq. (5) cancels these spurious parts of ψ_0.

Equation (5) can be solved in terms of a formal effective interaction that operates on ψ_0 to generate the full ψ. Using the notation

$$G=(E^+-H_0)^{-1}\ , \tag{7}$$

we obtain

$$\psi=[1-G(Q-1)V_{np}]^{-1}\psi_0\ , \tag{8}$$

$$=\{1+G(Q-1)V_{np}[1-G(Q-1)V_{np}]^{-1}\}\psi_0\ , \tag{9}$$

$$=[1+G(Q-1)V_{np}^{\rm eff}]\psi_0\ . \tag{10}$$

Here the effective interaction in Eq. (10) is

$$V_{np}^{\rm eff}\equiv V_{np}[1-G(Q-1)V_{np}]^{-1}\ . \tag{11}$$

We see that $V_{np}\psi$ in Eq. (5) has been replaced by $V_{np}^{\rm eff}\psi_0$ in Eq. (10); thus our definition of $V_{np}^{\rm eff}$ fulfills the usual relation

$$V_{np}\psi=V_{np}^{\rm eff}\psi_0\ . \tag{12}$$

Evidently Pauli corrections to stripping are accomplished if the product $V_{np}\psi$ in the stripping matrix element is replaced by the equivalent expression $V_{np}^{\rm eff}\psi_0$, using the uncorrected wave function ψ_0. We now consider practical approximations for the complicated nonlocal operator $V_{np}^{\rm eff}$.

Considerable simplification is obtained by use of the separable approximation

$$V_{np}=-|f\rangle\langle f|\ , \tag{13}$$

in which we later introduce the Yamaguchi form

$$\langle\vec{r}|f\rangle=Ne^{-\beta r}/r\ . \tag{14}$$

Insertion of Eq. (13) in Eq. (11) leads to the factorized expressions

$$V_{np}^{\rm eff}=V_{np}(r)[1+P_f(\vec{R},\vec{R}')]^{-1}\ , \tag{15a}$$

or

$$V_{np}^{\rm eff}=-|f\rangle[1+P_f(\vec{R},\vec{R}')]^{-1}\langle f|\ . \tag{15b}$$

The nonlocal operator

$$P_f(\vec{R},\vec{R}')\equiv\langle f|G(Q-1)|f\rangle\ , \tag{16}$$

which acts only on the center of mass coordinate $\vec{R}$, is the correction for Pauli blocking. A linearized version of Eqs. (15a) and (15b),

$$V_{np}^{\rm eff}\approx-|f\rangle[1-P_f(\vec{R},\vec{R}')]\langle f|\ , \tag{17}$$

allows easy interpretation of P_f, either as a wave operator that projects ψ_0 on to Pauli forbidden states, or as a scattered amplitude generated from the perturbation $(Q-1)V_{np}\psi_0$. We note the reasonableness of separable approximations for the short-range interaction V_{np}. However, care must be taken that the Yamaguchi form does not misrepresent the high momentum matrix elements of V_{np}.

Approximations for P_f are facilitated by the finite ranges of $\langle\vec{r}|f\rangle$ and of the bound s.p. orbitals in $(Q-1)$. On the other hand, the factors $\langle f|$, $|f\rangle$ select values of the three-body Green's function G near $r=0$. This suggests a need for careful treatment of V_{np} in H_0 in the Green's function.

It is convenient to remove V_{np} for separate treatment by the steps

$$G_1\equiv(E^+-h_n-h_p)^{-1}\ ,$$

$$G=G_1+G_1V_{np}G\ ,$$

$$=G_1-G_1|f\rangle\langle f|G\ , \tag{18}$$

so that

$$\langle f|G=\langle f|G_1-\langle f|G_1|f\rangle\langle f|G\ ,$$

and

$$\langle f|G=[1+\langle f|G_1|f\rangle]^{-1}\langle f|G_1\ . \tag{19}$$

By this transformation we obtain

$$P_f(\vec{R},\vec{R}')=\Lambda P_0\ , \tag{20}$$

in which

$$P_0(\vec{R},\vec{R}')\equiv\langle f|G_1(Q-1)|f\rangle\ , \tag{21}$$

with

$$\Lambda\equiv[1+\langle f|G_1|f\rangle]^{-1}\ , \tag{22}$$

the operator that corrects for short range correlations.

Approximations suitable for use in Λ are the Yamaguchi interaction of Eq. (14) and the short-range expression

$$G_1\approx[E^+-K_r-K_R-\bar{V}(R)]^{-1}\ , \tag{23}$$

in which K_r, K_R are kinetic energy operators, and

$$\bar{V}(R) \equiv U_n(R) + U_p(R) .$$

With these approximations, integration over $\vec{r}$ yields

$$\Lambda = \left[1 - \left[\frac{\beta+\alpha}{\beta + \{\alpha^2 - \frac{1}{4}[k_d{}^2(R) + \nabla_R{}^2]\}^{1/2}} \right]^2 \right]^{-1} , \tag{24}$$

in which

$$k_d{}^2(R) \equiv (4M/\hbar^2)[E_0^+ - \bar{V}(R)] , \tag{25}$$

α is the reciprocal of the deuteron radius, and E_0 is the incident deuteron kinetic energy. The parameter N of Eq. (14) has been adjusted so that Λ has a pole if the relative energy equals the deuteron binding energy. The infinitesimal positive imaginary part of $k_d{}^2$ determines the phase of the square root in Eq. (24), such that its imaginary part must be negative.

We recognize Λ as an $r \approx 0$ projection of the three-body Green's function G_1; we must expect that the operator P_f obtained from the application of Λ on P_0 in Eq. (20) is of rather long range in $|\vec{R} - \vec{R}'|$, even if P_0 itself is more localized.

We now consider the nonlocality of $P_0(\vec{R},\vec{R}')$. This operator is used in the expression

$$V_{np}^{\text{eff}} \psi_0 \approx - |f\rangle [1 - P_f] \langle f | \psi_0 \rangle , \tag{26}$$

in which $P_f = \Lambda P_0$ of Eq. (20) operates on $\langle f | \psi_0 \rangle$. We reduce the nonlocality of $P_0(\vec{R},\vec{R}')$ by introducing local Wentzel-Kramers-Brillouin (WKB) approximation in Eq. (26) to replace $\langle f | \psi_0 \rangle$ at $\vec{R}'$ by the approximation

$$\langle f | \psi_0 \rangle_{\vec{R}'} \approx \langle f | \psi_0 \rangle_{\vec{R}} \exp i \vec{k}_d \cdot (\vec{R}' - \vec{R}) . \tag{27}$$

Here $\vec{k}_d(\vec{R})$ is the local momentum whose magnitude is defined in Eq. (25). The exponential factor of Eq. (27) is combined with $P_0(\vec{R},\vec{R}')$ to define the local counterpart

$$\hat{P}_0(\vec{R}) \equiv e^{-i\vec{k}_d \cdot \vec{R}} \int d^3R' P_0(\vec{R},\vec{R}') e^{i\vec{k}_d \cdot \vec{R}'} . \tag{28}$$

The reasonableness of the local WKB approximation is examined later.

To continue the explicit evaluation of $\hat{P}_0$, we recall[9,20] that $(Q-1)$ is a sum of single- and double-exchange terms. Double exchange is usually rather weak. In the present case it requires an overlap between both nucleons of the incident deuteron and occupied states in the Fermi sea; momentum mismatch causes the contribution from this overlap to be negligible. The single-exchange terms of $(Q-1)$ can be expressed in terms of density matrices, so that

$$(Q-1) \approx \tfrac{1}{2}[\rho_p(\vec{r}_p,\vec{r}'_p)\delta(\vec{r}_n - \vec{r}'_n) + \rho_n(\vec{r}_n,\vec{r}'_n)\delta(\vec{r}_p - \vec{r}'_p)] . \tag{29}$$

There is a factor $\frac{1}{2}$ in Eq. (29) because only half the occupied states have spins parallel to those of the incident nucleons. We now insert Eq. (29) in Eq. (21) for P_0 and we allow h_p of the Green's function to operate on the s.p. orbitals in the first term of Eq. (29) and h_n to operate on the s.p. orbitals in the second term. Both these operations are replaced approximately by multiplications by $\bar{\epsilon}$, an average s.p. energy for the occupied orbitals. By this series of steps $\hat{P}_0(\vec{R})$ reduces to

$$\hat{P}_0(\vec{R}) \approx -e^{-i\vec{k}_d \cdot \vec{R}} \int d^3R' \langle f | (E^+ - \bar{\epsilon} - h_n)^{-1} \rho_p(\vec{r}_p,\vec{r}'_p) | f \rangle e^{i\vec{k}_d \cdot \vec{R}'} , \tag{30}$$

where the two terms of Eq. (29) are assumed to contribute equally to $\hat{P}_0(\vec{R})$. The bra and ket vectors in Eq. (30) imply integrations over the relative coordinates $\vec{r},\vec{r}'$, respectively.

To complete the calculation of $\hat{P}_0$ we insert the factorized density matrix of Negele and Vautherin[9,21]

$$\rho_p(\vec{r}_p,\vec{r}'_p) \approx \rho_p(r_p) C(\vec{r}_p - \vec{r}'_p) , \tag{31}$$

with

$$C(\vec{r}_p - \vec{r}'_p) = 3j_1(k_F |\vec{r}_p - \vec{r}'_p|)/k_F |\vec{r}_p - \vec{r}'_p| , = \frac{3}{4\pi k_F{}^3} \int_{k<k_F} d^3k\, e^{i\vec{k}\cdot(\vec{r}_p - \vec{r}'_p)} . \tag{32}$$

Under the approximation that ρ_p is slowly varying and can be removed from the integral, Eq. (30) becomes

$$\hat{P}_0(\vec{R}) \approx (3\rho(R)/8\pi k_F{}^3) \int_{k<k_F} d^3k [\tilde{f}(\vec{k} + \tfrac{1}{2}\vec{k}_d)]^2 [(\vec{k} + \vec{k}_d)^2 - \kappa^2 - i\epsilon]^{-1} , \tag{33}$$

where $\rho(R)$ is now the total density of nucleons, $\rho \approx 2\rho_p$. The momentum $\hbar\kappa(R)$ is the local momentum of the nucleon that is *not exchanged* into an occupied state. It contains the potential from h_n in Eq. (30), so that

$$(\hbar^2\kappa^2/2M) = E_\kappa(R) = E - \bar{\epsilon} - U_n(R) \ . \tag{34}$$

The form factors $\tilde{f}(\vec{k}+\frac{1}{2}\vec{k}_d)$ are Fourier transforms of the interaction functions $\langle \vec{r} \mid f \rangle$ in Eq. (30),

$$\tilde{f}(\vec{q}) \equiv \int d^3r \, e^{i\vec{q}\cdot\vec{r}} \langle \vec{r} \mid f \rangle \ . \tag{35}$$

The two form factors in Eq. (33) originate from the two coordinate transformations implied in Eq. (30),

$$\vec{r}', \vec{R}' \rightarrow \vec{r}'_p, \vec{r}_n, \quad \vec{r}_p, \vec{r}_n \rightarrow \vec{r}, \vec{R} \ .$$

Our final formal step returns to the Λ factor in the local WKB analysis of Eq. (26). Because the derivative operator in Λ now appears in the product

$$\Lambda \hat{P}_0(\vec{R}) \langle f \mid \psi_0 \rangle_{\vec{R}} \ ,$$

we simply replace $\nabla_R{}^2$ in Eq. (24) by $-(\vec{q}+\vec{k}_d)^2$ and obtain

$$\Lambda \hat{P}_0(\vec{R}) \langle f \mid \psi_0 \rangle_{\vec{R}} \approx \left[\int d^3q \, \Lambda(-(\vec{q}+\vec{k}_d)^2) \tilde{P}_0(\vec{q}) e^{i\vec{q}\cdot\vec{R}} \right] \langle f \mid \psi_0 \rangle_{\vec{R}} \ , \tag{36}$$

where $\tilde{P}_0(\vec{q})$ is the Fourier transform

$$\tilde{P}_0(\vec{q}) \equiv (2\pi)^{-3} \int d^3R \, e^{-i\vec{q}\cdot\vec{R}} \hat{P}_0(\vec{R}) \ . \tag{37}$$

In evaluating $\tilde{P}_0(\vec{q})$ we take the R dependence of $\hat{P}_0(\vec{R})$ to be that of $\rho(R)$ in Eq. (33), with $\rho(R)$ given by a Woods-Saxon form, for which an accurate expression for the Fourier transform is well known. Equation (36) can also be written in terms of a folding integral

$$\Lambda \hat{P}_0(\vec{R}) = \int d^3R' \tilde{\Lambda}(\vec{R}-\vec{R}') \hat{P}_0(\vec{R}') \ , \tag{38}$$

where

$$\tilde{\Lambda}(\vec{x}) \equiv (2\pi)^{-3} \int d^3q \, \Lambda(-q^2) e^{i\vec{x}\cdot(\vec{q}-\vec{k}_d)} \ . \tag{39}$$

As a guide to the properties of $\tilde{\Lambda}(\vec{x})$ we approximate Λ in Eq. (24) by the form obtained by neglecting the small quantity α, to obtain

$$\Lambda(-q^2) \approx 1 + \beta^2 [\beta(q^2-k_d{}^2)^{1/2} + \tfrac{1}{4}(q^2-k_d{}^2)]^{-1} \ , \tag{40}$$

so that

$$\tilde{\Lambda}(\vec{x}) = \delta(\vec{x}) + e^{-i\vec{k}_d\cdot\vec{x}} \left[\frac{\beta^2}{2\pi^2 x} \right] \times \int_0^\infty dq \frac{q \sin qx}{\beta(q^2-k_d{}^2)^{1/2} + \frac{1}{4}(q^2-k_d{}^2)} \ , \tag{41}$$

$$\approx \delta(\vec{x}) + e^{-i\vec{k}_d\cdot\vec{x}} \left[\frac{i\beta k_d}{4\pi x} \right] H_1^{(1)}(k_d x) \ . \tag{42}$$

In obtaining Eq. (42) the small second term in the denominator in Eq. (41) has been omitted. The second term in Eq. (42) now accounts for short range correlations in the original Green's function in Eq. (16).

We see that the function $\tilde{\Lambda}(\vec{x})$ is strongly anisotropic and that in some directions it becomes rather long ranged, since as $\vec{x}$ takes different directions the oscillations of $H_1^{(1)}$ can be reinforced or cancelled by those of $\exp(-i\vec{k}_d\cdot\vec{x})$. The short range correlations in Λ propagate farthest for $\vec{x}$ in the forward direction. These properties are appropriate, since the Pauli correction term in Eq. (26) not only generates short range modifications inside the target nucleus, it also generates Pauli modified scattered waves. Thus, the effective interaction V_{np}^{eff} does *not* approach V_{np} as $R \rightarrow \infty$, as can also be seen formally from the definitions in Eqs. (15) and (16). (Calculations of effective interactions for bound states are not subject to these long range complications.) In this work emphasis is placed on the short range modifications, which are of most relevance to stripping. Calculations of the Pauli modified elastically scattered waves are reported in Refs. 15 and 22. We finally note that the long range properties of Λ do not conflict with the use of local WKB, because the WKB approximation for $\langle f \mid \psi_0 \rangle_{\vec{R}}$ is only applied in the source function for Λ, in its limited region of overlap with $\hat{P}_0(\vec{R})$. The calculations reported in the next section make use of the expression Eq. (36) without the approximations to Λ that lead to Eq. (40).

III. DISCUSSION AND APPLICATIONS

Physical applications of the above theory are based on the approximate effective *np* interaction of Eq. (26),

$$V_{np}^{\text{eff}} \approx V_{np}(1+\Lambda\hat{P}_0)^{-1} , \tag{43}$$

with the correction term $\Lambda\hat{P}_0$ given by Eqs. (33), (36), and (37). We now compute $\Lambda\hat{P}_0$ for some cases of practical interest.

A. Local WKB

The weakest step of our analysis probably has been the use of local WKB approximation in the projected wave function $\langle f \mid \psi_0 \rangle_{\vec{R}}$, to reduce the Green's function in P_0 to the simple energy denominator of Eq. (33). Local WKB is sometimes seriously misleading. For example, it was shown long ago that a local WKB evaluation of the finite-range correction to stripping greatly exaggerates that effect.[23] However, the finite-range analysis concerns a rapidly-varying form factor, and it makes essential use of relations between the directions of local momenta. In the present case we apply the local momentum in a Green's function, and it is only combined vectorially with the Fermi gas momentum $\vec{k}$, which is averaged over all directions. Our use of the local momentum seems sufficiently accurate for this application.

Corrections to local WKB would be obtained by treating $\langle f \mid \psi_0 \rangle_{\vec{R}}$ as a distribution of momenta, rather than as a single local plane wave of momentum $\vec{k}_d$. Because such corrections are likely to be most important in the surface region, we consider our theory to be unreliable in the nuclear surface and we only place emphasis on our calculated results at $R=0$.

Although we ignore the possible distribution of momenta $\vec{k}_d$ in Eq. (33), the introduction of such a distribution would have helpful implications. With independent averages over $\vec{k}_d$ and over the momentum $\vec{k}$ in the Fermi sea, each momentum average would tend to reduce complications that might be associated with the other momentum. For example, the d^3k integration would tend to eliminate modifications of $\vec{k}_d$ due to absorption. On the other hand, if k_F were treated as density dependent and allowed to become small in the surface region, we see that an average over $\vec{k}_d$ in Eq. (33) would tend to compensate for the lost average over $\vec{k}$. Once again, we ignore these aspects and emphasize $R=0$.

B. Numerical results for ^{208}Pb target at $R=0$

For a first application of the Pauli correction theory, we evaluate $\Lambda\hat{P}_0$ of Eq. (43) at $R=0$. We apply the correlation factor Λ to a simple shape function $\rho(\vec{R})$ in Eq. (37), in place of the accurate $\hat{P}_0(\vec{R})$, so that

$$P_f(0)=(\Lambda\hat{P}_0)_{R=0}$$
$$=\hat{P}_0(0)\left\{\frac{\int d^3q\,\Lambda(-(\vec{q}+\vec{k}_d)^2)\tilde{\rho}(\vec{q})}{\int d^3q\,\tilde{\rho}(\vec{q})}\right\}, \tag{44}$$

with $\hat{P}_0(0)$ calculated from Eq. (33) at $R=0$. Equation (33) is integrated semianalytically, using the form factor obtained for the Yamaguchi interaction of Eq. (14)

$$[\tilde{f}(q)]^2=[\tilde{f}(0)]^2[1+(q/\beta)^2]^{-2} , \tag{45}$$

with

$$[\tilde{f}(0)]^2=(8\pi\hbar^2/\beta M)\left[1+\frac{\alpha}{\beta}\right]^2$$
$$=1070\ \text{MeV fm}^3 . \tag{46}$$

Reasonable estimates for the other parameters are

$$\rho(0)=0.17\ \text{fm}^{-3},\quad E_F=38.5\ \text{MeV} ,$$
$$k_F=1.36\ \text{fm}^{-1},\quad \beta=1.34\ \text{fm}^{-1} . \tag{47}$$

The Pauli correction at $R=0$ is now calculated numerically for a ^{208}Pb target nucleus, using Eqs. (33) and (44). The local momenta $k_d(0)$ and $\kappa(0)$, defined by Eqs. (25) and (34), are calculated assuming U_n and $\bar{V}$ to be the nucleon optical potential and the sum of the neutron and proton optical potentials, respectively. These are taken from the tabulation of Becchetti and Greenlees.[24] For $\rho(R)$ we choose a Woods-Saxon form derived from electron scattering.[25] Finally the local energy $E_\kappa(0)$ is obtained from Eq. (34) using the estimate $\bar{\epsilon}\approx -20$ MeV. (Explicit calculations show that our results are not sensitive to change in $\bar{\epsilon}$ in the range -10 to -30 MeV, except at the lowest incident energies.)

The results for $P_f(0)$ and $[1+P_f(0)]^{-1}$ at several typical bombarding energies are given in Table I.

TABLE I. Values of the Pauli correction factor for ^{208}Pb at $R=0$, as a function of incident deuteron kinetic energy E_0. This factor relates V_{np}^{eff} and V_{np} in Eq. (15).

E_0 (MeV)	$P_f(0)\equiv(\Lambda\hat{P}_0)_{R=0}$	$[1+P_f(0)]^{-1}$
0	$0.38+i0.95$	$0.49-i0.34$
40	$0.34+i0.60$	$0.62-i0.28$
100	$0.30+i0.27$	$0.74-i0.15$

The effective interaction $V_{np}[1+P_f]^{-1}$ at $R=0$ is seen to increase slowly with energy, as expected. It is interesting that the correlation factor Λ approximately doubles the value of $P_f(0)$ in this calculation.

For very high energy we have $k_d \gg k_F$ and κ, therefore Eq. (33) reduces to

$$\hat{P}_0(R)=\rho(R)[\tilde{f}(\tfrac{1}{2}k_d)]^2/2k_d^2 \,. \tag{48}$$

Although this approximation is not very useful at moderate deuteron energies, it does bring out the important role played by the momentum dependence of the V_{np} form factor in determining the magnitude of the Pauli correction. We comment further on this effect in Sec. IV.

C. Effect of R dependence: Application to ^{24}Mg

An estimate of the R dependence of P_f can be obtained by endowing the quantities $E_d(R)$, $E_\kappa(R)$, and $E_F(R)$ with the R dependence obtained from Eqs. (25) and (34), using physically reasonable potentials $\bar{V}(R)$ and $U_n(R)$, and an assumed nucleon density distribution $\rho(R)$. As an illustration we have chosen the case of 80 MeV deuterons incident on ^{24}Mg, which has been the subject of considerable recent interest.[26]

Our results for $(1+P_F)^{-1}$ are shown in Fig. 1 and the effect of inserting this factor into a stand-

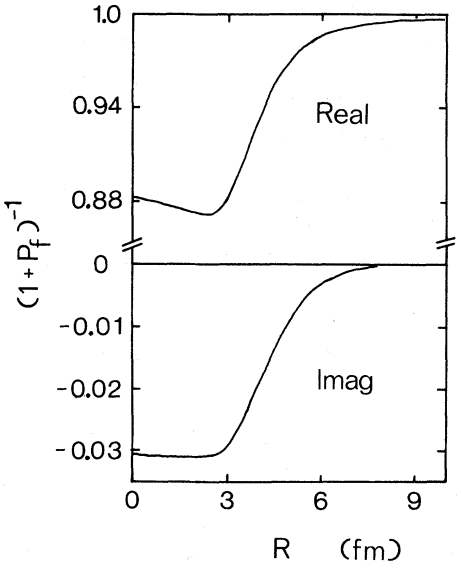

FIG. 1. Pauli correction factor $(1+P_f)^{-1}$ as function of R, for 80 MeV deuterons incident on ^{24}Mg, as given by Eq. (36) (spherically symmetric part only) and Eqs. (25) and (34).

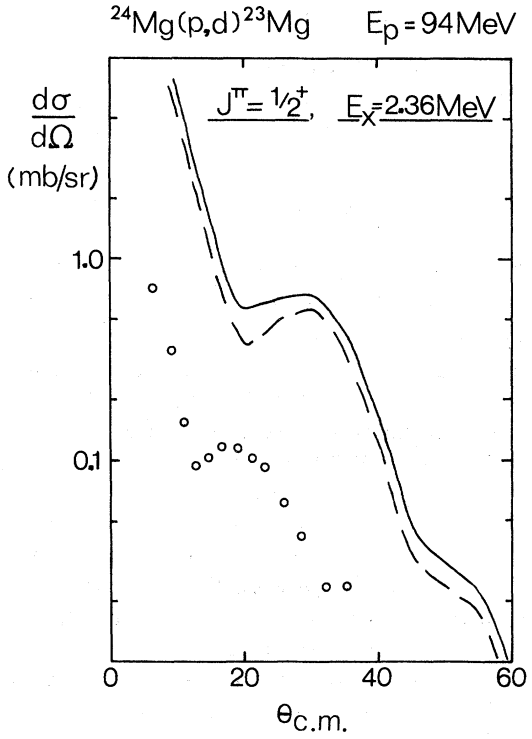

FIG. 2. DWBA zero-range calculation for ^{24}Mg$(p,d)^{23}$Mg (2.36 MeV, $\frac{1}{2}^+$ state) at $E_p=94$ MeV with (dashed curve) and without (solid curve) the Pauli correction factor of Fig. 2. The data are from Ref. 31.

TABLE II. Potential parameters. Coulomb radius $=1.3$ fm, spectroscopic factor $=0.655$ (Ref. 30), and neutron binding energy $=18.891$ MeV.

		Proton (Ref. 24)	Deuteron (Ref. 29)	Neutron
Real	V (MeV)	25.58	61.50	68.23
	r_v (fm)	1.17	1.25	1.31
	a_v (fm)	0.75	0.70	0.70
	Shape	WS	WS	WS
Imag.	W (MeV)	18.0	13.5	
	r_w (fm)	1.32	1.12	
	a_w (fm)	0.51	0.69	
	Shape	WS	WS Der.	

ard DWBA zero range calculation[27] of the reaction ^{24}Mg(p,d) at 94 MeV is shown in Fig. 2. Table II lists the parameters used for this calculation. It is clear that the Pauli correction alone cannot account for the order of magnitude discrepancy with experiment reported in Ref. 26.

D. Angle dependence of Λ

As a final refinement of the calculation of $P_F(R)$, we return to the observation made following Eq. (42), concerning the dependence of $\Lambda\hat{P}_0$ in Eq. (36) on the angle between $\vec{k}_d$ and $\vec{R}$. This effect has been ignored in the calculations reported so far, except at $R=0$, where, of course, only the spherically symmetrical part of $\Lambda\hat{P}_0$ survives.

The $L=1$ contribution to this angular dependence is readily calculated. In Fig. 3 the resulting modified factor $[1+P_f]^{-1}$ is shown for three values of the angle θ between $\vec{k}_d$ and $\vec{R}$ for the same case as Fig. 1. Clearly, the angular dependence of P_f will play an important role in a precise evaluation of the Pauli effects in the nuclear surface, but it is unlikely to modify the qualitative nature of the results shown in Fig. 2.

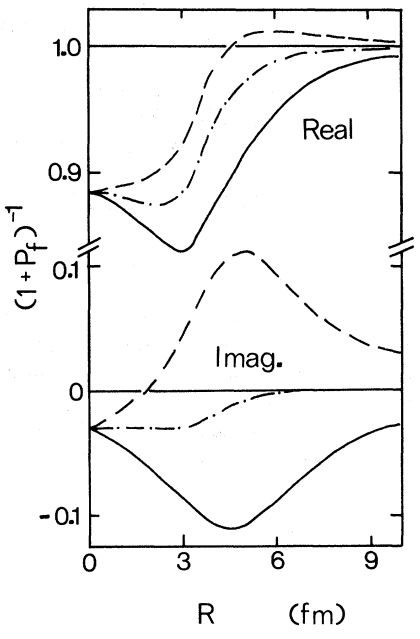

FIG. 3. The Pauli correction factor $(1+P_f)^{-1}$ for 80 MeV deuterons incident on ^{24}Mg. These calculations include both the $L=0$ and $L=1$ parts of the dependence on $\cos\theta$ $(=\vec{k}_d\cdot\vec{R}/k_dR)$ in Eq. (36). The solid, dashed-dotted, and dashed curves correspond, respectively, to $\cos\theta=+1$, 0, and -1.

IV. SUMMARY

Pauli modifications of the three-body neutron plus proton plus nucleus relative wave function are expressed in terms of an effective neutron-proton interaction V_{np}^{eff} whose strength is reduced in the nuclear interior. Although this effective interaction also leads to Pauli modifications of elastic scattering and breakup cross sections, we give particular attention to stripping, for which the amplitude is directly a matrix element of V_{np}^{eff} between unmodified wave functions. In an application to ^{24}Mg the real part of V_{np}^{eff} in the nuclear interior is approximately seventy percent of V_{np} at low energy, but it is about ninety percent at 100 MeV. Thus the Pauli effects do not seem relevant to recent investigations[26] of the ^{24}Mg(p,d) reaction at 94 MeV. However, Pauli suppression of interior contributions at low energy may be significant in cases of poor momentum matching, which would otherwise allow interior contributions. Similar results are obtained in an alternative analysis by Tostevin, Lopes, and Johnson,[22] based on an adiabatic approximation of an intermediate Green's function.

The small value of the Pauli correction obtained here is greatly influenced by the small value of the squared form factor in Eq. (33), as indicated by the ratio

$$[\tilde{f}(k_d/2)/\tilde{f}(0)]^2 ,$$

which ranges from 0.21 at low energy to 0.10 at 100 MeV. Although it may be objected that our $l=0$ Yamaguchi form factor might fall too rapidly at high momenta, we can note that important finite range reductions of $\tilde{f}$ are inherently reasonable. The Pauli corrections to stripping remove spurious transitions from the continuum to Pauli forbidden bound states. These forbidden transitions involve much larger momentum differences, for example, than those that appear in corresponding analyses of Pauli modifications of the t matrix for elastic scattering.[28] Therefore, smaller Pauli effects are expected in stripping than in scattering.

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