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$P-A$ in intermediate energy inelastic scattering

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The difference between the polarization, P, and analyzing power, A, in inelastic scattering by a spin $\frac{1}{2}$ projectile is shown to be proportional to the reaction Q-value. This follows from parity and time reversal symmetry only. At intermediate energy (500−1000 MeV) analyticity suggests that Q scales with $E_{\text{c.m.}}$ so that $|P-A| \leq Q/E_{\text{c.m.}}$, and hence that $|P-A|$ is very small.

[NUCLEAR REACTIONS General restrictions on polarization minus analyzing power with application to intermediate energy inelastic scattering.]

Spin observables in intermediate energy proton-nucleus scattering are currently of great interest, both because they give promise of being a direct and sensitive measure of interesting dynamic features,[1] and because of remarkable experimental developments that make the spin observables accessible. Recently there has been interest in both the polarization P and the analyzing power A in inelastic scattering to discrete nuclear states.[2] For elastic scattering, if time reversal invariance is valid, it is well known that $P=A$, but for inelastic scattering there is no such restriction, and hence there is interest in the difference between P and A $(P-A)$ for these reactions.[3] We will show here that for scattering to states of excitation energy Q by a spin $\frac{1}{2}$ projectile, $P-A$ is proportional to Q. (Note that this reduces to zero for elastic scattering, $Q=0$, as it should.) Our proof is very general and uses only rotational invariance, parity and time reversal invariance, without any particular dynamical model.

Since P and A are dimensionless, we can ask what energy scales Q. For medium energies, where amplitudes vary slowly with bombarding energy, we invoke analyticity to argue that it is $E_{\text{c.m.}}$ that sets the scale. Since for a typical medium energy experiment Q is of the order of 1−5 MeV and $E_{\text{c.m.}}$ of the order of 500−1000 MeV, we expect $P-A$ to be no more than 0.1% to 1%. This would seem to place $P-A$ outside the range of easily accessible measurement.[4] At somewhat lower energy (100−200) MeV) exchange forces can be important. These will lead to a smaller energy scale ($\sim E_{\text{Fermi}}$) and hence a larger bound for $P-A$.[5]

We first calculate $P-A$ for a particularly simple set of target spins $(0^+ \rightarrow 1^+)$. We expand the scattering amplitude in invariants and show how parity and time reversal restrict the amplitude. We calculate $P-A$ and show from the decomposition into invariants that it must vanish like Q. We do this example in detail, even though it involves only textbook applications of invariance principles, since our experience is that such arguments are only obvious after they have been given.[6] We abstract the essential features of the example to give a general proof that $|P-A| \sim Q$. We use analyticity and the slow variation of amplitudes with $E_{\text{c.m.}}$ to argue that at intermediate energy $|P-A| \lesssim Q/E_{\text{c.m.}}$.

Consider the inelastic scattering of a spin $\frac{1}{2}$ projectile, a proton say, from a 0^+ ground state, to an excited state of spin parity 1^+. We wish to compare the polarization P with the analyzing power A in this reaction. We begin by decomposing the scattering amplitude for the process in terms of invariants. Using rotation symmetry only, we see that there are two initial spin states and 2×3 final spin states, and we therefore expect a total of 12 independent terms, each characterized by a different scalar invariant. The operators at our disposal are the polarization vector of the final state $\vec{\epsilon}$, the three Pauli spin operators $\vec{\sigma}$ of the projectile, and three orthogonal spatial vectors $\vec{V}_i$. In the center-of-mass for scattering from incident relative momentum $\vec{p}$ to final relative momentum $\vec{p}\,'$, these are conveniently taken to be

$$\vec{k}=\vec{p}+\vec{p}\,'\equiv\vec{V}_3 ,$$

$$\vec{q}=\vec{p}-\vec{p}\,'-\frac{\vec{k}\cdot(\vec{p}-\vec{p}\,')\vec{k}}{k^2}\equiv V_1 ,$$

and

$$\vec{n}=\vec{k}\times\vec{q}\equiv\vec{V}_2 .$$

The numbering is taken to correspond to the usual choice of average direction along z and normal to the scattering plane along y. $\vec{q}$ is defined as the transverse momentum rather than as the momentum transfer, to ensure that $\vec{k}\cdot\vec{q}=0$. For elastic scattering $\vec{q}$ reduces to the momentum transfer since then $\vec{k}\cdot(\vec{p}-\vec{p}\,')=0$.

The scattering amplitude m must be linear in $\vec{\epsilon}$, since the target goes from 0^+ to 1^+, and the most general form for the amplitude consistent with rotational invariance linear in $\vec{\epsilon}$ and no more than linear in $\vec{\sigma}$ is then

$$m=\sum_i \vec{\epsilon}\cdot\vec{V}_i a_i+\sum_{ij}\vec{\epsilon}\cdot\vec{V}_i\vec{\sigma}\cdot\vec{V}_j a_{ij} , \quad (1)$$

where the a_i and a_{ij} are invariant functions of the energy and momentum transfer or scattering angle. For reasons that will become clear, we write $a_i(p^2,p'^2,\vec{p}\cdot\vec{p}\,')$, and similarly for a_{ij}. Since i,j run from 1 to 3, there are 12 terms in (1), as we expect. One might ask about the apparently missing term $\vec{\epsilon}\cdot\vec{\sigma}$, but it is a linear combination of the nine last terms and is therefore not independent.

We now impose parity and time reversal invariance on m. (We assume throughout that these are good symmetries.) Under parity $\vec{V}_1\to-\vec{V}_1$ and $\vec{V}_3\to-\vec{V}_3$, but $\vec{V}_2\to\vec{V}_2$, while $\vec{\sigma}\to\vec{\sigma}$ and $\vec{\epsilon}\to\vec{\epsilon}$. Furthermore, all the a_i and a_{ij} go into themselves. Thus requiring that m be invariant under parity removes a number of terms and we are left with

$$m=\vec{\epsilon}\cdot\vec{V}_2 a_2+\sum_i\vec{\epsilon}\cdot\vec{V}_i\vec{\sigma}\cdot\vec{V}_i a_{ii}$$
$$+\vec{\epsilon}\cdot\vec{V}_1\vec{\sigma}\cdot V_3 a_{13}+\vec{\epsilon}\cdot\vec{V}_3\vec{\sigma}\cdot\vec{V}_1 a_{31} . \quad (2)$$

Time reversal corresponds to $\vec{p}\to-\vec{p}\,'$ and $\vec{p}\,'\to-\vec{p}$. Under time reversal $\vec{V}_1\to\vec{V}_1$, $\vec{V}_3\to-\vec{V}_3$, and $\vec{V}_2\to-\vec{V}_2$, while $\vec{\sigma}\to-\vec{\sigma}$ and $\vec{\epsilon}\to-\vec{\epsilon}$. Because time reversal is $\vec{p}\rightleftarrows-\vec{p}\,'$, time reversal takes $a(p^2,p'^2,\vec{p}\cdot\vec{p}\,')$ into $a(p'^2,p^2,\vec{p}\cdot\vec{p}\,')$. Thus, the requirement that m be invariant and the transformation properties of $\vec{\sigma}$, $\vec{\epsilon}$, and $\vec{V}_i$ require that a_2 and a_{ii} be even under interchange of p^2 and p'^2, while a_{31} and a_{13} must be odd. We see immediately that for "elastic" scattering, i.e., $p^2=p'^2$, this requirement gives $a_{13}=a_{31}=0$. The time reversal oddness requirement for a_{13} and a_{31} can be realized by writing

$$a_{13}(p^2,p'^2,\vec{p}\cdot\vec{p}\,')=\frac{p^2-p'^2}{p^2+p'^2}$$
$$\times B_{13}(p^2,p'^2,\vec{p}\cdot\vec{p}\,') , \quad (3)$$

and similarly for a_{31}, where B is even under interchange of p^2 and p'^2, and hence need not vanish for $p'^2=p^2$. For the inelastic reaction we are considering $p^2=p'^2+2\mu Q$, where Q is the "Q" value or excitation energy and μ the reduced mass, and hence, the factor preceding B is of the order of $Q/E_{\text{c.m.}}$, the ratio of Q to the center-of-mass kinetic energy. For typical intermediate energy reactions this is a number of the order of $1-0.1\%$ and will make $P-A$ correspondingly small. The assumption that it is $p^2+p'^2$ that scales the difference in (3) or that $E_{\text{c.m.}}$ scales Q, or equivalently, the assumption that B is of the same order as the other amplitudes, is based on analyticity, the expansion of

$$a_{13}(p^2,p^2+2\mu Q,\vec{p}\cdot\vec{p})$$

in powers of Q, and the observation that the energy (not momentum transfer) variation of a typical intermediate energy amplitude is slow. At 500 MeV and above these assumptions are probably valid. At lower energy ($\sim 100-200$ MeV) exchange processes can make B proportional to $E_{\text{c.m.}}/E_{\text{Fermi}}$ and hence lead to a less restrictive bound on $P-A$.

We now calculate the polarization P and analyzing power A for the amplitude m of (2). We have

$$A=\frac{\text{tr}\,m\,\vec{\sigma}\cdot\vec{\Pi}\,m^+}{\text{tr}\,mm^+} \quad (4a)$$

and

$$P=\frac{\text{tr}\,m^+\vec{\sigma}\cdot\vec{\Pi}\,m}{\text{tr}\,mm^+} , \quad (4b)$$

where in A, $\vec{\Pi}$ is the polarization of the incident beam, while in P it is a unit vector in the direction of the polarization measurement. We see from (4) that if time reversal symmetry is valid, P is just the A of the inverse reaction. Since, for elastic scattering, the reaction is its own inverse, this gives $P=A$. For inelastic scattering we expect that only the amplitudes odd under time reversal will contribute to $P-A$. A straightforward calculation using (4) and (2) yields, in terms of the cross section σ,

$$P-A=\frac{4\vec{\Pi}\cdot\vec{V}_2(V_1{}^2\text{Im}a_{11}a_{13}^*+V_3{}^2\text{Im}a_{31}a_{33}^*)}{\sigma} , \quad (5)$$

as we expect $P-A$ is proportional to $\vec{\Pi}\cdot\vec{V}_2$ (only the component of $\vec{\Pi}$ normal to the scattering plane contributes) and is linear in a_{13} or a_{31}, the time reversal odd invariant amplitudes. These vanish as $Q\rightarrow 0$, and hence so does $P-A$. The limit $Q=0$ is called the adiabatic limit and some restrictions on $P-A$ in this limit have been discussed before.[7] It is the presence of the time-reversal odd amplitudes that will make $P-A$ of order or less than $Q/E_{\text{c.m.}}$ for intermediate energies in the absence of exchange. It is always possible that dynamics will make $P-A$ much smaller.

Consider the general problem of inelastic scattering by a spin $\frac{1}{2}$ projectile from an arbitrary spin initial state to an arbitrary spin final state. Assuming the validity of rotational symmetry and parity, we decompose the scattering amplitude m in terms of invariants I_j and the corresponding invariant amplitudes a_j

$$m=\sum_j I_j a_j(p^2,p'^2,\vec{p}\cdot\vec{p}\,') , \tag{6}$$

where we use the same kinematics as in Sec. II; $\vec{p}$ is the center-of-mass initial relative momentum and $\vec{p}\,'$ the final relative momentum. The I_j are scalar, even parity operators in the spin space of the projectile and target and are constructed from the projectile Pauli spin operator, the polarization operators of the initial and final state spins, and the three kinematic vectors $\vec{V}_i$ defined in Sec. II. In (6) the sum over j runs over the full set of linearly independent invariants required.

Under time reversal, the invariants I_j may be classified as either even $I_j^{(e)}$, or odd $I_j^{(o)}$. In order that m remain invariant under time reversal, the corresponding a_j's must be even; we call these E_j, or odd O_j. We can then write

$$m=\sum I_j^{(e)}E_j(p^2,p'^2,\vec{p}\cdot\vec{p}\,') + \sum I_j^{(o)}O_j(p^2,p'^2,\vec{p}\cdot\vec{p}\,') . \tag{7}$$

We now look at $P-A$ for the case in which the target spin is initially unpolarized and finally unobserved. On general invariance grounds, P and A must be proportional to $\vec{\Pi}\cdot\vec{V}_2$ times bilinear forms of the structure $a_k a_l^*$. $\vec{\Pi}$, the initial proton polarization in the A measurement or the direction of the final polarization in the P measurement, transforms like $\vec{\sigma}$ and is therefore odd under time reversal. $\vec{V}_2$, the normal to the scattering plane, is as well. Hence $\vec{\Pi}\cdot\vec{V}_2$ is even. Since $P-A$ is simply a measure of the difference in polarization (or equivalently analyzing power) between the reaction and its inverse, only terms in the bilinear $a_k a_l^*$ odd under $\vec{p}\rightleftarrows-\vec{p}\,'$ will contribute to $P-A$. These are terms of the form $E_k O_l^*$. (Note that terms bilinear in O are even.) Hence $P-A$ is linear in the O amplitudes. By definition these satisfy

$$O_k(p^2,p'^2,\vec{p}\cdot\vec{p}\,')=-O_k(p'^2,p^2,\vec{p}\cdot\vec{p}\,')$$

and therefore vanish like Q.

The spin of the target plays no essential role in this discussion except to provide the invariants I_j. However, the spin structure of the target must be rich enough to allow for the existence of time reversed odd invariants $I_j^{(o)}$. For example, in the inelastic scattering from a state of 0^+ to another of spin parity 0^+, $I_j^{(o)}$ vanishes by rotational invariance and parity alone, and $P-A$ must also vanish. However, so long as the target spin structure permits the construction of even parity, time reversal odd invariants $P-A$ can be nonzero, but linear in O_k and, therefore, will be proportional to Q. At intermediate energy we expect this to bound $P-A$ by $\sim Q/E_{\text{c.m.}}$.

In summary, the difference between the polarization P and analyzing power A in intermediate energy inelastic proton nucleus scattering is currently of interest. Using only invariance principles we show that $P-A\sim Q$, the excitation energy of the reaction. At intermediate energy ($\gtrsim 500$ MeV) we expect the slow variation of amplitudes with energy to bound $P-A$ by quantities of order $Q/E_{\text{c.m.}}$ and hence expect $P-A$ to be very small. At somewhat lower energy (100–200 MeV) exchange processes permit a larger $P-A$. Our analysis makes clear which amplitudes contribute to $P-A$, but is too general to shed any light on the question of whether there is any new dynamics to be learned from that difference.

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[1]Cf. R. D. Amado, J. McNeil, and D. Sparrow, Phys. Rev. C 23, 2114 (1981).
[2]G. Igo (private communication).
[3]We will assume parity and time reversal invariance are valid.
[4]Preliminary experimental evidence bears this out. G. Igo (private communication).
[5]As recently observed, J. Moss (private communication).
[6]Our methods follow closely the very clear exposition in J. R. Taylor, *Scattering Theory* (Wiley, New York, 1972).
[7]Cf. E. J. Squires, Nucl. Phys. 6, 504 (1958); G. R. Satchler, Phys. Lett. 19, 312 (1965).