PHYSICAL REVIEW C VOLUME 26, NUMBER 6 DECEMBER 1982

Possible existence of a $\pi^- nn$ bound state

Humberto Garcilazo*
Physics Department, Texas A & M University, College Station, Texas 77843
(Received 6 July 1982)

We have applied the relativistic Faddeev formalism to look for a bound state of a negative pion and two neutrons, using as input the pion-nucleon P_{33} channel and the nucleon-nucleon $^1S_0$ channel. We found that if the pion-nucleon interaction is sufficiently short ranged, the system can be bound. We represented the pion-nucleon interaction with two different models of rank-one separable potentials, while for the nucleon-nucleon interaction we used the Yamaguchi potential and three different models of rank-two separable potentials.

[NUCLEAR REACTIONS $\pi^- nn$ bound state; calculated, Fredholm determinant at $\sqrt{S}=2018$ MeV.]

The possibility that a $\pi^- nn$ bound state may be formed in the laboratory is suggested by the fact that the two neutrons alone are almost bound in the $^1S_0$ channel, while the $\pi^- n$ interaction is dominated by the P_{33} resonance, which is attractive, so that the combined effect of these two channels may produce enough attraction to bind the three-body system. This bound state, which would have total isospin $T=2$ and $T_Z=-2$, cannot decay by electromagnetic or strong interactions, so that it will have a lifetime similar to that of the charged pion.

Several attempts have been made in the past to search theoretically for this state, which have not been successful, although they were performed without a complete three-body theory. Thus Gale and Duck[1] solved the Faddeev equations without including the very important nucleon-nucleon interaction, while Kalbermann and Eisenberg,[2] as well as Ueda,[3] simply applied the Heitler-London-Pauli variational method in a nonrelativistic approach. In this paper we will present the results of the first calculation based in the relativistic Faddeev equations in which we have included both the pion-nucleon P_{33} channel and the nucleon-nucleon $^1S_0$ channel.

The most favorable state to form the bound state is that with total angular momentum and parity $J^P=1^-$ [we take parity as $P=(-)^{l+\lambda}$, where l is the orbital angular momentum of a pair and λ is the orbital angular momentum of the third particle with respect to the pair], since in a minimal coupling scheme the two neutrons which are in a state of $l=0$ and $j=0$ can couple to the pion in a state with $\lambda=1$ to give total angular momentum 1 and parity -1. Similarly, the pion and one of the neutrons which are in a state of $l=1$ and $j=\frac{3}{2}$ can couple to the spin of the other neutron in a state with $\lambda=0$ to give total angular momentum 1 and parity -1. Of course, we will not restrict ourselves to the minimal coupling, but include all possible couplings leading to $J^P=1^-$.

The relativistic Faddeev equations are based in the sum of all possible sets of ladder diagrams in which two particles interact while the third particle acts as spectator.[4–7] As one has conservation of total four momentum, these equations depend on eight continuous variables. However, by applying a Blankenbecler-Sugar reduction,[5] one can eliminate two of these variables so that after performing an angular momentum decomposition one is left with integral equations in two continuous variables. These equations can then be reduced to one-variable integral equations by making the isobar or separable approximation. The input of these equations are the two-body amplitudes that are obtained by solving the Blankenbecler-Sugar equation for the two-body subsystems. In the case of the nucleon-nucleon interaction, this amplitude can be related to the nonrelativistic Lippmann-Schwinger T matrix by using the prescription of minimal relativity.[8] Thus we will use for this subsystem the rank-one separable potential of Yamaguchi,[9,10] which is purely attractive, as well as the three models of rank-two separable potentials of Mongan,[11] which have both attraction and repulsion. For the case of the pion-nucleon interaction in the P_{33} channel, we will use the rank-one separable models that have been proposed recently by Garcilazo, Mathelitsch, and Verwest,[12] which fit the scattering volume and the phase shift from 0 to 350 MeV. These potentials are of the form

$$V(p,p')=pg(p)\gamma p'g(p') \ , \qquad (1)$$

where the strength γ is determined by requiring that the system have a resonance at an invariant mass of 1232.2 MeV, and the form factors for the two models are

$$g_{\rm I}(p) = A/(1+p^2/\alpha_1^2) + (1-A)p^2/(1+p^2/\alpha_2^2)^2 \ ;$$
$$A = 0.938\,15, \quad \alpha_1 = 2.141\ \text{fm}^{-1} \ , \quad \alpha_2 = 6.072\ \text{fm}^{-1} \ , \tag{2}$$

$$g_{\rm II}(p) = Ae^{-p^2/\alpha_1^2} + (1-A)/(1+p^2/\alpha_2^2) \ ;$$
$$A = 0.556\,06, \quad \alpha_1 = 1.238\ \text{fm}^{-1}, \quad \alpha_2 = 50.677\ \text{fm}^{-1} \ . \tag{3}$$

In order to perform the partial-wave decomposition of the equations, we have used the fully relativistic angular momentum formalism of the three-body helicity states.[13,14] Since we considered the pion-nucleon amplitude in the P_{33} channel and the nucleon-nucleon amplitude in the $^1S_0$ channel, the relativistic Faddeev equations for total angular momentum and parity $J^P = 1^-$ consist of three coupled integral equations for the configuration where the nucleon is the spectator, and two coupled integral equations for the configuration where the pion is the spectator. We used a 40-point Gauss mesh to represent each of these integrations, which gives a numerical accuracy of better than 0.1%.

In order to look for bound states, we have to calculate the Fredholm determinant of the system. If the two-body interactions are repulsive, then the Fredholm determinant is larger than one, while if they are attractive, it is less than one. If the Fredholm determinant passes through zero at an energy below the three-body threshold, then there is a bound state. Thus the simplest way to find out whether there is a bound state in the $\pi^- nn$ system is to see if the Fredholm determinant is negative at the three-body threshold $\sqrt{S} = 2018$ MeV.

We show in Table I the values of the Fredholm determinant at threshold for the various models of the pion-nucleon and nucleon-nucleon interactions. We see that if we neglect the nucleon-nucleon interaction or include it using the Yamaguchi potential which has only attraction, then the Fredholm determinant is negative so that the system is bound. However, when we include the Mongan potentials which have both attraction and repulsion, the system becomes unbound for the model I of the pion-nucleon interaction, although it is still bound for the model II of the pion-nucleon interaction. The actual binding energies for the various models of Table I go from a few MeV up to more than 1 GeV, which shows that probably some of the models have already too much attraction. It also indicates the large sensitivity of the binding energy to the form of the two-body interactions.

The large effect that the nucleon-nucleon short-range repulsion has on the Fredholm determinant can be understood if we look at the pion-nucleon potentials I and II in coordinates space, where we see that they are strongly attractive at short distances. Thus, when we switch on the nucleon-nucleon short-range repulsion, it becomes harder for the pion-nucleon attraction to act, since this requires that the pion be close to the two neutrons at the same time, but the nucleon-nucleon repulsion tries to keep them apart. It is interesting, however, that for the model II of the pion-nucleon interaction the pion-nucleon attraction is stronger than the nucleon-nucleon repulsion, so that the system can still be bound. Since the main mechanism that binds the system is the short-range pion-nucleon attraction, it is clear that if the nucleon-nucleon interaction has a repulsive hard core, this mechanism will not be allowed to act and, consequently, we probably will not have a bound state in that case.

The calculations that we have described were performed for a system with isospin 2. Similar calculations for the case of isospin 0 or 1 in the continuum region have been performed in the past. Thus extensive work has been done for the pion-deuteron system[15–22] which has isospin 1. Similarly, the relativistic Faddeev method has been applied by Kloet and Silbar to describe nucleon-nucleon scattering, taking into account the inelastic pion-production channel.[23–26] More recent calculations have included all the various couplings between the NN, πNN, and πd channels, in a unified description that preserves two- and three-body unitarity.[27–30]

Recently, solutions of nucleon-nucleon phase-shift analyses above pion-production threshold[31–34] have revealed what appears to be dibaryon resonances in

TABLE I. Values of the Fredholm determinant at threshold for several models of the pion-nucleon and nucleon-nucleon interactions.

	Without *NN* force	Yamaguchi	Mongan I	Mongan II	Mongan III
P_{33} model I	−0.013	−0.069	1.215	1.478	0.442
P_{33} Model II	−0.559	−0.524	−0.117	−0.566	−0.343

several isospin 1 nucleon-nucleon channels. These states are similar to our $\pi^- nn$ bound state, except that in this case the Fredholm determinant is zero not below threshold, but at some energy above threshold on the unphysical sheet not far from the real axis.

To conclude, we have shown that it is possible to have a $\pi^- nn$ bound state with some of the existing models of the pion-nucleon and nucleon-nucleon interaction which reproduce well the two-body data. However, given the uncertainty in our knowledge of the short-range part of the pion-nucleon and nucleon-nucleon interactions, we cannot really establish whether the bound state exists or not.

This work was supported in part by the United States Department of Energy under Contract No. DE-AS05-76ER05223.

---

[*]On leave from Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, México 14 D.F., México.

[1]W. A. Gale and I. M. Duck, Nucl. Phys. B8, 109 (1968).

[2]G. Kalbermann and J. M. Eisenberg, J. Phys. G 5, 35 (1977).

[3]T. Ueda, Phys. Lett. 74B, 123 (1978).

[4]V. A. Alessandrini and R. L. Omnes, Phys. Rev. 139, B167 (1965).

[5]R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).

[6]D. Z. Freedman, C. Lovelace, and J. M. Namyslowski, Nuovo Cimento 43A, 258 (1966).

[7]R. Aaron, R. D. Amado, and J. E. Young, Phys. Rev. 174, 2022 (1968).

[8]G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976).

[9]Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).

[10]J. M. Rivera and H. Garcilazo, Nucl. Phys. A285, 505 (1977).

[11]T. R. Mongan, Phys. Rev. 178, 1597 (1969).

[12]H. Garcilazo, L. Mathelitsch, and B. J. Verwest, Phys. Rev. C (to be published).

[13]G. C. Wick, Ann. Phys. (N.Y.) 18, 65 (1962).

[14]H. Garcilazo, Nucl. Phys. A360, 411 (1981).

[15]A. S. Rinat and A. W. Thomas, Nucl. Phys. A282, 365 (1977).

[16]A. S. Rinat, E. Hammel, Y. Starkand, and A. W. Thomas, Nucl. Phys. A329, 285 (1979).

[17]N. Giraud, G. H. Lamot, and C. Fayard, Phys. Rev. Lett. 40, 438 (1978).

[18]C. Fayard, G. H. Lamot, and T. Mizutani, Phys. Rev. Lett. 45, 524 (1980).

[19]H. Garcilazo, Phys. Rev. Lett. 45, 780 (1980).

[20]H. Garcilazo, Phys. Rev. Lett. 48, 577 (1982).

[21]A. Matsuyama and K. Yazaki, Nucl. Phys. A364, 477 (1981).

[22]A. Matsuyama, Nucl. Phys. A379, 415 (1982).

[23]W. M. Kloet, R. R. Silbar, R. Aaron, and R. D. Amado, Phys. Rev. Lett. 39, 1643 (1977).

[24]W. M. Kloet and R. R. Silbar, Nucl. Phys. A338, 281(1980).

[25]R. R. Silbar and W. M. Kloet, Nucl. Phys. A338, 317 (1980).

[26]W. M. Kloet and R. R. Silbar, Phys. Rev. Lett. 45, 970 (1980).

[27]M. Araki, Y. Koike, and T. Ueda, Prog. Theor. Phys. 63, 335 (1980).

[28]M. Araki, Y. Koike, and T. Ueda, Prog. Theor. Phys. 63, 2133 (1980).

[29]B. Blankleider and I. R. Afnan, Phys. Rev. C 24, 1572 (1981).

[30]A. S. Rinat, Y. Starkand, and E. Hammel, Nucl. Phys. A364, 486 (1981).

[31]N. Hoshizaki, Prog. Theor. Phys. 60, 1796 (1978).

[32]R. Bhandari, R. A. Arndt, L. D. Roper, and B. J. Verwest, Phys. Rev. Lett. 46, 1111 (1981).

[33]R. A. Arndt, L. D. Roper, B. J. Verwest, R. B. Clark, R. A. Bryan, and P. Signell (unpublished).

[34]J. Bystricky, J. Deregel, F. Lehar, C. R. Newsom, J. Yonnet, J. M. Fontaine, J. Gosset, F. Perrot, C. Lechanoine-Leluc, W. R. Leo, Y. Onel, P. Winternitz, S. Colautti-Dalla Torre, A. Penzo, and H. Azaiez, Saclay Report No. DPhPE 82-09, 1982 (unpublished).