Isosinglet K^+N S-wave scattering length

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Faddeev equations are used to analyze the available data of low-energy elastic K^+d scattering in order to determine the isosinglet K^+N S-wave scattering length. For 107 MeV kaon lab kinetic energy the extracted $I=0$ S-wave scattering length is $a_0=-0.015$ fm. The need of more precise experiments at lower energies is made clear.

[NUCLEAR REACTIONS Faddeev equations; low-energy elastic K^+d scattering; calculated a_0.]

Considerable interest on low- and intermediate-energy kaon-nucleus scattering has arisen in recent years.[1,2] K^- beams are used in the formation of hypernuclei, whereas K^+ experiments offer the possibility of testing both reaction dynamics and nuclear structure. The construction of kaon factories in the near future[3,4] will undoubtedly increase that interest.

The reliability of the information to be drawn from the second type of experiments is conditioned, as in every many-body process, to a good knowledge of the two-body system.[4] It seems, therefore, worthwhile to try to reduce the present uncertainties in the K^+N interaction. The scattering length and effective range in S wave for the isospin $I=1$ channel are reasonably well established through K^+p scattering experiments.[5,6] For the $I=0$ channel, instead, the situation is much more unsatisfactory. Since the information on this channel must be drawn mainly from K^+d data, the obtained $I=0$ parameters can be affected by important errors due to the approximation made in handling the three-body problem. In the analysis of the K^+d data, use is made, generally, of the impulse approximation. Multiple scattering terms, however, may be very important at low energies[7]; neglecting them can introduce appreciable errors in the parameters fitting the experimental results.

Faddeev equations provide an (in principle) exact method of relating three-body data with two-body parameters. The only limitation in their applicability is the use (for the sake of computational simplicity and feasibility) of separable potentials to represent the two-body interactions. This is not a severe limitation, as such potentials can reproduce, reasonably well, the existing two-body data.

We have used the Faddeev formalism to obtain the S-wave scattering length for the $I=0$ channel of the K^+N system from low-energy K^+d scattering data. We have analyzed the results of elastic K^+d scattering at 342 MeV/c K^+ incident laboratory momentum reported by Glasser.[8] These are, to our knowledge, the most recent low-energy K^+d experiments.

Rank-one separable potentials of the form (units $\hbar=c=1$ are used)

$$V(p,p';E)=g(p)[\lambda(E)/2\mu]g(p') \ , \tag{1}$$

where μ denotes the reduced mass, have been used to describe all the two-body interactions (only S-wave has been considered). Form factors $g(p)$ of the Yamaguchi type,

$$g(p)=(p^2+\beta^2)^{-1} \ , \tag{2}$$

have been taken and a dependence on the energy for the intensity parameter $\lambda(E)$ is allowed. For the NN system, this dependence has been chosen of the type recently proposed by Garcilazo,[9]

$$\lambda(E)=\lambda_{NN}\tanh(1-E/E_c) \ , \tag{3}$$

with $\lambda_{NN}=-9.411\,127\,3$ fm^{-3} and $E_c=0.816$ fm^{-1}. This potential, with range parameter $\beta_{NN}=1.632\,384$ fm^{-1}, fits the triplet scattering length ($a_t=5.39$ fm) and the deuteron binding energy ($E_B=-2.225$ MeV). The K^+N interaction in the isospin $I=1$ channel can be fairly well represented at low energies by the potential suggested in a recent paper.[10] Its energy dependence has been taken of the form

$$\lambda(E)=\lambda_1\exp(-2\mu C_1E) \ , \tag{4}$$

with $\lambda_1=371.055\,33$ fm^{-3} and $C_1=-0.151\,953$ fm^2. For the range, the value $\beta_1=4.740\,140$ fm^{-1} has been selected. With this rank-one separable potential, both the scattering length ($a_1=-0.309$ fm) and the effective range ($r_0=0.32$ fm) in S wave can be fitted.

Previously analyses of the $I=0$ channel K^+N interaction indicate that it is considerably weaker than the $I=1$ one. The second term in the effective range expansion can be neglected at the energy under consideration.[11] So, a dependence on the energy in the intensity parameter is not needed and we have taken $\lambda(E)=\lambda_0$. The two parameters, λ_0 and β_0,

entering in the potential have not been considered to be independent; they have been so related as to produce a vanishing effective range. In this manner, there remains only one adjustable parameter, which we have taken to be the S-wave scattering length a_0.

In the solution of the Faddeev equations, the contour rotation method[12] has been used. At the energy under consideration, a mesh of 48 points in every two-body channel has been needed to obtain an error of less than 1%. As it can be seen in Fig. 1, it becomes immediately apparent from our Faddeev analysis that the theoretical elastic K^+d differential cross section depends linearly on a_0 for a range of values of a_0 containing previous determinations of this parameter.[13–18] This fact, due to the above mentioned relation between the intensities of the $I=0$ and 1 interaction, obviously indicates that it is enough to retain only the first two terms of a Taylor expansion of the cross section around the point $a_0=0$ (no interaction). In this situation the minimization procedure to determine a_0 from the experimental data becomes extremely simple. The best fit is obtained for

$$a_0=-0.015\pm0.023 \text{ fm} , \tag{5}$$

with a χ^2 per degree of freedom of 0.96.

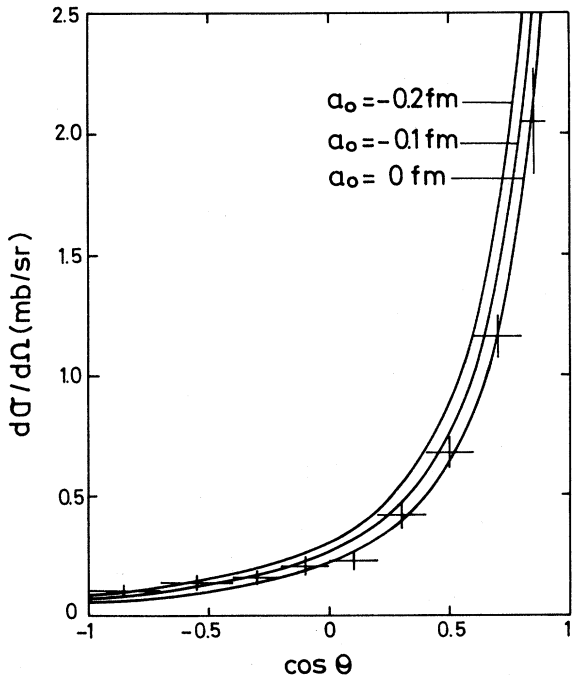

FIG. 1. Elastic K^+d differential cross section in the laboratory frame at 342 MeV/c K^+ incident momentum. The experimental data are from Glasser (Ref. 6). The curves represent theoretical predictions obtained in the Faddeev formalism with the two-body potentials quoted in the text and for three different values of the $I=0$ K^+N S-wave scattering length a_0.

In order to make evident the importance of multiple scattering terms at low energies, we have represented in Fig. 2 the theoretical differential cross sections obtained, respectively, in a Faddeev treatment and in the single-scattering impulse approximation,[8,13] with the value of a_0 given in Eq. (5). The discrepancy observed amounts to a 15% at low angles, reaching a 25% for laboratory angles above 70°. It seems, therefore, absolutely necessary to take into account multiple scattering terms. Besides Faddeev equations, other procedures of incorporating rescattering effects exist. Recently, Landau and coworkers[2,19] have developed, for the analysis of scattering of pions, nucleons and kaons from nuclei, a method that consists in solving the Lippman-Schwinger equation with an optical potential constructed from the elementary two-body amplitudes. To our knowledge, it has not been applied to the K^+d system and a contrast of such a method with the Faddeev one, though desirable, is not possible at present.

For comparison, we list in Table I the values of a_0 obtained previously by other authors with different methods. The large errors, induced by the experimental uncertainties, in the various determinations of

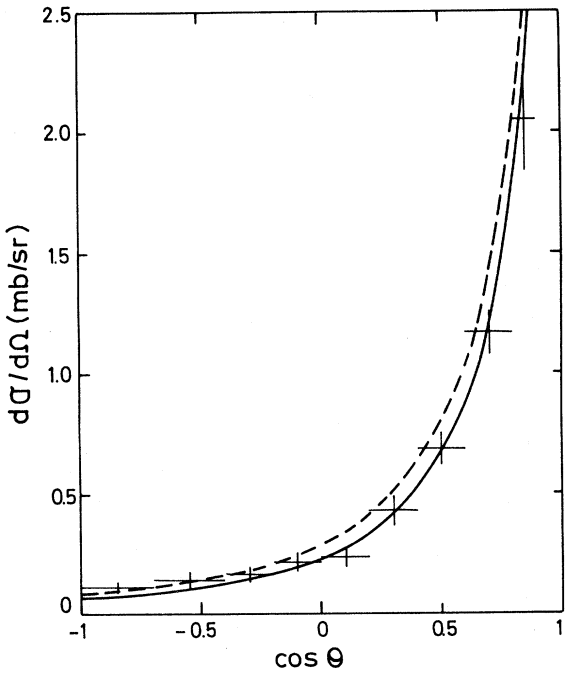

FIG. 2. Comparison of the elastic K^+d differential cross sections in the laboratory frame evaluated by means of the Faddeev equations (continuous curve) and in the single-scattering impulse-approximation (dashed curve) at 342 MeV/c K^+ incident momentum. The two-body potentials used in both calculations are those mentioned in the text, with a $I=0$ K^+N S-wave scattering length $a_0=-0.015$ fm. The experimental points are the same as in Fig. 1.

TABLE I. Previous determinations of isosinglet S-wave scattering lengths.

a_0 (fm)	Reference	Method
0.04 ± 0.04	13	Impulse-approximation analysis of K^+d data
$-0.11^{+0.06}_{-0.04}$	14	K^+d multiple scattering with zero-range boundary-condition formalism
-0.005	15	KN phase shift analysis with dispersion relation constraints
-0.23 ± 0.18	16	KN forward dispersion relation
0.02	17	$\bar{K}N$ multichannel analysis with dispersion relation constraints
-0.17	18	KN effective Lagrangian

a_0 make all of them compatible. However, in view of the importance of multiple scattering terms, we find our determination more reliable than that of Ref. 13, where such terms have been ignored, or that of Ref. 14, where they have been evaluated approximately by assuming a zero-range KN interaction. The methods used in Refs. 15–18 correspond to a theoretical approach quite different from ours and, therefore, it is difficult to draw any conclusion from a comparison of the corresponding values of a_0.

There are two main simplifications in our treatment of the K^+d system. First, we have ignored Coulomb interaction and, second, our two-body potentials are effective only in S wave. The inclusion of Coulomb interaction in the Faddeev equations would modify the theoretical cross section only in the region of low angles, where the experimental errors are larger. The effect on the fitted scattering length would then be unimportant. As far as we are considering a low-energy process, the inclusion of D-wave interaction in the NN system and P wave in K^+N interaction would be enough to avoid our second simplification. Having at our disposal also the P-wave low-energy parameters of the $I=0$ K^+N channel, a better fit of the experimental data would be obtained. However, the size of the matrices (144 rows of complex elements, in the case of purely S wave) to be inverted in the obtention of the Faddeev amplitudes increases considerably. In view of the inaccuracy of the experimental data, we have considered it unnecessary to introduce such complications which would result in small corrections to our determination of a_0. Obviously, more precise experiments at lower energies are needed to obtain a satisfactory value of the $I=0$ K^+N S-wave scattering length.

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