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Nuclear parameters for $\pi^{\pm}$-^{12}C scattering from Coulomb-nuclear interference

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Forward nuclear amplitude for scattering of charged pions from ^{12}C nuclei has been calculated by extrapolation of the available data to the forward angle in the $(\frac{3}{2},\frac{3}{2})$ resonance region. The analysis is independent of any nuclear model. The values of σ_{el}, σ_{inel}, and $[(d\sigma_N/d\Omega)(0^\circ)]_{el}$ are reported. Substantial difference, indicative of Coulomb distortion, is found for oppositely charged pions.

[NUCLEAR REACTIONS ^{12}C$(\pi^{\pm},\pi^{\pm})$, $T^{lab}=115$–280 MeV; calculated forward nuclear amplitude, σ_{el}, σ_{inel}, $[(d\sigma_N/d\Omega)(0^\circ)]_{el}$, pole subtraction method.]

In a recent paper,[1] the differential cross section data for charged pion scattering from ^{4}He nuclei have been analyzed by a pole subtraction method[2] and nuclear parameters in the forward direction have been obtained independent of any specific nuclear model. The real parts of the residual nuclear amplitude $f_N(0)$ at different energies have been obtained both in sign and magnitude quite accurately by extrapolation to the forward electromagnetic pole where actual experiments cannot be precisely performed. The imaginary parts of $f_N(0)$ have also been obtained by a converging method of successive iteration. With the extensive data coming out of CERN and other laboratories, it has become incumbent to extend the analysis to still heavier target nuclei like ^{12}C. Data for small angle scattering of both π^+ and π^- in the $(\frac{3}{2},\frac{3}{2})$ resonance region are available.[3–5] The methods of analysis adopted earlier[3,5–9] are more or less nuclear model dependent and also depend on the available total cross section data. Under such circumstances, pole subtraction method plays a good role in obtaining the residual nuclear amplitudes in the forward direction from Coulomb-nuclear interference experiments in a model independent manner.

Elliptic mapping[10] is used to achieve maximum convergence of the polynomial expansion for the nuclear part. For scattering of $\pi^{\pm}$ from ^{12}C nuclei, the analytic structure of the strong interaction amplitude, which is closely associated with the rate of convergence of the series, is shown in Fig. 1(a) along with the Lehmann ellipse.[11] There is a branch cut (x_+,∞)

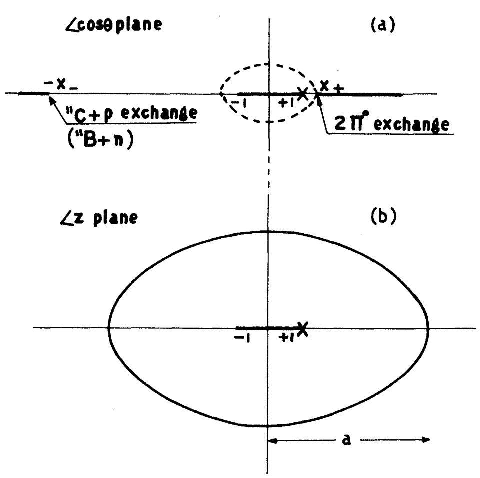

FIG. 1. (a) Analytic structure of $\pi^{\pm}$-^{12}C scattering amplitude in $\cos\theta$ plane; X is the Coulomb pole at $\cos\theta=1$; dashed ellipse is the Lehmann ellipse with semimajor axis x_+. (b) Ellipse of convergence in the mapped ($z-$) plane with semimajor axis a; X is the Coulomb pole at $x=1$.

TABLE I. T^{lab} = pion kinetic energy in the laboratory; x_+ = semimajor axis of the Lehmann ellipse; a = semimajor axis of the mapped ellipse; L = order of the polynomial where the series is truncated and NDF = number of degrees of freedom.

T^{lab} (MeV)	x_+	a	$\cos\theta$ plane L	$\cos\theta$ plane χ^2/NDF	z plane L	z plane χ^2/NDF
120	1.796	6.310	9	0.96	7	0.88
150	1.595	5.409	10	1.43	7	1.01
180	1.466	4.804	10	1.21	7	1.39
200	1.403	4.495	11	2.33	8	2.36
230	1.331	4.138	8	2.74	7	2.63
260	1.278	3.856	9	1.11	7	1.18
280	1.250	3.700	8	0.45	7	0.41
115	1.840	6.506	2	1.00	2	1.00
167	1.516	5.038	1	0.92	1	0.92
242	1.308	4.015	2	1.00	2	1.05

TABLE II. Results of pole extrapolation in z plane for π^--^{12}C scattering.

T^{lab} (MeV)	$\text{Re}f_N^-(0)$ (fm)	$\left[\frac{d\sigma_N^-}{d\Omega}(0°)\right]_{\text{el}}$ (mb/sr)	$\text{Im}f_N^-(0)$ (fm)	σ_{tot}^- (mb)	σ_{el} (mb)	σ_{inel}^- (mb)
120	1.94 ± 0.09	342.3 ± 5.5	5.52 ± 0.02	640.05 ± 2.32	219.4 ± 2.4	420.65 ± 3.34
150	1.10 ± 0.52	520.5 ± 10.0	7.13 ± 0.01	715.01 ± 1.01	272.0 ± 6.0	443.01 ± 6.08
180	0.50 ± 0.13	607.0 ± 6.5	7.77 ± 0.03	689.86 ± 2.93	258.6 ± 2.4	431.26 ± 3.79
200	-1.80 ± 0.75	713.0 ± 16.0	8.25 ± 0.26	680.94 ± 21.51	264.0 ± 6.5	416.94 ± 22.47
230	-2.75 ± 1.40	744.0 ± 48.0	8.18 ± 0.76	611.97 ± 57.23	...	...
260	-1.71 ± 0.16	764.5 ± 9.5	8.57 ± 0.09	588.14 ± 5.99	...	...
280	-2.38 ± 1.65	686.0 ± 40.0	7.93 ± 0.75	515.84 ± 48.52	...	...

for the $2\pi^0$ exchange process in t channel, so that

$$x_+ = 1 + 2m_{\pi^0}^2/k^2 \ .$$

Exchange of ^{11}C (^{11}B) $+p(n)$ in the u channel corresponds to the branch cut $(-\infty, -x_-)$, when

$$x_- = 1 + \frac{[m_{^{11}\text{C}(^{11}\text{B})} + m_{p(n)}]^2}{2k^2} - \frac{[m_{\pi^\pm}{}^2 - m_{^{12}\text{C}}{}^2]^2}{2k^2 s} \ .$$

The mapping increases the size of the ellipse of convergence as shown in Fig. 1(b). This reduces the order of the polynomial expansion by two or three and hence provides a better fit as is indicated in Table I.

The results of this analysis are presented in Tables II and III. The errors correspond to a variation of minimum χ^2 value by ± 1. The real parts of $f_N^\pm(0)$ versus pion kinetic energy in the laboratory are plotted in Fig. 2. Since we allow for Coulomb distortion, our results are sometimes significantly different from those of Binon *et al.* and Mutchler *et al.* or those obtained from forward dispersion calculations. These values are also shown in Fig. 2. The quantity $\frac{1}{2}[\text{Re}f_N^+(0) + \text{Re}f_N^-(0)] =$ (real part of the pure nuclear amplitude in the forward direction) $+ O(\alpha^2)$ agree with the results obtained for pure strong interaction by the previous authors. The large errors at 200, 230, and 280 MeV are due to the lack of data sufficiently close to the forward direction.

The energy variation of the total cross section results of this analysis is shown in Fig. 3. σ_{tot}^- are close to the total cross section measurements of Binon *et al.* and Wilkin *et al.*[12] The agreement for σ_{tot}^+ is not so good, specifically at 167 MeV. This is be-

TABLE III. Results of pole extrapolation in z plane for π^+-^{12}C scattering.

T^{lab} (MeV)	$\text{Re}f_N^+(0)$ (fm)	$\left[\frac{d\sigma_N^+}{d\Omega}(0°)\right]_{\text{el}}$ (mb/sr)	$\text{Im}f_N^+(0)$ (fm)	σ_{tot}^+ (mb)
115	-0.01 ± 0.27	231.0 ± 31.0	4.81 ± 0.32	572.64 ± 38.48
167	-0.94 ± 0.46	418.0 ± 46.0	6.40 ± 0.43	597.05 ± 39.86
242	-2.82 ± 0.58	815.0 ± 120.0	8.58 ± 0.89	619.14 ± 64.26

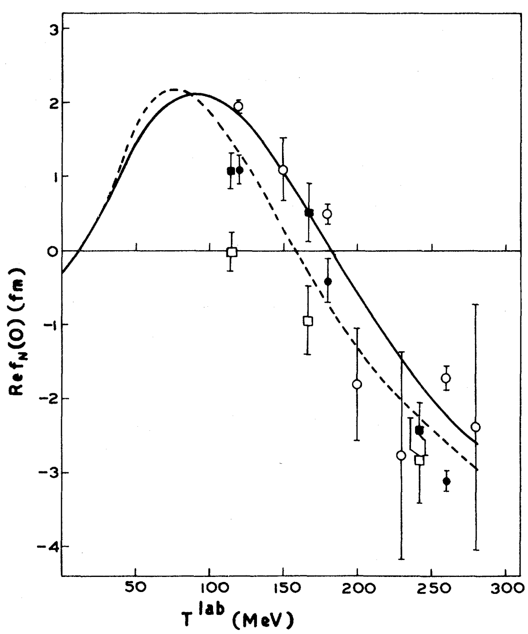

FIG. 2. Real parts of the residual nuclear amplitude in the forward direction from this analysis for π^-(○) and π^+(□); the curves are calculated using forward dispersion relation by Ericson and Locher (solid curve) and by Landau and Locher (dashed curve); results obtained by Binon *et al.* (●) and Mutchler *et al.* (■) for π^- and π^+, respectively.

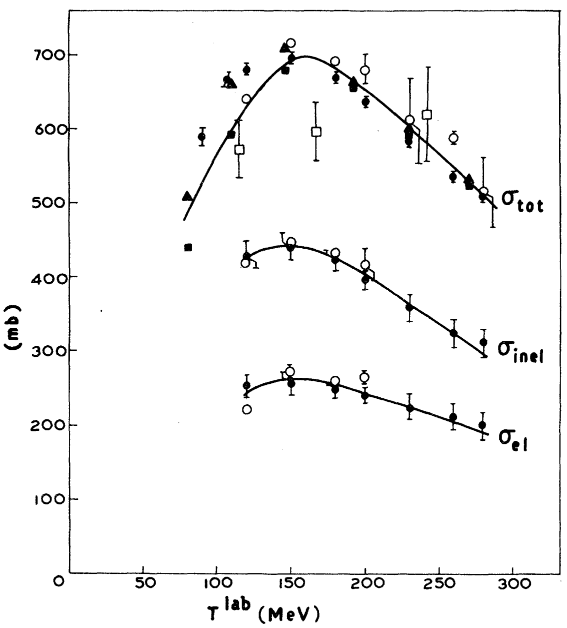

FIG. 3. Results of this analysis for π^-(○) and π^+(□); experimental results of Binon *et al.* for π^-(●) and of Wilkin *et al.* for π^-(▲) and π^+(■). The curves are to guide the eye only.

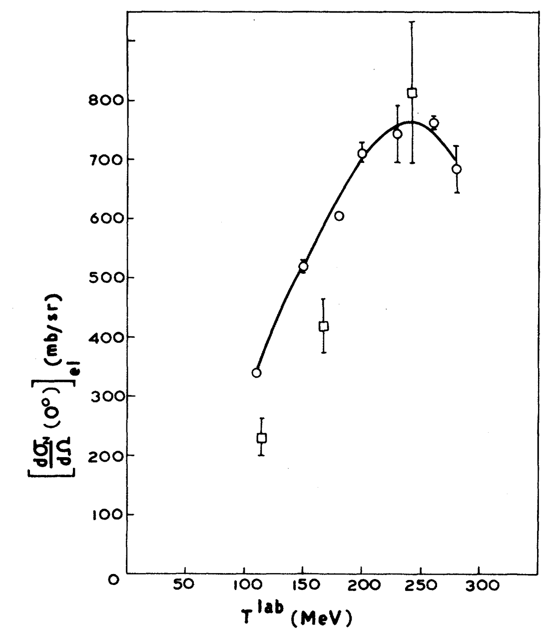

FIG. 4. Elastic scattering cross section for strong interaction at 0° for π^-(○) and π^+(□). The curve is to guide the eye.

cause the data are confined to a limited physical region and appear to vary erratically with large errors. A good fit is obtained with only one term, and cannot be reliable.

Predictably, the values of the total elastic cross sections σ_{el}, which are independent of the charge of the pion, agree well with the results of Binon *et al.* in the energy range 120–200 MeV. This is shown in Fig. 3. At higher energies, the extrapolation results for σ_{el} should not be that reliable since data at backward angles are not there for these energies. In Fig. 3, we have also plotted the total inelastic cross sections which are again close to the values obtained experimentally. In Fig. 4 the values of $[(d\sigma_N/d\Omega) \times (0°)]_{el}$ are plotted versus the pion kinetic energy. There seems to be a large difference between these values for scattering of oppositely charged pions.

An accurate estimate of the Coulomb distortion could have been made if the experimental data were available at the same energy for both types of pions. The average of the amplitudes $f_N^+(0)$ and $f_N^-(0)$ would equal to the pure nuclear amplitude and the difference would be a convenient measure of the Coulomb distortion at that energy.

To conclude, we mention that the pole subtraction method is an efficient way of extracting both the real and imaginary parts of the forward nuclear amplitude without going into the details of a specified nuclear model. The only input are a good set of differential scattering cross section data fairly close to the forward region. Given a set of data spreading over the backward region, the integrated elastic and inelastic cross sections can also be obtained quite precisely by this method.

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