# Analyzing powers in the $^{58}$Ni($\vec{p}$,$p'$) reaction calculated with the statistical multistep direct emission theory

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The fully quantum mechanical statistical multistep direct theory of Feshbach, Kerman, and Koonin is used to calculate the analyzing powers of the $^{58}$Ni($\vec{p}$,$p'$) reaction at 65 MeV to the continuum. The overall behavior of the analyzing powers and differential cross sections as a function of energy and angle is reasonably described by the theory and the relative contributions of the single and multistep processes are evaluated.

NUCLEAR REACTIONS $^{58}$Ni($\vec{p}$,$p'$), $E_p$=65 MeV. Statistical multistep direct emission. Calculated differential cross sections and analyzing powers.

## I. INTRODUCTION

At the present time there is increasing interest in nuclear reactions to the continuum and many attempts have been made to understand the measured cross sections.[1,2] The energy spectra of the emitted particles and also the excitation functions are often reproduced quite well by the exciton model,[3] and more recently, considerable progress has been made toward understanding the differential cross sections.[4–8] In particular, we have used the formalism of Feshbach, Kerman, and Koonin[9] to make a fully quantum-mechanical analysis of the differential cross sections of $(p,n)$ reactions on $^{48}$Ca, $^{90}$Zr, $^{120}$Sn, and $^{208}$Pb from 25 to 45 MeV.[10] This analysis accounted very well for the variations of the cross sections with energy and with angle, and showed the magnitudes of the contributions of the single step and multistep processes.

The next stage in this work is to apply the same description to the analyzing powers which are now being measured with increasing precision. Data depending on polarization are always more sensitive than the differential cross section to the details of the interaction mechanism and therefore provide a severe test of any theory of the reaction. The analyzing power is very likely to depend on the number of steps in the reaction; as the number of steps increases, the magnitude of the analyzing power decreases as the memory of the initial polarization is gradually lost, until finally it becomes zero for the evaporated particles. The ($\vec{p}$,$p'$) measurements of Sakai *et al.*[11] show, however, that the analyzing power in the continuum is large and positive at backward angles. This result is unexpected on the basis of the conventional reaction theories. As pointed out by Sakai *et al.*,[11] the slope of the energy spectra at backward angles resembles that of an evaporation spectrum. It would be, however, very difficult to attribute the reaction in this angular range principally to a compound nucleus evaporation process, because of the large values of the analyzing power. On the other hand, a simple one-step direct process would hardly predict the steep energy dependence of the emitted particle spectra measured at backward angles.

It is therefore natural to think of a multistep direct process to be particularly important here. Following these lines of reasoning, Tamura and Udagawa[12] have applied their multistep direct reaction theory[6] to calculate some of these analyzing powers. The DWBA calculations were made with a collective form factor having an $l$-independent radial shape. They found that the calculations overestimate the experimental analyzing power by about a factor of 3, and suggested that this discrepancy would be eliminated by properly taking into account the $l$ dependence of the form factors, as is naturally

 

done in the framework of the microscopic model of the inelastic scattering.

In this paper we extend our previous work[10] to calculate the cross sections and the analyzing powers of the reaction $^{58}$Ni($\vec{p},p'$) at 65 MeV.[11] The theoretical formulation is described in Sec. II, the choice of parameters is shown in Sec. III, and the results of the calculations are presented and discussions are given in Sec. IV.

## II. THEORY AND APPLICATION

The quantum mechanical theory of the statistical multistep direct emission (SMDE) and its application to calculate the continuum cross sections for nucleon-induced reactions have already been described.[10] In what follows, we shall concentrate on the calculation with this theory of the analyzing power.

The analyzing power $A_y$ is commonly defined as:

$$A_y=\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R}\ , \tag{1}$$

$\sigma_L$ and $\sigma_R$ being the left and right cross sections, respectively. As an immediate generalization of the SMDE theory, each left and right cross section can be written as

$$\sigma_{L,R}=\sigma_{L,R}^{\text{single step}}+\sigma_{L,R}^{\text{multistep}}\ . \tag{2}$$

The multistep contribution has the same formal expression as that given in Ref. 10 [Eq. (2.2)]. The appropriate left and right transition probabilities are used in constructing that expression, limiting the integration over the intermediate angles to the $\phi$ values for which each left and right cross section is defined. The major problem of the above calculation is therefore to obtain the theoretical left and right individual DWBA cross sections for a particular value of the angular momentum transfer and of the energy loss. These cross sections are used for evaluating, by means of the averaging procedure described in Ref. 10, both the single-step left and single-step right cross sections [Eq. (2.6) of Ref. 10], and the left and right transition probabilities from any $(n-1)$th stage to the next $n$th stage [Eq. (2.3) of Ref. 10]. This finally makes it possible to calculate, following Eqs. (1) and (2), the contribution to the analyzing power of the single and multistep processes. The theoretical left and right cross sections can be easily extracted by any DWBA computer program giving the analyzing power as well as the unpolarized cross section, within a certain model of the inelastic scattering. For our calculations, following the philosophy underlying the SMDE theory, we use a microscopic model of inelastic scattering. The excitation of the target nucleus is described in terms of the number of excitons and the level density of any particle-hole state at any step of the multistep direct process is given by the Ericson formula,[13] as described in a previous paper.[10] The important innovation in the calculations presented here is the use of a complete microscopic model which includes the noncentral components of the two-body residual interaction, and particularly the $\vec{l}\cdot\vec{s}$ term. This interaction was indeed found essential in the analyzing power calculations in previous work on reactions to discrete states[14–19] and is also important in reproducing the cross sections of inelastic scattering reactions, particularly in the energy range we are considering here.[20,21] We have therefore used the code DWBA76 (Ref. 22) which can include the effects of central, tensor, and spin-orbit interactions and the knockon exchange amplitude.[23] The effective two-body interaction used in this code has the form:

$$\begin{aligned}V(r)=&V_c(r)+V_0(r)+V_\sigma(r)\vec{\sigma}_1\cdot\vec{\sigma}_2\\&+V_\tau(r)(\vec{\tau}_1\cdot\vec{\tau}_2)\\&+V_{\sigma\tau}(r)(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)\\&+[V_{LS}(r)+V_{LS\tau}(r)(\vec{\tau}_1\cdot\vec{\tau}_2)]\vec{L}\cdot\vec{S}\\&+[V_T(r)+V_{T\tau}(\vec{\tau}_1\cdot\vec{\tau}_2)]S_{12}\ ,\end{aligned} \tag{3}$$

where the suffixes 1 and 2 refer to the two interacting nucleons. The radial dependence has the Yukawa form:

$$V(r)=Ve^{-r/\mu}/(r/\mu)\ . \tag{4}$$

## III. CHOICE OF PARAMETERS AND CALCULATION

Such a complete form of the interaction increases the number of parameters in the DWBA calculations, and it is highly desirable to fix all of them *a priori*. This can indeed be done for most of them, but not for all. The optical model parameters used in the calculation of the incoming and outgoing distorted waves can all be taken from independent work on elastic scattering. We have chosen to use the best-fit parameters of Sakaguchi[24] for polarized protons on $^{58}$Ni at 65 MeV to describe the initial interaction needed for the first step of the SMDE chain. All the other distorted waves with kinetic energies corresponding to the different excitation

TABLE I. Optical model parameters used in the DWBA calculations at different energies. Depths are in MeV, radii and diffusenesses are in Fermis.

| $E_p$ (MeV) | $V_R$ | $r_R$ | $a_R$ | $W_v$ | $r_{wv}$ | $a_{wv}$ | $W_S$ | $r_{ws}$ | $a_{ws}$ | $V_{LS}$ | $r_{LS}$ | $a_{LS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | 33.654 | 1.205 | 0.708 | 11.341 | 1.052 | 0.881 | 2.606 | 1.341 | 0.392 | 5.582 | 1.058 | 0.625 |
| 59–43 | 53.7 | 1.16 | 0.75 | 1.2 | 1.37 | 0.774 | 4.73 | 1.37 | 0.774 | 6.04 | 1.064 | 0.78 |
| | $-0.22\times E$ | | | $+0.09\times E$ | | $-0.008\times E$ | $-0.05\times E$ | | $-0.008\times E$ | | | |

energies considered here ($6 \leq U \leq 22$ MeV) have been calculated with the global optical model parameters of Menet *et al.*[25] All these parameters are shown in Table I. For simplicity, the bound state shell-model wave functions have been generated with a harmonic oscillator potential, after checking that the cross sections and the analyzing powers obtained in this particular case are practically indistinguishable from those given by the more realistic Woods-Saxon potential. As far as the effective two-body interaction is concerned, it is customary, in the work on discrete states with the microscopic model, to construct each term of the interaction (central, tensor, and spin orbit) by means of a sum of a certain number of Yukawa potentials. The ranges and depths of these potentials are fixed by fitting the Fourier transforms of the free nucleon-nucleon potential as a function of the transferred angular momentum.[16,26,27] To minimize the number of parameters and for calculational simplicity, we decided to use a single potential for each term of the interaction [Eq. (3)]. Therefore the interaction adopted should be regarded as an entirely "effective" one.

After a careful analysis of the work previously done on the effective two-body interaction, we chose the values given by Austin[28] for the central components. These values are quite reliable since they are obtained as averages over many similar results, and are consistent with the free nucleon-nucleon potential.[28]

Unfortunately, this is not the case for the noncentral components (tensor and spin orbit)[28,29] for which only few rather scattered results are avail-

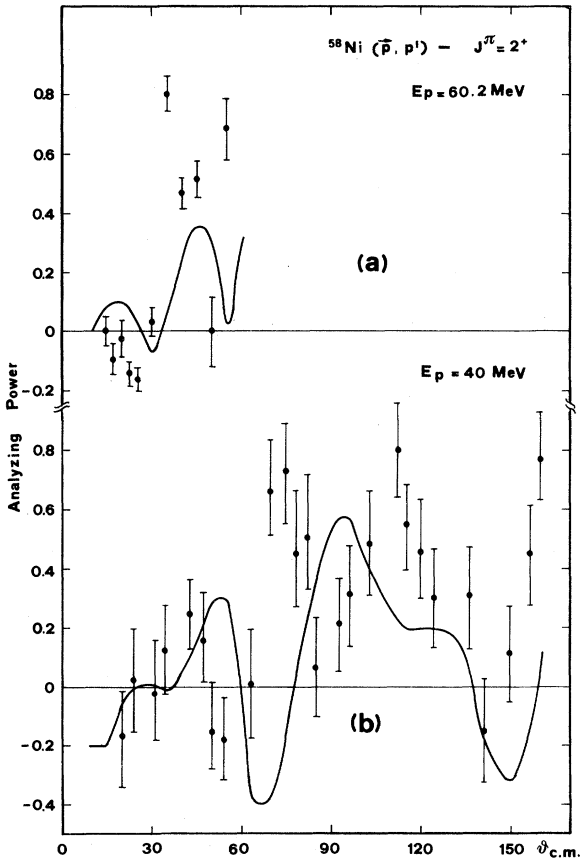


FIG. 1. Results of microscopic DWBA calculations of the analyzing power of the $2^+$, 1.45 MeV state populated by the $^{58}$Ni($\vec{p}$,$p'$) reaction at (a) $E_p=60.2$ MeV (Ref. 31) and (b) $E_p=40$ MeV (Ref. 30).

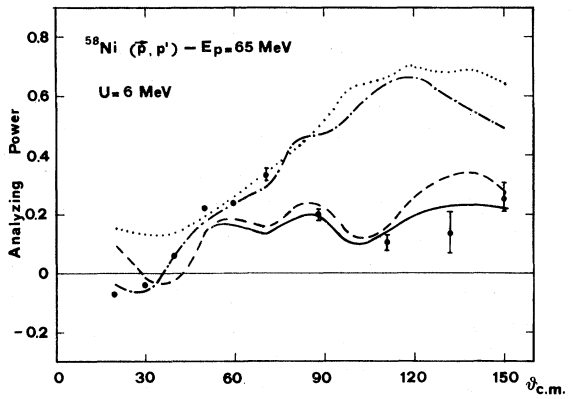


FIG. 2. Different contributions in the DWBA microscopic calculations of the $^{58}$Ni($\vec{p}$,$p'$) reaction at 65 MeV to the continuum, compared with the experimental data at $U=6$ MeV (Ref. 11). $\cdots$ single step contribution, only direct term in the DWBA matrix elements; $\cdot-\cdot-\cdot$ single step contribution, direct + exchange terms, only central potential in the two-body interaction [Eq. (3)]; – – – same as above, with inclusion of noncentral potentials; and —— same as above, with inclusion of the multistep contribution (up to the fourth step).

TABLE II. Parameters of the two-body effective interaction [Eq. (3)]. Depths are in MeV, ranges in Fermis.

| $V_0$ | $V_\tau$ | $\mu_{\text{cent.}}$ | $V_\sigma$ | $V_{\sigma\tau}$ | $\mu_{\sigma\cdot\sigma}$ | $V_{LS}$ | $V_{LS\tau}$ | $\mu_{L\cdot S}$ | $V_T$ | $V_{T\tau}$ | $\mu_{\text{tens.}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −24.5 | +13 | 1 | +11.5 | +11.7 | 1 | −4 | +12 | 0.7 | −7.75 | +6.75 | 0.8 |

able. We therefore decided to fix the corresponding values by simultaneously fitting the analyzing power of the discrete states of the reaction and of a part of the continuum at very low excitation energy, where the high-order effects should be not very important. We therefore concentrated on the $2^+$ level at 1.45 MeV excitation energy, whose analyzing power was measured in a detailed experiment by Fricke *et al.*[30] at 40 MeV, and by Kocker *et al.*[31] at 60.2 MeV.

As a starting set of parameters for the noncentral components of the interaction, we chose those extracted by Hosono *et al.*[17] in their analysis of $^{12}C(\vec{p},p')$ at 65 MeV, using the same DWBA formalism.[22] The final values we adopted are the same as those of Hosono *et al.*, as far as the tensor components are concerned (this term was found to be not critical at all as far as the outcomes of the calculations for the present case are concerned). The spin-orbit components have been changed in depth, retaining the same signs and ranges, in order to acceptably fit the data of the $2^+$ level (Fig. 1). The same parameters also give a reasonable fit to the analyzing power in the continuum part of the spectrum at 6 MeV, measured by Sakai *et al.*[11] (Fig. 2). All the two-body interaction parameters are shown in Table II.

As a final comment, we note that the only parameters we fixed in these ways, namely the spin orbit ones, have reasonable values; that adopted for $V_{LS\tau}$ in particular, which was found especially important in determining the analyzing power at backward angles where the exchange contribution is relevant (see Fig. 2), is consistent with the free nucleon-nucleon force (see, e.g., the *LS* values of the Elliot force as reported by Bertsch *et al.*,[26] when normalized to the same range $\mu$ of the interaction).

In this way we extended the calculations up to 22 MeV residual nucleus excitation energy, for four steps of the SMDE chain. To reduce the number of DWBA calculations, only neutron particle-hole configurations have been taken into account. The interaction we used would indeed give a much smaller cross section for proton configurations, since the proton-neutron potential is larger than the proton-proton one. Despite this simplification, about 350 DWBA calculations were performed for different combinations of the nuclear configurations, angular momentum transfer, and incoming-outgoing energy values. The final results are presented in Figs. 3 and 4, both for the cross section and the analyzing power.

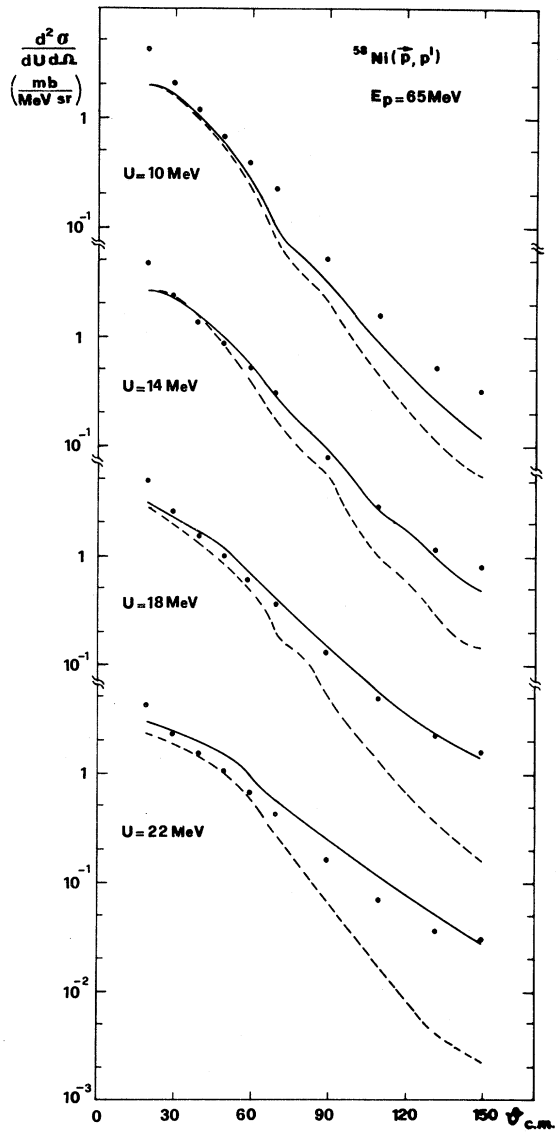


FIG. 3. Comparison between calculated and experimental differential cross sections for the $^{58}Ni(\vec{p},p')$ reaction at several energies of the residual nucleus. — — — single step contribution; and —— single + multistep contribution. Note the increasing importance of the multistep contribution with increasing excitation energy and scattering angle.

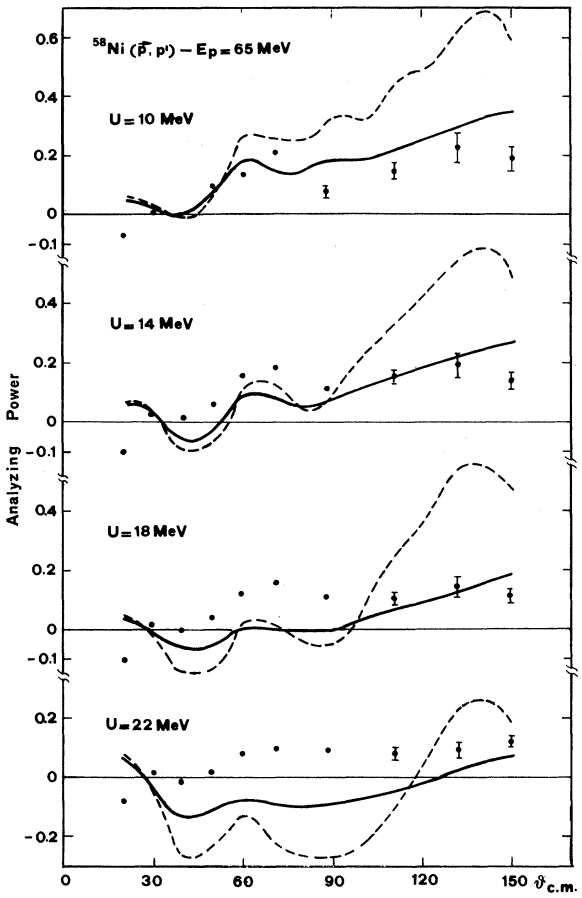


FIG. 4. Comparison between calculated and experimental analyzing power for the $^{58}$Ni($\vec{p}$,$p'$) reaction at several excitation energies of the residual nucleus. — — — single step contribution; and —— single + multistep contribution.

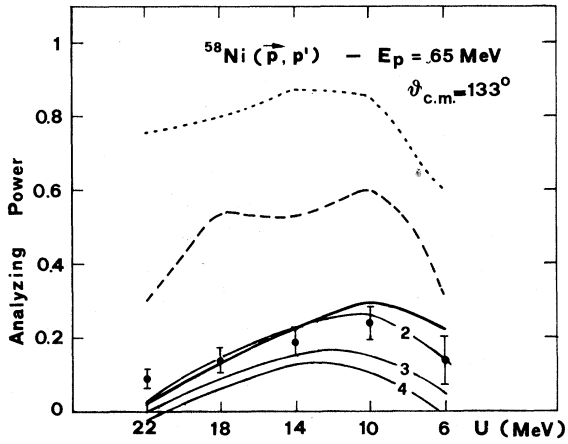


FIG. 5. Comparison between calculated and experimental analyzing power at a backward angle ($\theta_{c.m.}=133°$) as a function of the excitation energy $U$. · · · single step contribution, only central potential in the two-body interaction [Eq. (3)]; — — — single step contribution, central + noncentral potentials; and —— same as above, with inclusion of the multistep contribution. The contributions of the different steps are also shown separately (——2 3 4——).

## IV. DISCUSSION OF RESULTS AND CONCLUSION

As Fig. 3 shows, the differential cross sections are reproduced quite well, the quality of the fits for the different excitation energy ranges being even better than those previously obtained for the ($p$,$n$) reactions at 45 MeV. This confirms the validity of the SMDE theory, as well as that of the model for the inelastic scattering adopted in the present work.

As far as the analyzing power is concerned, the calculations reproduce the overall feature of the experimental data for $U \lesssim 18$ MeV. In particular, the magnitude in the backward direction is correctly given together with the rather uniform variation with angle. This is the combined result of the high value for the analyzing power in the first stage and the progressively smaller values of the subsequent stages (see Fig. 5).

As Fig. 5 shows, it is not possible to reproduce the magnitude and the energy variation of the analyzing power in the backward direction, even by including the effect of the microscopic spin-orbit potential in a calculation limited to the first step. The experimental results can only be understood by including the contributions of subsequent steps in the reaction. On the other hand, the analyzing power in the forward direction is essentially determined by that of the first step, since the cross section of this step dominates in this angular region (see Fig. 3). The fits to the data in this angular region could probably be improved by further parameter variations or by releasing the approximations described in Sec. III, but we did not think this would be worthwhile at the present stage of the investigations. The poor fit to the analyzing power at high excitation energy ($U=22$ MeV) could be due to having neglected any secondary particle emission, which is present in these ($\vec{p}$,$xp'$) inclusive measurements and is likely to be more and more important as the excitation energy increases.

If we compare the results of these calculations with the previous work on ($p$,$n$) reactions, we see that the relative contributions of the different stages of the reaction are quite similar. In particular, one would expect the contribution of the first stage in the reaction to pass through a minimum as the incident energy increases. At lower energies there is

insufficient energy for many stages, while at high energies many stages become less likely again because the value of the interaction matrix falls with increasing energy. These qualitative considerations are borne out by the results of the work on the $(p,n)$ reactions, which give the contributions of the first stage as 82% and ~55% at 25 and 45 MeV, respectively, and by the present result (~70%) for the $(\vec{p},p')$ at 65 MeV. These results can be compared with the corresponding figure of about 90% for the $(p,p')$ at 62 MeV in the same excitation energy range, obtained by Tamura *et al.*[6]

In conclusion, the more stringent test given by the simultaneous fit of cross sections and analyzing powers presented here has confirmed the validity of the SMDE theory. These results, together with those previously obtained for the $(p,n)$ reactions, have pointed out the importance of high-order effects in direct reactions induced by medium-energy nucleons. In particular, these contributions have been found to be of crucial importance in explaining the analyzing power, especially at backward angles, where their effect on the differential cross section is not always so relevant (see, for example, the calculations at $U=10$ MeV, Figs. 3 and 4).

It might also be worthwhile to point out the importance of the spin-orbit interaction in the two-body residual potential (see Fig. 5). The possibility of including this interaction in the calculations can only be given by a fully quantum-mechanical description of the reaction, such as the SMDE. This remark naturally rules out the previous semiclassical preequilibrium models[3] in any description of data depending on polarization.

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