$N+{}^6$Li system with flexible cluster wave function

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The $n+{}^6$Li and $p+{}^6$Li systems are studied with the single-channel resonating-group method. The ^{6}Li internal wave function used is either a single translationally-invariant harmonic-oscillator shell-model function or a superposition of two such functions. The result shows that the main features of this system do not depend sensitively on which of these functions is employed, although significant differences in cross-section values do appear at backward angles. The fit to experimental data is only fair, indicating that the present calculation should be refined by including the $N+{}^6$Li$^*(3^+)$ inelastic channel, by taking into better account $d+\alpha$ clustering in ^{6}Li, by carefully considering the effect of specific distortion, and by, perhaps, also adopting a noncentral nucleon-nucleon potential in the formulation.

NUCLEAR REACTIONS ^{6}Li(p,p), ^{6}Li(n,n); calculated phase shifts and $\sigma(\theta)$. Resonating-group method with complex-generator-coordinate technique.

I. INTRODUCTION

The development of the complex-generator-coordinate technique[1-3] (CGCT) has made it feasible to utilize flexible cluster internal functions in resonating-group studies of the interactions between composite nuclei. Especially for systems in which clusters with diffuse density distributions are involved, it is reasonable to expect that the use of such internal functions may be necessary for a detailed explanation of the measured results. In this investigation, we illustrate this by considering the $N+{}^6$Li system. This particular system is chosen because the nucleus ^{6}Li is lightly bound and has a rms radius even larger than that of the heavier nucleus ^{12}C,[4] indicating that the simplified choice of a *single,* translationally-invariant harmonic-oscillator shell-model function (hereafter referred to as a $1G$ function), which is the type of internal function commonly adopted in resonating-group and generator-coordinate calculations of relatively heavy two-cluster systems such as $\alpha+{}^{16}$O, ^{16}O $+{}^{40}$Ca, and so on,[5,6] may not be sufficient to describe properly the essential behavior of this diffuse nonclosed-shell light cluster.

For a preliminary investigation,[7] hereafter referred to as SLT, we previously considered the case of $n+{}^6$Li scattering using a $1G$, ^{6}Li internal function. With such a function, it was found possible to describe reasonably well the charge-form-factor behavior in the low-q^2 region up to about 1.5 fm^{-2}. In this study, we shall adopt a more flexible function consisting of a superposition of two harmonic-oscillator shell-model functions (referred to as a $2G$ function), which can yield satisfactory form-factor values for q^2 even as large as 6 fm^{-2}. With this choice, the description of the behavior of the ^{6}Li nucleus is certainly improved, but the amount of computational effort required for the $N+{}^6$Li calculation will also be substantially increased; however, the power of the CGCT is such that the calculation can still be carried out in a straightforward manner.

In the next section, we discuss the modifications made in the single-channel resonating-group formulation of the $N+{}^6$Li problem described in SLT. The results obtained with both the $1G$ and the $2G$ ^{6}Li internal functions will be presented in Sec. III. Here also, a comparison with experimental data will be discussed. Finally, in Sec. IV, we summarize the findings of this investigation and make some concluding remarks.

II. FORMULATION

The main modification of the formulation given in SLT is the replacement of the $1G$, ^{6}Li internal

function ϕ_{1G} [denoted as ϕ_6 in Eq. (3) of SLT, with $\alpha=0.305$ fm^{-2}] by a $2G$ internal function ϕ_{2G}; that is, instead of ϕ_{1G}, we now use[8]

$$\phi_{2G}=(\vec{r}_5-\vec{R}_6)\cdot(\vec{r}_6-\vec{R}_6)\times\left\{\exp\left[-\tfrac{1}{2}\alpha_1\sum_{i=1}^{6}(\vec{r}_i-\vec{R}_6)^2\right]+c\exp\left[-\tfrac{1}{2}\alpha_2\sum_{i=1}^{6}(\vec{r}_i-\vec{R}_6)^2\right]\right\} \quad (1)$$

in the $N+^6$Li calculation. The parameters α_1, α_2, and c are then adjusted to yield a best overall agreement with the experimental charge-form-factor data[9,10] in the q^2 region up to about 6 fm^{-2}. The resultant values are

$$\alpha_1=0.169\ \text{fm}^{-2},\quad \alpha_2=0.458\ \text{fm}^{-2},\quad c=145.1, \quad (2)$$

with the corresponding rms charge radius being equal to 2.48 fm. In Fig. 1, we show a comparison between the empirical values for $F_{\text{ch}}{}^2$ and the values obtained with the $2G$ (solid curve) and the $1G$ (dashed curve) functions. Here one finds that the fit to experiment obtained with the $2G$ function is indeed quite good, while that obtained with the $1G$ function is satisfactory only in the low-q^2 region below about 1.5 fm^{-2}.

Another modification is the use of the nucleon-nucleon potential given by Eqs. (9)–(11) of a previous publication,[11] rather than the nucleon-nucleon potential given by Eqs. (12)–(14) of SLT. This modification is made in order to conform with recent resonating-group calculations[12] in other light systems. In this potential, there exists an exchange-mixture parameter u, the value of which will be determined by using the experimental information[13] that there exist in ^{7}Li $J^\pi=\frac{5}{2}^-$ and $\frac{3}{2}^-$ states at 7.47 and 10.25 MeV with $l=1$, $S=\frac{3}{2}$ and a predominant $n+^6$Li cluster configuration. This means that, in a calculation where the nucleon-nucleon potential employed is purely central, one should consider that the ^4P state in ^{7}Li lies at 1.89 MeV above the $n+^6$Li threshold,[14] a value obtained by simply averaging the experimental excitation energies according to $\langle \vec{l}\cdot\vec{S}\rangle$ weighting. Using this information, one then finds that the values of u should be taken to be 0.98 and 1.00 in the $1G$ and $2G$ cases, respectively.

Because of the adoption of a rather complicated internal function ϕ_{2G}, the expression for the kernel

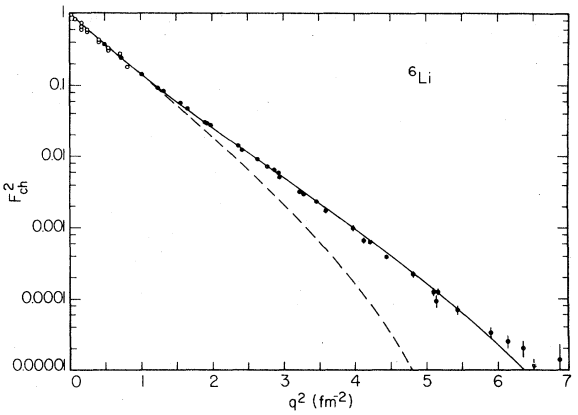

FIG. 1. Charge form factors of ^{6}Li. The solid and dashed curves represent results obtained with the $2G$ and $1G$ functions, respectively.

function becomes quite lengthy, but, as mentioned in the Introduction, can be straightforwardly derived by using the CGCT. For the exchange-Coulomb kernel, we use the exact expression in the $n+^6$Li case but an approximate one in the $p+^6$Li case. This latter expression is obtained by employing a self-consistent procedure described previously.[15] This procedure has been found to work well in many light systems and should be accurate enough for our present purposes.

III. RESULTS

A. Phase shifts

In Fig. 2, we show the $n+^6$Li phase shifts calculated with the $2G$ (solid curves) and $1G$ (dashed curves) internal functions, respectively. From this figure, one notes the following interesting features:

(i) The main difference between the phase-shift results in these two cases occurs in the $l=0$ state. However, this difference arises to a large extent from the fact that there appear, as expected, spurious resonances[3,16] in the $2G$ case (shown as breaks in the phase-shift curves) and, hence, is not particularly meaningful. A more useful comparison would result when such resonances are suppressed by the introduction of a phenomenological imaginary potential into the resonating-group formulation.[17] This will be discussed in the next subsection.

(ii) The phase-shift rise near the $n+^6$Li threshold in the $l=0$, $S=\frac{3}{2}$ state may be associated with the experimental observation of a broad ^4S resonance at 2.4 MeV.[18] We should caution, however, that our present wave function is probably not flexible

enough to describe properly the behavior of this broad resonance state; in addition, the existence of the spurious resonance does complicate the interpretation. This is a situation which requires further extensive investigation.

(iii) In orbital-angular-momentum states where spurious resonances do not exist (i.e., $l \geq 1$), the calculated phase-shift values in the $1G$ and $2G$ cases turn out to be not too different. As is seen from Fig. 2, the $l=1$ phase shifts differ substantially from each other only in the energy region around the ^4P resonance. For higher-l values, the differences are, in fact, even smaller; for example, the ^4D phases in the $2G$ case differ from those in the $1G$ case by only $-2.7°$ and $1.4°$ at 8 and 50 MeV, respectively.

B. Differential cross sections

To achieve a meaningful comparison between the $1G$ and $2G$ $l=0$ phases, and to see the quality of fit to experiment, we introduce phenomenological absorptive potentials into the resonating-group formulation. These potentials have the form given by Eqs. (18) and (19) of SLT, and contain adjustable strength parameters W_{02} and W_{04} in the channel spin $\frac{1}{2}$ and $\frac{3}{2}$ states, respectively.

A comparison between calculated and experimental[19] differential cross sections for $n+{}^6$Li scattering at 12 MeV is shown in Fig. 3. In this figure, the solid curve represents the $2G$ result with $W_{04}=3.5$ MeV, $W_{02}=6.5$ MeV, while the dashed curve represents the $1G$ result with $W_{04}=4.0$ MeV, $W_{02}=6.0$ MeV. The total reaction cross sections σ_R in the $2G$ and $1G$ cases are equal to 537 and 530 mb, respectively. Here one sees that, because of rather large errors associated with the experimental data, there seems to be no clear preference for adopting the internal function ϕ_{2G} over the internal function ϕ_{1G}. On the other hand, it is evident that the oscillatory behaviors in the backward angular region are somewhat different in these two calculations. For instance, the peak-to-valley ratio (i.e., the ratio of the cross section at 180° to the minimum cross section at an angle near 138°) is equal to 1.8 in the $2G$ case, which is significantly smaller than the value of 2.6 in the $1G$ case.

In Fig. 4, we compare $p+{}^6$Li differential cross sections calculated in the $2G$ case with experimental data[20] at 22.2, 30, and 38.9 MeV. For this calculation, we have made the additional simplification of setting

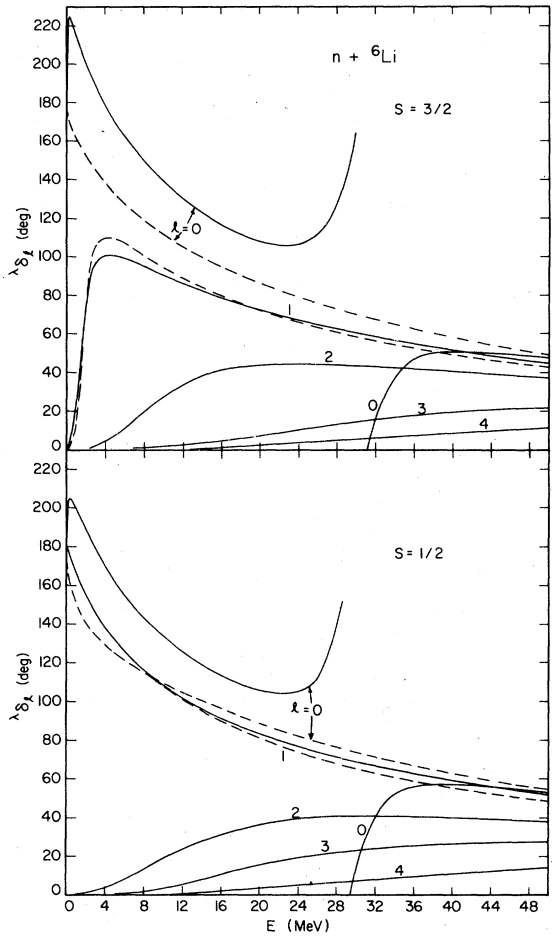

FIG. 2. Phase shifts $^\lambda\delta_l$ for $n+{}^6$Li scattering, calculated with the $2G$ (solid curves) and $1G$ (dashed curves) functions.

$$W_{02}=W_{04}=W_0\ ,$$

and adjusted the only parameter W_0 to yield reaction-cross-section values measured at nearby energies.[21,22] As is seen, the agreement between calculation and experiment is reasonable at angles smaller than about 130°. Considering the fact that no adjustment is made to fit the differential-cross-section data, one can view the calculated result as fairly satisfactory. The major discrepancy occurs in the large-angle region, where it is seen that the calculated cross-section values are too large.[23] This can probably be attributed to the various approximations made in our calculation. These are the following: (i) the use of a ^{6}Li internal function, which may not have a sufficient degree of $d+\alpha$ clustering; (ii) the lack of explicit consideration of the 3^+, first excited state in ^{6}Li; (iii) the adoption of a purely central nucleon-nucleon potential; and (iv) a rather

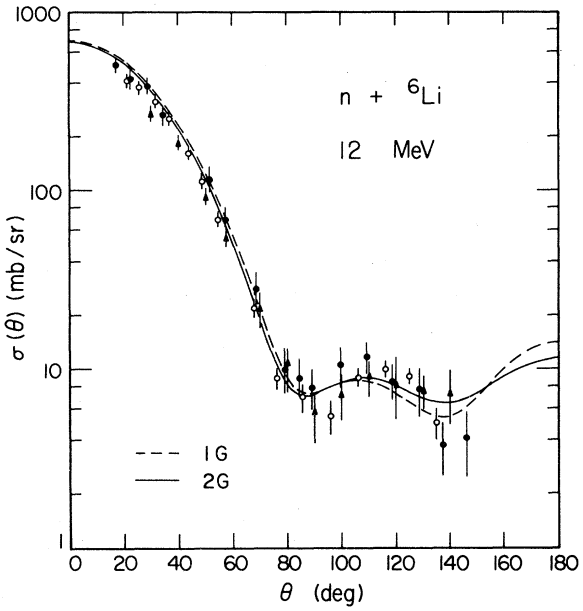

FIG. 3. Comparison of calculated and experimental differential cross sections for $n+^6$Li scattering at 12 MeV. The solid and dashed curves represent results obtained with the $2G$ and $1G$ functions, respectively.

crude treatment of the specific distortion effect by simply adjusting the u value. The deficiencies associated with employing these approximations can all be properly corrected within the resonating-group framework, although it should be mentioned that the resulting calculation will certainly be very tedious.

Calculations have also been made with the $1G$ function for $p+^6$Li scattering at 22.2 and 30 MeV, using the same absorptive strength as that employed in the $2G$ case. Here again, one finds that, as in the 12-MeV $n+^6$Li comparison, the positions of the cross-section minima and maxima are essentially the same in the $1G$ and $2G$ cases, and the cross-section values are noticeably different only in the backward angular region. For example, the 180° cross sections are equal to 2.9 and 1.4 mb in the $1G$ case at these two energies, which should be compared with the corresponding values of 3.4 and 2.0 mb in the $2G$ case.

Complex phase shifts calculated in the $1G$ and $2G$ cases are tabulated in Table I for $n+^6$Li scattering at 12 MeV and $p+^6$Li scattering at 22.2 MeV. From this table, it is seen that the differences are indeed not too large. For the real part of the phase shift, the largest difference occurs in the ^2S state. But, even here, the difference is only 7.1° in the 12-MeV case and 6.0° in the 22.2-MeV case.

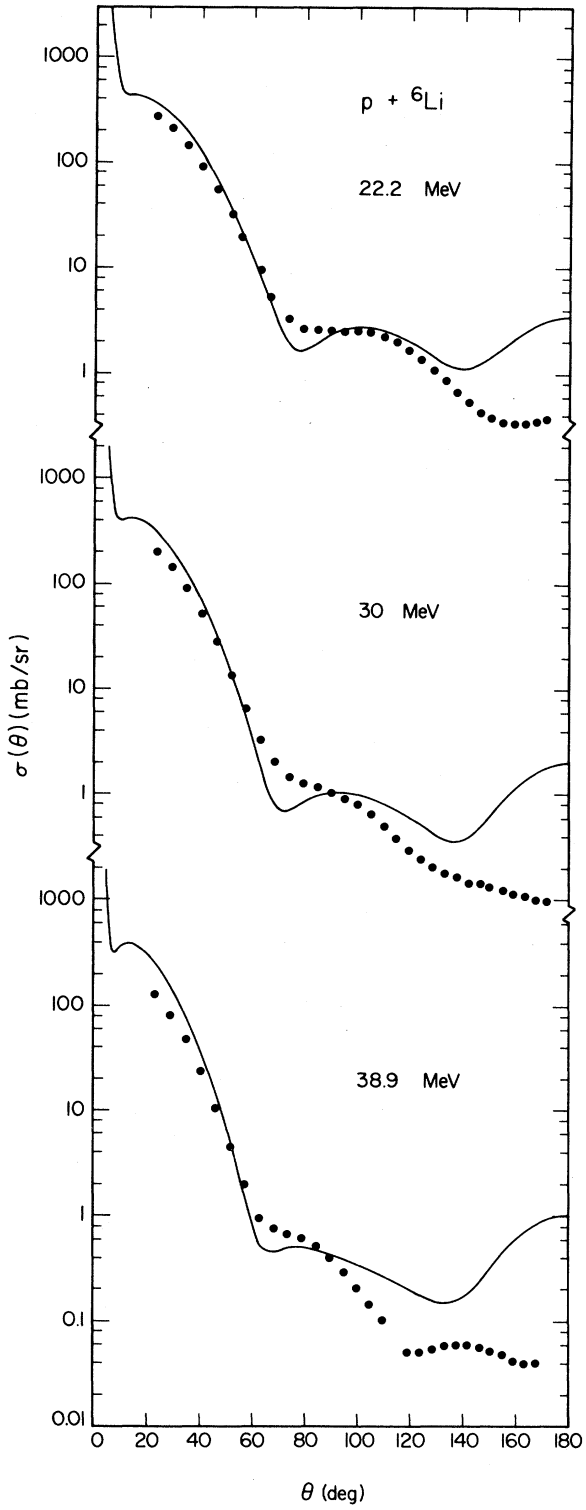

FIG. 4. Differential cross sections for $p+^6$Li scattering at various energies. The solid curves represent results obtained with the $2G$ function. Experimental data shown are those of Ref. 20.

TABLE I. Values of ${}^{\lambda}\delta_l$ (in degrees), calculated in the $1G$ and $2G$ cases, for the $n+{}^6$Li system at 12 MeV and the $p+{}^6$Li system at 22.2 MeV.

E (MeV)	12		22.2	
case	$1G$	$2G$	$1G$	$2G$
${}^4\delta_0$	$105.0+i9.8$	$105.1+i18.3$	$88.7+i8.0$	$86.6+i15.9$
${}^4\delta_1$	$91.1+i10.4$	$88.4+i11.2$	$71.7+i8.2$	$72.0+i8.9$
${}^4\delta_2$	$32.1+i20.5$	$31.8+i18.7$	$43.4+i6.8$	$44.2+i16.8$
${}^4\delta_3$	$0.9+i3.3$	$2.3+i3.0$	$6.3+i9.4$	$7.2+i9.2$
${}^4\delta_4$	$0.5+i0.6$	$0.5+i0.5$	$2.2+i2.6$	$2.5+i2.7$
${}^4\delta_5$	$0\ +i0.1$	$0.1+i0.1$	$0.3+i0.7$	$0.5+i0.7$
${}^4\delta_6$	0	0	$0.1+i0.2$	$0.1+i0.2$
${}^2\delta_0$	$105.3+i19.5$	$98.2+i23.7$	$90.9+i9.4$	$84.9+i16.9$
${}^2\delta_1$	$100.4+i10.0$	$100.9+i10.5$	$80.4+i7.1$	$82.8+i7.2$
${}^2\delta_2$	$8.9+i32.8$	$10.7+i33.2$	$39.3+i22.0$	$37.6+i21.7$
${}^2\delta_3$	$4.6+i5.8$	$4.5+i6.3$	$14.5+i10.4$	$15.0+i10.5$
${}^2\delta_4$	$0.4+i0.9$	$0.7+i1.0$	$2.8+i2.8$	$3.4+i2.8$
${}^2\delta_5$	$0.1+i0.1$	$0.1+i0.2$	$0.7+i0.7$	$0.9+i0.7$
${}^2\delta_6$	0	0	$0.1+i0.2$	$0.2+i0.2$

C. Odd-even behavior

In Fig. 5, we show the l-dependent behavior of the real part of the phase shift for $p+{}^6$Li scattering at 22.2 MeV (i.e., 25.9 MeV/nucleon). Here one sees that, except for the ${}^2S$ phase-shift points, the odd-even characteristics, discussed in SLT, seem to be quite similar in the $1G$ and $2G$ cases. This indicates that core-exchange contributions[24] of type a and type d do not depend sensitively on whether ϕ_{1G} or ϕ_{2G} is used in the resonating-group formulation, which explains in turn the observation that at the 180° peaks the cross-section values in the $1G$ and $2G$ calculations are not greatly different.

To demonstrate even more clearly the similarity of the odd-even behavior in the $1G$ and $2G$ cases, we depict in Fig. 6 the values of $C_{l\lambda}$, defined in Eq. (24) of SLT, obtained for $n+{}^6$Li scattering at 30 MeV. In these calculations we have not considered absorptive effects and, hence, the results for $C_{0\lambda}$ are not shown. From this figure, it is evident that the l-dependent and S-dependent features of $C_{l\lambda}$ are qualitatively the same and the effect of employing either ϕ_{1G} or ϕ_{2G} does not seem to be too important.

IV. CONCLUSION

In this investigation, the main purpose was to examine, in the case of $N+{}^6$Li scattering, the effect of using for the ^{6}Li internal function either a single translationally-invariant harmonic-oscillator shell-model function (i.e., ϕ_{1G}) or a superposition of two such functions (i.e., ϕ_{2G}). These two internal functions are distinguished by the fact that with ϕ_{1G} one can obtain a reasonable fit to the empirical charge-form-factor data only in the restricted range of q^2 less than about 1.5 fm^{-2}, while with ϕ_{2G} a good form-factor fit for q^2 even as large as 6 fm^{-2} can be accomplished. The result shows, however, that the main features of the $N+{}^6$Li system do not seem to depend sensitively on which of these functions is employed. This means that if one wishes to study only the essential properties of this system, then the

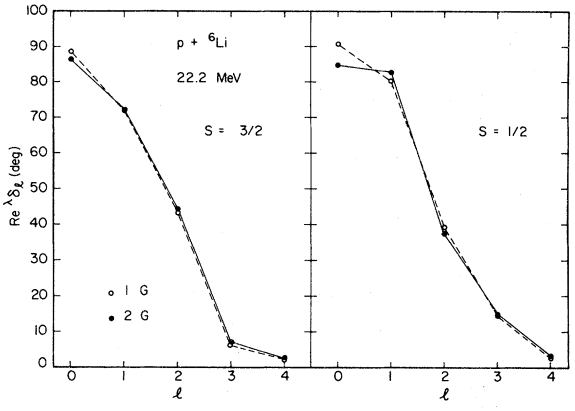

FIG. 5. Real part of the phase shift as a function of the orbital angular momentum in the $p+{}^6$Li case at 22.2 MeV. The solid and open circles represent results obtained with the $2G$ and $1G$ functions, respectively.

adoption of the simple $1G$, ^{6}Li function is sufficient; on the other hand, if the purpose is to achieve a detailed and quantitative understanding, then the use of the computationally more tedious $2G$ function will become necessary.

With our present formulation, it is only possible to obtain a fair agreement with experiment. A particularly noticeable discrepancy occurs in the large-angle region where the calculated cross sections, especially at higher energies, are too large. In this respect, it is interesting to note that, because of the rather large nucleon-number difference between the interacting nuclei, the odd-even effect in this system is not as prominent as that in many other light systems.[3] Thus, we are presently of the opinion that this discrepancy may arise as the cumulative result of adopting a number of simplifying assumptions (see the discussion in Sec. III B) and the correction of this discrepancy may require a careful examination of all these assumptions.

From this and other[25] investigations, we come to the conclusion that, from the resonating-group viewpoint, it would be a tedious task to achieve a quantitative and detailed explanation of the experimental features observed in any system involving a ^{6}Li nucleus. The reason for this is that the nucleus ^{6}Li has many complicated properties; it is a nonclosed-shell nucleus, has low-lying excited states, is easily distortable, and contains a strong degree of nucleon clustering. On the other hand, these very properties also make it interesting and challenging to perform refined studies in ^{6}Li + nucleus systems, because a successful conclusion of such studies would certainly serve as a convincing argument for the usefulness of the resonating-group approach in treating nuclear many-body problems.

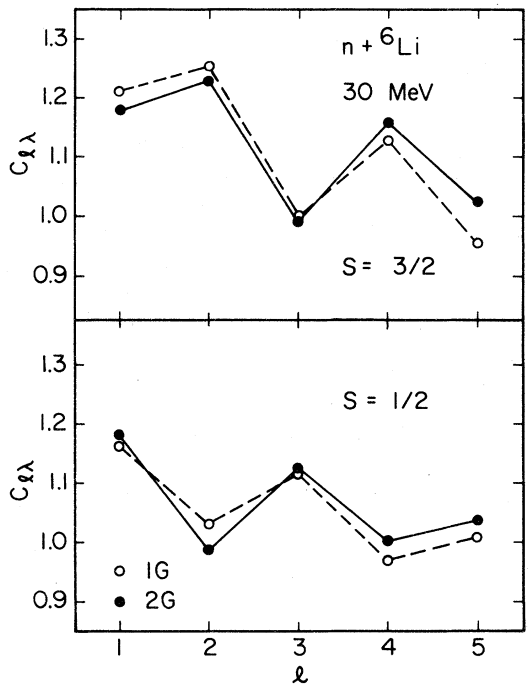

FIG. 6. $C_{l\lambda}$ as a function of l for $n+{}^6$Li scattering at 30 MeV. The solid and open circles represent results obtained with the $2G$ and $1G$ functions, respectively.

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[22]The values of W_0 are equal to 4.0, 4.0, and 3.5 MeV at $p+{}^6$Li c.m. energies of 22.2, 30, and 38.9 MeV, respectively. The corresponding reaction cross sections are calculated to be 387, 344, and 280 mb.
[23]Various forms for the absorptive potential have been employed in our attempts to improve the cross-section behavior in the large-angle region. However, such attempts have not been successful.
[24]For an explanation of the meaning of various exchange types, see M. LeMere, D. J. Stubeda, H. Horiuchi, and Y. C. Tang, Nucl. Phys. A320, 449 (1979).
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