Cross section and polarization in deuteron photodisintegration: General formulas

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The differential cross section and the outgoing nucleon polarization in the disintegration of unpolarized deuterons by polarized photons are given as expansions in Legendre functions.

[NUCLEAR REACTIONS $d(\vec{\gamma},n)p$. Differential cross section and polarization. Legendre function expansion.]

I. INTRODUCTION

Recently, we made an attempt[1] to resolve the long standing discrepancy between theory and experiment in the forward differential cross section, $d\sigma/d\Omega$ (0°) (Ref. 2), for the $\vec{\gamma}d\rightarrow np$ reaction, taking into account the relativistic corrections to the charge density ρ coming from the nonrelativistic reduction of the nucleon Dirac current. The commonly used analytic expressions of the differential cross section $d\sigma/d\Omega$ and the polarization $\vec{P}$ of the outgoing nucleons are those given by Partovi,[3] even if extensive work carried out by the Yale group[4] is also worth mentioning. Partovi's paper has been often quoted by both theoreticians and experimentalists since it gives the general expressions for $d\sigma/d\Omega$, $\sigma_{\rm tot}$, and $\vec{P}$ valid for arbitrary photon polarization and for all the electromagnetic (em) multipoles, and neglects only the possibility of a polarized deuteron target. However, these quantities are given in Ref. 3 by expansions in terms of reduced rotation matrices combined in a complicated way which depends on the multipolarities of the transitions involved. This results from expressing them by means of summations over the magnetic quantum numbers of the initial and final states. Instead, it is possible to sum over these quantum numbers and arrive at an expansion in Legendre functions, $P_n^m(\cos\theta)$, of $d\sigma/d\Omega$ and of most of the functions of θ defining $\vec{P}$, θ being the center of mass (c.m.) angle between the outgoing nucleon and the incoming photon (see Fig. 1). This was the purpose of our work, which is organized as follows. The Legendre function expansions of $d\sigma/d\Omega$ and $\vec{P}$ are deduced in Secs. II and III, respectively, and our conclusions are stated in Sec. IV. The coefficients of our expansions are compared with those appearing in Partovi's expressions in the Appendix.

II. DIFFERENTIAL CROSS SECTION

The differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\mathrm{Tr}(T\rho_{\rm in}T^{\dagger}) , \quad (1)$$

where T is the transition matrix and $\rho_{\rm in}$ is the density matrix of the initial state. Since the deuteron target is unpolarized, the matrix elements of $\rho_{\rm in}$ are

$$\begin{aligned}(m_d\mu\,|\,\rho_{\rm in}\,|\,m_d'\mu')&=\tfrac{1}{3}\delta_{m_d,m_d'}(\mu\,|\,\rho_\gamma\,|\,\mu')\\ &=\tfrac{1}{6}\delta_{m_d,m_d'}(\mu\,|\,1-\vec{\Sigma}\cdot\vec{\sigma}\,|\,\mu') , \quad (2)\end{aligned}$$

in the c.m. frame with the polar axis coincident with the photon momentum $\vec{q}$. In Eq. (2) $|\,m_d\mu)$ is the state with photon helicity $\mu=\pm1$ and projection $m_d=\pm1,0$ of the deuteron total angular momentum on $\vec{q}$; $\vec{\Sigma}$ is the Stokes vector describing the polarization of the photon beam, and $\vec{\sigma}$ are the Pauli matrices. Shown in Fig. 1 is the relative orientation of

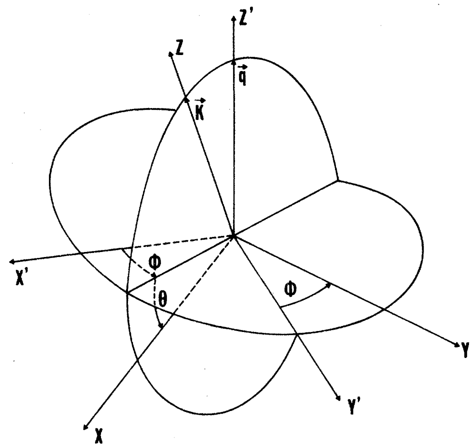

FIG. 1. The two c.m. coordinate systems for deuteron photodisintegration.

the two c.m. coordinate systems $(x'y'z')$ and (xyz) necessary for the description of the deuteron photodisintegration process, with the z' axis parallel to the photon momentum $\vec{q}$, and the z axis parallel to the relative momentum of the NN system,

$$\vec{k}=(\vec{k}_1-\vec{k}_2)/2 \; .$$

As in the work of Partovi[3] we take the y axis in the direction $\vec{q}\times\vec{k}$ and the x' axis in the direction of the linear photon polarization (if any) so that $\Sigma_{y'}=0$, $\Sigma_{x'}=\Sigma_l$ is the degree of linear polarization, and $\Sigma_{z'}=\Sigma_c$ the degree of circular polarization. The wave function of the deuteron is given by the usual combination of S and D states

$$|\, m_d)=\sum_{l_d=0,2}\frac{u_{l_d}(r)}{r}\mathcal{Y}_{l_d 1}{}^{1m_d}(\hat{r}) \; , \tag{3}$$

with the spin-angle function defined by

$$\mathcal{Y}_{ls}^{jm}(\hat{r})=\sum_{m_l m_s}(lm_l,sm_s\,|\,jm)Y_{m_l}^{l}(\hat{r})\chi_{m_s}^{s} \; , \tag{4}$$

in terms of Pauli spinors $\chi_{m_s}^{s}$ and spherical harmonics $Y_m^l(\hat{r})$, with $\hat{r}=\vec{r}/r$.

The final np state is characterized by $\vec{k}$, the spin s, and its projection m_s on $\vec{k}$. In the (xyz) frame we have the Blatt-Biedenharn expansion in partial waves of the final wave function with ingoing asymptotic behavior

$$|\, sm_s)=\sum_{\lambda jl}\sqrt{4\pi\hat{l}}\,(l0,sm_s\,|\,jm_s)e^{-i\delta_\lambda^j}U_{ls\lambda}^{j}\,|\,\lambda jm_s) \; , \tag{5}$$

with

$$|\,\lambda jm_s)=\sum_{l's'}i^{l'}U_{l's'\lambda}^{j}\frac{v_{l's'\lambda}^{j}(kr)}{kr}\mathcal{Y}_{l's'}^{jm_s}(\hat{r}) \; , \tag{6}$$

where $\hat{l}=2l+1$ and the radial wave functions are defined by the asymptotic behavior

$$v_{l's'\lambda}^{j}(kr)\rightarrow\sin\left[kr-\frac{l'\pi}{2}+\delta_\lambda^j\right] . \tag{7}$$

δ_λ^j are the eigenphases and $U_{ls\lambda}^{j}$ the coupling matrix defined in terms of the coupling parameter ϵ^j by

$$U_{ls\lambda}^{j}=\begin{array}{l} \\ s=1,\; l=j-1 \\ s=0,\; l=j \\ s=1,\; l=j+1 \\ s=1,\; l=j \end{array}\begin{array}{cccc} \lambda=1 & \lambda=2 & \lambda=3 & \lambda=4 \\ \end{array}\left[\begin{array}{cccc} \cos\epsilon^j & 0 & \sin\epsilon^j & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\epsilon^j & 0 & \cos\epsilon^j & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] . \tag{8}$$

Obviously, there are only two $j=0$ states and the corresponding coupling matrix reduces to a 2×2 matrix.

In the first order perturbation theory, the T matrix is proportional to the em transition Hamiltonian and, in Coulomb gauge, we have

$$(sm_s\,|\,T\,|\,\mu m_d)=-\left[\frac{kM}{4\pi\omega}\right]^{1/2}\int d^3x\, e^{i\vec{q}\cdot\vec{x}}(sm_s\,|\,\vec{\epsilon}_\mu\cdot\vec{j}(\vec{x})\,|\,dm_d) \; , \tag{9}$$

where $\omega=|\,\vec{q}\,|$ is the energy of the photon, $\vec{\epsilon}_\mu$ is its polarization vector, and $\vec{j}(\vec{x})$ is the nuclear current density.

Following the conventions of Edmonds[5] for the rotation matrices $D(R)$, the multipole expansion of Eq. (9) reads

$$(sm_s\,|\,T\,|\,\mu m_d)=\left[\frac{kM}{2\omega}\right]^{1/2}\sum_{LM}i^L\sqrt{\hat{L}}D_{M\mu}^{L}(R)(sm_s\,|\,E^{LM}+\mu M^{LM}\,|\,m_d) \; , \tag{10}$$

where $R\equiv(0,\theta,\Phi)$ and (θ,Φ) are the polar angles of $\vec{k}$ with respect to $\vec{q}$. In Eq. (10) the magnetic M^{LM} and electric E^{LM} multipole operators are the usual ones

$$M^{LM}=\int d^3x\ \vec{j}(\vec{x})\cdot\vec{A}^{LM}_{\rm mag}(\vec{x})\ , \tag{11a}$$

$$E^{LM}=\int d^3x\ \vec{j}(\vec{x})\cdot\vec{A}^{LM}_{\rm el}(\vec{x})\ , \tag{11b}$$

with the transverse ML and EL vector potentials given by

$$\vec{A}^{LM}_{\rm mag}(\vec{x})=j_L(qx)\vec{Y}^{LM}_{L1}(\hat{x})\ , \tag{12a}$$

$$\vec{A}^{LM}_{\rm el}(\vec{x})=\frac{1}{q}\vec{\nabla}\times\vec{A}^{LM}_{\rm mag}(\vec{x})\ . \tag{12b}$$

Here, $\vec{Y}^{LM}_{l1}(\hat{x})$ are the vector spherical harmonics, defined as the spin-angle functions in Eq. (4).

Inserting Eqs. (10) and (11) in (9) we obtain

$$(sm_s\,|\,T\,|\,\mu m_d)=\sum_{LM\lambda jl}\sqrt{4\pi}\hat{l}\,\hat{j}\,e^{i\delta^j_\lambda}U^j_{ls\lambda}(-)^{l+s+j}\times\begin{pmatrix} l & s & j\\ 0 & m_s & -m_s\end{pmatrix}\begin{pmatrix} j & L & 1\\ -m_s & M & m_d\end{pmatrix}D^L_{M\mu}(R)(\mathscr{T}^L_{\lambda j}+\mu\mathscr{S}^L_{\lambda j}) \tag{13}$$

in terms of the electric $\mathscr{T}^L_{\lambda j}$ and magnetic $\mathscr{S}^L_{\lambda j}$ reduced matrix elements defined as in the work of Partovi. Explicitly we have

$$\mathscr{T}^L_{\lambda j}+\mu\mathscr{S}^L_{\lambda j}=\left[\frac{kM}{2\omega}\right]^{1/2}\sum_{l's'l_d} i^{L-l'}\sqrt{\hat{L}}\,U^j_{l's'\lambda}(l's'j\lambda\,||E^L+\mu M^L||\,l_d)\ , \tag{14}$$

with the definition

$$(l's'j\lambda\,||E^L+\mu M^L||\,l_d)=\int dr\, r^2\frac{v^j_{l's'\lambda}(kr)}{kr}(l's'j\,||E^L+\mu M^L||\,l_d 11)\frac{u_{l_d}(r)}{r}\ . \tag{15}$$

Because of the time-reversal invariance, $\mathscr{T}^L_{\lambda j}$, $\mathscr{S}^L_{\lambda j}$ are real, and, because of the parity conservation, they satisfy the following symmetry relations:

$$\mathscr{T}^L_{\bar\lambda j}=(-)^{L+\lambda+j}\mathscr{T}^L_{\lambda j}\ ,\qquad \mathscr{S}^L_{\bar\lambda j}=(-)^{L+\lambda+j+1}\mathscr{S}^L_{\lambda j}\ . \tag{16}$$

Working out the trace in Eq. (1) and using expression (2) the differential cross section takes the form

$$\frac{d\sigma}{d\Omega}=\frac{1}{3}\sum_{\substack{sm_sm_d\\ \mu\mu'}}(sm_s\,|\,T\,|\,\mu m_d)(sm_s\,|\,T\,|\,\mu' m_d)^*(\mu\,|\,\rho_\gamma\,|\,\mu')\ . \tag{17}$$

The insertion of Eq. (14) in Eq. (17) leads to an expression containing the product of two D matrices, for which the Clebsch-Gordan series[5] yields

$$D^L_{M\mu}(R)D^{L'}_{M'\mu'}{}^*(R)=(-)^{M-\mu}\sum_J\hat{J}\begin{pmatrix} L & L' & J\\ M & -M' & -\epsilon'\end{pmatrix}\begin{pmatrix} L & L' & J\\ \mu & -\mu' & -\epsilon\end{pmatrix}D^J_{\epsilon'\epsilon}(R)\ . \tag{18}$$

The sum over the magnetic quantum numbers in Eq. (17) can be easily made twice exploiting a well known relation between the $6-j$ and $3-j$ symbols (see, for example, formula 6.2.8 of Edmonds[5]), giving the result

$$\frac{d\sigma}{d\Omega}=\sum_{Js\beta\beta'\mu\mu'}(-)^N W^{Js}_{\beta\beta'}(\mu,\mu')D^J_{0,\mu-\mu'}(R)\cos(\delta^j_\lambda-\delta^{j'}_{\lambda'})(\mu\,|\,\rho_\gamma\,|\,\mu')(\mathscr{T}^L_{\lambda j}+\mu\mathscr{S}^L_{\lambda j})(\mathscr{T}^{L'}_{\lambda' j'}+\mu'\mathscr{S}^{L'}_{\lambda' j'})\ , \tag{19}$$

where $N=L+L'+J$ and β,β' indicate the set of quantum numbers $(Ljl\lambda)$ and $(L'j'l'\lambda')$, respectively. In Eq. (19) we have defined

$$W^{Js}_{\beta\beta'}(\mu,\mu')=\frac{4\pi}{3}(-)^{J+j+j'+s}\hat{J}\sqrt{\hat{l}\hat{j}\hat{l}'\hat{j}'}\,U^{j}_{ls\lambda}U^{j'}_{l's\lambda'}\begin{bmatrix} L & L' & J \\ \mu & -\mu' & \mu'-\mu \end{bmatrix}\begin{bmatrix} l' & J & l \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} l' & J & l \\ j & s & j' \end{Bmatrix}\begin{Bmatrix} j & j' & J \\ L' & L & 1 \end{Bmatrix} . \tag{20}$$

Performing the sum over $\mu=\pm 1$, the part of $d\sigma/d\Omega$ proportional to the photon circular polarization Σ_c vanishes because of the symmetry relations (16) and the condition $l+l'+J=$even, following from

$$\begin{bmatrix} l' & J & l \\ 0 & 0 & 0 \end{bmatrix} .$$

Thus, we again recover Partovi's expression for $d\sigma/d\Omega$:

$$\begin{aligned}\frac{d\sigma}{d\Omega}&=I_0(\theta)+\Sigma_l\cos(2\Phi)I_1(\theta)\\ &=I_0(\theta)(1+\Sigma_l\Sigma(\theta)\cos(2\Phi)) ,\end{aligned} \tag{21}$$

where the second equation defines the asymmetry function $\Sigma(\theta)$. The angular distributions $I_0(\theta)$ and $I_1(\theta)$ have the following expansions in terms of Legendre polynomials and associated Legendre functions

$$\begin{aligned}I_0(\theta)&=\sum_{J=0} A_J P_J(\cos\theta) ,\\ I_1(\theta)&=\sum_{J=2} B_J P_J{}^2(\cos\theta) ,\end{aligned} \tag{22}$$

with

$$\begin{aligned}A_J&=\sum_{s\beta\beta'} W^{Js}_{\beta\beta'}(1,1)\cos(\delta^{j}_{\lambda}-\delta^{j'}_{\lambda'})[\delta_{N,\mathrm{even}}(\mathcal{T}^{L}_{\lambda j}\mathcal{T}^{L'}_{\lambda'j'}+\mathcal{S}^{L}_{\lambda j}\mathcal{S}^{L'}_{\lambda'j'})-\delta_{N,\mathrm{odd}}(\mathcal{T}^{L}_{\lambda j}\mathcal{S}^{L'}_{\lambda'j'}+\mathcal{S}^{L}_{\lambda j}\mathcal{T}^{L'}_{\lambda'j'})] ,\\ B_J&=\sum_{s\beta\beta'} W^{Js}_{\beta\beta'}(1,-1)\left[\frac{(J-2)!}{(J+2)!}\right]^{1/2}\cos(\delta^{j}_{\lambda}-\delta^{j'}_{\lambda'})\\ &\quad\times[\delta_{N,\mathrm{even}}(\mathcal{S}^{L}_{\lambda j}\mathcal{S}^{L'}_{\lambda'j'}-\mathcal{T}^{L}_{\lambda j}\mathcal{T}^{L'}_{\lambda'j'})+\delta_{N,\mathrm{odd}}(\mathcal{S}^{L}_{\lambda j}\mathcal{T}^{L'}_{\lambda'j'}-\mathcal{T}^{L}_{\lambda j}\mathcal{S}^{L'}_{\lambda'j'})] ,\end{aligned} \tag{23}$$

where $\delta_{N,\mathrm{even}}$, $\delta_{N,\mathrm{odd}}$ are shorthand notations for

$$\frac{(1+(-)^N)}{2}$$

and

$$\frac{(1-(-)^N)}{2} ,$$

respectively.

The total cross section and the zero degree differential cross section follow immediately from Eqs. (21) and (22) because of the normalization and the orthogonality properties of the Legendre functions

$$\begin{aligned}\sigma_{\mathrm{tot}}&=4\pi A_0 ,\\ \frac{d\sigma}{d\Omega}(0^\circ)&=\sum_J A_J .\end{aligned} \tag{24}$$

As a comment to our expressions for $d\sigma/d\Omega$, we may compare them with those given by Partovi.[3] As already recalled in the Introduction, the expressions for $I_0(\theta)$ and $I_1(\theta)$ in Ref. 3 are expansions in combinations of reduced rotation matrices of the form

$$d^{L}_{1,M}(\theta)d^{L'}_{1,M}(\theta)\pm d^{L}_{1,-M}(\theta)d^{l'}_{1,-M}(\theta)$$

and

$$(-)^M(d^{L}_{1,M}(\theta)d^{L'}_{1,-M}(\theta)\pm d^{L}_{1,-M}(\theta)d^{L'}_{1,M}(\theta)) ,$$

respectively, L and L' being the multipolarities of the transitions considered and $0\le M\le 2$. From these rather involved expansions, Partovi calculates the coefficients in the expansion of $I_0(\theta)$ and $I_1(\theta)$ in a series of circular functions

$$I_0(\theta)=a+b\sin^2\theta+c\cos\theta+d\sin^2\theta\cos\theta+e\sin^4\theta , \tag{25a}$$

$$I_1(\theta)=\sin^2\theta(f+g\cos\theta+h\sin^2\theta) , \tag{25b}$$

having truncated the multipole expansion to the dipole-octupole interferences. Incidentally, we may note that the dependence of $I_1(\theta)$ on $P_J{}^2(\cos\theta)$, [see Eq. (22)], makes clear the factor $\sin^2\theta$ in Eq. (25b).

Of course, Partovi does not give the expression connecting the coefficients $a,b,\ldots,$ and the reduced transition matrix elements $\mathscr{T}^L_{\lambda j}$, $\mathscr{S}^L_{\lambda j}$, because of their complexity. Now, besides pointing out the simplicity of our expansions (22) for $I_0(\theta)$ and $I_1(\theta)$ in Legendre functions, together with the explicit expressions (23) for the coefficients A_J and B_J, we would also like to emphasize that our expansions in orthogonal functions are the most appropriate for fitting the experimental results. In fact, as noted by various authors,[6] the coefficients of Partovi's expansion (25) cannot be determined unambiguously from the data. Indeed, the number of terms in a polynomial required to give a statistically valid fit to the data depends upon the number and distribution of data points, and the lack of forward and backward data leads to different and equally possible sets of parameters. On the contrary, expansion (22) allows the determination of a unique set of coefficients thanks to the orthogonality of the Legendre functions. In conclusion, the parameters experimentally derived without ambiguity are the coefficients A_J, B_J of our expansion and not the coefficients $a,b,c,\ldots,$ of Partovi's expansion. On the other hand, the last ones are easily connected to the first ones, through the expansion in Legendre functions of $\sin^n\theta\cos^m\theta$, and the formulas relating the two sets of coefficients are given in the Appendix.

Expressions (23) are rather transparent for picking out the main multipoles contributing to A_J and B_J. The case of A_0 is particularly important since it is directly related to the total cross section [see Eq. (24)]. Its expression in terms of $\mathscr{T}^L_{\lambda j}$ and $\mathscr{S}^L_{\lambda j}$ is immediately obtained putting $J=0$ in Eq. (20). It follows that $L'=L$, $j'=j$, $l'=l$, and then $N=$even in Eq. (23). It is a simple matter to obtain

$$\sigma_{\text{tot}}=\frac{16\pi^2}{3}\sum_{L\lambda j}\frac{1}{\hat{L}}[\,|\mathscr{T}^L_{\lambda j}|^2+|\mathscr{S}^L_{\lambda j}|^2]\,, \tag{26}$$

using the orthogonality of the U matrix,

$$\sum_{ls} U^j_{ls\lambda}U^j_{ls\lambda'}=\delta_{\lambda'\lambda}\,.$$

For the other coefficients, one has simply to read the formulas, remembering the decreasing importance of the various electric and magnetic multipoles with increasing L. The first contributions to A_1 deriving from $L=L'=1$ multipoles signify $N=$odd in Eq. (23), and then $E1-M1$ interferences. In the case $J=2$, the same $L=L'=1$ multipoles give $N=$even and then separated $E1$ and $M1$ contributions to A_2 and B_2. The first multipoles contributing to A_3, B_3, are $L=1$, $L'=2$ through the $E1-E2$ and $M1-M2$ interferences, and so on. Obviously, the exact percentage of importance of the first contributions thus identified in determining the value of the coefficients can be established only by the calculation.

III. POLARIZATIONS OF THE OUTGOING NUCLEONS

The polarization $(d\sigma/d\Omega)\vec{\mathrm{P}}(\alpha)$ of the outgoing nucleon $\alpha=1,2$ is given by

$$\frac{d\sigma}{d\Omega}\vec{\mathrm{P}}(\alpha)=\mathrm{Tr}(\vec{\sigma}_\alpha T\rho_{\text{in}}T^*)\,, \tag{27}$$

where $\vec{\sigma}_\alpha$ is twice the spin operator for the nucleon in the representation of the final np scattering states $|sm_s)$.

Explicating the trace in (27) we have, for the spherical component q of $\vec{\mathrm{P}}$

$$\frac{d\sigma}{d\Omega}P_q=\tfrac{1}{3}\sum_{\substack{sm_s s'm_s' m_d\\ \mu\mu'}}(s'm_s'\,|\,\sigma_q\,|\,sm_s)(sm_s\,|\,T\,|\,\mu m_d)(\mu\,|\,\rho_\gamma\,|\,\mu')(s'm_s'\,|\,T\,|\,\mu' m_d)^*\,. \tag{28}$$

With respect of the calculation of $d\sigma/d\Omega$, there are some complications, i.e., the two T-matrix elements involve different spin states, so that $M'\neq M$, and there is one more matrix element (that of σ_q).

Following the same procedure as above, we first use the Clebsch-Gordan series [Eq. (18)] and then we perform the sum over M,M',m_d in Eq. (28) by means of the same relation 6.2.8 of Edmonds.[5] Finally, the last summation over m_s,m_s' of the product of four $3-j$ symbols can be made with the sum rule obtained by de-Shalit,[7] resulting in

$$\frac{d\sigma}{d\Omega}P_q(1)=\sum_{\substack{J\alpha\alpha'\\ \mu\mu'}}X_J(\alpha,\alpha';q,\mu,\mu')D^J_{-q,\mu-\mu'}(R)e^{i(\delta^j_\lambda-\delta^{j'}_{\lambda'})}(-)^{N+F}(\mu\,|\,\rho_\gamma\,|\,\mu')(\mathscr{T}^L_{\lambda j}+\mu\mathscr{S}^L_{\lambda j})(\mathscr{T}^{L'}_{\lambda' j'}+\mu'\mathscr{S}^{L'}_{\lambda' j'})\,, \tag{29}$$

with

$$X_J(\alpha,\alpha';q,\mu,\mu')=\sum_x \frac{4\pi}{3}\hat{x}\hat{J}(6\hat{l}\hat{s}\hat{j}\hat{l}'\hat{s}'\hat{j}')^{1/2}U^{j}_{ls\lambda}U^{j'}_{l's'\lambda'}(-)^{l+s+j'} \times \begin{pmatrix} l' & l & x \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x & 1 & J \\ 0 & -q & q \end{pmatrix}\begin{pmatrix} L & L' & J \\ \mu & -\mu' & \mu'-\mu \end{pmatrix}\begin{Bmatrix} \frac{1}{2} & s & \frac{1}{2} \\ s' & \frac{1}{2} & 1 \end{Bmatrix}\begin{Bmatrix} j & j' & J \\ L' & L & 1 \end{Bmatrix}\begin{Bmatrix} l' & l & x \\ s' & s & 1 \\ j' & j & J \end{Bmatrix}, \quad (30)$$

where $F=l+l'+J$ and α and α' indicate the set of quantum numbers $(Ljl\lambda s)$ and $(L'j'l'\lambda's')$, respectively.

The x,y,z components of the polarization vector $\vec{P}$ easily follow from Eq. (29) and can be written in Partovi's form

$$\frac{d\sigma}{d\Omega}P_x(1)=\Sigma_l\sin(2\Phi)Q_{x2}(\theta)+\Sigma_c Q_{x3}(\theta)\,,$$

$$\frac{d\sigma}{d\Omega}P_y(1)=I_0(\theta)P_{y0}(\theta)+\Sigma_l\cos(2\Phi)Q_{y1}(\theta)\,,$$

$$\frac{d\sigma}{d\Omega}P_z(1)=\Sigma_l\sin(2\Phi)Q_{z2}(\theta)+\Sigma_c Q_{z3}(\theta)\,. \quad (31)$$

In deriving Eq. (31) use has been made of the symmetry relations (16) together with the selection rules coming from the $(3-j)$ symbols in Eq. (30). This is the cause of the annihilation of two terms, one proportional to Σ_c in P_y and one independent from the photon polarization in P_x and P_z.

The polarization functions in Eq. (31) are given by

$$I_0(\theta)P_{y0}(\theta)=\sum_{J=1}C_J P_J{}^1(\cos\theta)\,,$$

$$Q_{y1}(\theta)=\sum_{J=2}[D_J(d^J_{2,1}(\theta)-d^J_{2,-1}(\theta))+E_J(d^J_{2,1}(\theta)+d^J_{2,-1}(\theta))]\,,$$

$$Q_{x2}(\theta)=\sum_{J=2}[E_J(d^J_{2,1}(\theta)-d^J_{2,-1}(\theta))+D_J(d^J_{2,1}(\theta)+d^J_{2,-1}(\theta))]\,,$$

$$Q_{x3}(\theta)=\sum_{J=1}F_J P_J{}^1(\cos\theta)\,,$$

$$Q_{z2}(\theta)=\sum_{J=2}G_J P_J{}^2(\cos\theta)\,,$$

$$Q_{z3}(\theta)=\sum_{J=0}H_J P_J(\cos\theta)\,. \quad (32)$$

As is easily seen, not all the polarization functions can be expressed by means of Legendre functions. In fact, $Q_{x2}(\theta)$ and $Q_{y1}(\theta)$ are combinations of reduced rotation matrices, which, however, do not depend on the multipolarities L,L' of the transition involved.

The coefficients in (32) are the following:

$$C_J=\sum_{\alpha\alpha'}\left[\frac{2}{J(J+1)}\right]^{1/2}X_J(\alpha,\alpha';1,1,1)\sin(\delta^j_\lambda-\delta^{j'}_{\lambda'}) \times[\delta_{N,\text{odd}}(\mathcal{T}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'}+\mathcal{S}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'})-\delta_{N,\text{even}}(\mathcal{T}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'}+\mathcal{S}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'})]\,,$$

$$D_J=\sum_{\alpha\alpha'}\frac{1}{\sqrt{2}}X_J(\alpha,\alpha';1,1,-1)\sin(\delta^j_\lambda-\delta^{j'}_{\lambda'}) \times[\delta_{N,\text{even}}(\mathcal{T}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'}-\mathcal{S}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'})+\delta_{N,\text{odd}}(\mathcal{T}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'}-\mathcal{S}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'})]\,,$$

$$E_J=\sum_{\alpha\alpha'}\frac{1}{\sqrt{2}}X_J(\alpha,\alpha';1,1,-1)\sin(\delta^j_\lambda-\delta^{j'}_{\lambda'}) \times[\delta_{N,\text{odd}}(\mathcal{T}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'}-\mathcal{S}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'})+\delta_{N,\text{even}}(\mathcal{T}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'}-\mathcal{S}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'})]\,,$$

$$F_j=\sum_{\alpha\alpha'}\left[\frac{2}{J(J+1)}\right]^{1/2}X_J(\alpha,\alpha';1,1,1)\cos(\delta^j_\lambda-\delta^{j'}_{\lambda'}) \times[\delta_{N,\text{even}}(\mathcal{T}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'}+\mathcal{S}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'})-\delta_{N,\text{odd}}(\mathcal{T}^L_{\lambda j}\mathcal{T}^{L'}_{\lambda' j'}+\mathcal{S}^L_{\lambda j}\mathcal{S}^{L'}_{\lambda' j'})]\,, \quad (33)$$

$$G_J=\sum_{\alpha\alpha'}\left[\frac{(J-2)!}{(J+2)!}\right]^{1/2} X_J(\alpha,\alpha';0,1,-1)\sin(\delta_\lambda^j-\delta_{\lambda'}^{j'})$$
$$\times[\delta_{N,\mathrm{odd}}(\mathscr{T}_{\lambda j}^L\mathscr{T}_{\lambda' j'}^{L'}-\mathscr{S}_{\lambda j}^L\mathscr{S}_{\lambda' j'}^{L'})+\delta_{N,\mathrm{even}}(\mathscr{T}_{\lambda j}^L\mathscr{S}_{\lambda' j'}^{L'}-\mathscr{S}_{\lambda j}^L\mathscr{T}_{\lambda' j'}^{L'})]\ ,$$

$$H_J=\sum_{\alpha\alpha'}X_J(\alpha,\alpha';0,1,1)\cos(\delta_\lambda^j-\delta_{\lambda'}^{j'})$$
$$\times[\delta_{N,\mathrm{even}}(\mathscr{T}_{\lambda j}^L\mathscr{S}_{\lambda' j'}^{L'}+\mathscr{S}_{\lambda j}^L\mathscr{T}_{\lambda' j'}^{L'})-\delta_{N,\mathrm{odd}}(\mathscr{T}_{\lambda j}^L\mathscr{T}_{\lambda' j'}^{L'}+\mathscr{S}_{\lambda j}^L\mathscr{S}_{\lambda' j'}^{L'})]\ .$$

Having chosen the y-axis coincident with $\vec{q}\times\vec{k}$, P_y is the only component of the ejected nucleon polarization, for an unpolarized photon beam, because of parity conservation. It is worth noting that the surviving term $I_0(\theta)P_{y0}(\theta)$ is simply expressed in (32) as an expansion in associated Legendre functions $P_J{}^1(\cos\theta)$. For the sake of comparison, let us recall that this polarization function is given by Partovi as an expansion in terms of the polarization patterns

$$d_{1,-M}^L(\theta)d_{1,1-M}^{L'}(\theta)\pm d_{1,M}^L(\theta)d_{1,M-1}^{L'}(\theta)\ ,$$

from which he draws the coefficients of the expression

$$I_0(\theta)P_{y0}(\theta)=\sin\theta(i+j\cos\theta+k\sin^2\theta+l\sin^2\theta\cos\theta)\ . \qquad (34)$$

Here, the $\sin\theta$ term is an immediate consequence of the expansion in $P_J{}^1(\cos\theta)$. As before, this form results because of the inclusion of the em multipoles up to the dipole-octupole interferences. Again, the formulas relating Partovi's parameters to our C_J coefficients are given in the Appendix.

Dealing with expression (33) of C_J as above for A_J, B_J, one easily sees that the first and fundamental contributions to C_1 are given by the $E1-M1$ interferences. In addition, since C_J is proportional to $\sin(\delta_\lambda^j-\delta_{\lambda'}^{j'})$, only transitions to different final states contribute.

IV. CONCLUSIONS

In this paper we have considered the disintegration of unpolarized deuterons by polarized photons below the pion production threshold, using the standard theory of the nuclear em interaction (first order perturbation theory with expansion in multipoles) and of the nuclear interaction (Schrödinger equation), with the aim of obtaining expressions for the measurable quantities in a way easier to handle than that of Partovi.[3] Indeed, by summing over the magnetic quantum numbers and exploiting the symmetry relations due to the time-reversal invariance and parity conservation, we have naturally arrived at expansions in series of Legendre functions for $d\sigma/d\Omega$ and for the nucleon polarization with unpolarized photons. All the coefficients of these expansions are explicitly given in terms of the reduced matrix elements of the em multipoles.

Our formulas are simpler and more useful than earlier results.[3] As expansions in orthogonal functions, they are particularly suitable for comparing with experimental data. Thus, our coefficients A_J, B_J, and C_J follow from experimental results with fewer uncertainties than the coefficients $a,b,c,\ldots,$ of the expansions of $d\sigma/d\Omega$ and $I_0(\theta)P_{y0}(\theta)$ in circular functions.

When the photon beam has arbitrary polarization, five additional functions of θ are necessary for determining the nucleon polarization. Three of these are expressible by means of Legendre functions, while for the other two the reduced rotation matrices are the natural basis.

APPENDIX: COMPARISON BETWEEN THE EXPANSIONS OF $d\sigma/d\Omega$ AND $I_0(\theta)P_{y0}(\theta)$ IN CIRCULAR AND LEGENDRE FUNCTIONS

Since Partovi's coefficients $a,b,c,\ldots,$ of the expansions of $I_0(\theta)$, $I_1(\theta)$, and $I_0(\theta)P_{y0}(\theta)$ in circular functions are largely used in the literature, we derive below formulas connecting them to the coefficients A_J, B_J, and C_J of our expansions in Legendre functions.

As for the differential cross section for unpolarized photons, it is convenient to start by rewriting the two expansions of $I_0(\theta)$

$$\sum_{J=0}A_JP_J(\cos\theta)=\sum_{n=0}(a_n+b_n\cos\theta)\sin^2\theta\ , \qquad (A1)$$

the right-hand side being the obvious generalization of Eq. (25a). One has just to exploit the relations

$$P_{2n}(x)=\frac{n!}{(2n-1)!!}\sum_{k=0}^{n}\frac{(-)^k}{(n-k)!}\frac{[2(n+k)-1]!!}{2^k(k!)^2}(1-x^2)^k\,,$$
$$P_{2n+1}(x)=\frac{n!}{(2n+1)!!}\sum_{k=0}^{n}\frac{(-)^k}{(n-k)!}\frac{[2(n+k)+1]!!}{2^k(k!)^2}x(1-x^2)^k\,, \tag{A2}$$

for getting the general relations

$$a_n=\frac{(-)^n}{2^n(n!)^2}\sum_{k=n}\frac{k!}{(k-n)!}\frac{[2(k+n)-1]!!}{(2k+1)!!}(2k+1)A_{2k}\,,$$
$$b_n=\frac{(-)^n}{2^n(n!)^2}\sum_{k=n}\frac{k!}{(k-n)!}\frac{[2(k+n)+1]!!}{(2k+1)!!}A_{2k+1}\,. \tag{A3}$$

To be explicit, the first coefficients of Partovi's expansion are given by

$$a\equiv a_0=\sum_{k=0}A_{2k}\,,$$
$$b\equiv a_1=-\tfrac{1}{2}(3A_2+10A_4+21A_6+36A_8+55A_{10}+\cdots)\,,$$
$$e\equiv a_2=\tfrac{1}{8}(35A_4+189A_6+594A_8+1430A_{10}+\cdots)\,, \tag{A4}$$
$$c\equiv b_0=\sum_{k=0}A_{2k+1}\,,$$
$$d\equiv b_1=-\tfrac{1}{2}(5A_3+14A_5+27A_7+44A_9+65A_{11}+\cdots)\,.$$

The simplest way for deriving the other relations we are interested in is to exploit the definition of $P_J^m(\cos\theta)$ in terms of Legendre polynomials. Indeed, applying the operator

$$\sin\theta\frac{d}{d(\cos\theta)}$$

to (A1), we obtain

$$\sum_{J=1}A_JP_J{}^1(\cos\theta)=\sin\theta\sum_{n=0}[(2n+1)b_n-2(n+1)b_{n+1}-2(n+1)a_{n+1}\cos\theta]\sin^{2n}\theta\,, \tag{A5}$$

which has to be compared with the two expressions of $I_0(\theta)P_{y0}(\theta)$

$$\sum_{J=1}C_JP_J{}^1(\cos\theta)=\sin\theta\sum_{n=0}[i_n+j_n\cos\theta]\sin^{2n}\theta\,. \tag{A6}$$

Here we have written the general form of the expansion (34) for the polarization. Thus, the coefficients i_n, j_n are given by

$$i_n=(2n+1)b_n-2(n+1)b_{n+1}\,,$$
$$j_n=-2(n+1)a_{n+1}\,, \tag{A7}$$

where a_n, b_n are those in (A3) with the substitution $A_J\to C_J$. Explicitly, we have

$$i_n=\frac{(-)^n}{2^nn!}\frac{1}{(n+1)!}\sum_{k=n}\frac{(k+1)!}{(k-n)!}\frac{[2(k+n)+1]!!}{(2k+1)!!}(2k+1)C_{2k+1}\,,$$
$$j_n=\frac{(-)^n}{2^nn!}\frac{1}{(n+1)!}\sum_{k=n+1}\frac{k!}{(k-n-1)!}\frac{[2(k+n)+1]!!}{(2k-1)!!}C_{2k}\,, \tag{A8}$$

and the first relations read

$$\begin{aligned}
i &\equiv i_0 = C_1 + 6C_3 + 15C_5 + 28C_7 + 45C_9 + 66C_{11} + \cdots \,, \\
j &\equiv j_0 = 3C_2 + 10C_4 + 21C_6 + 36C_8 + 55C_{10} + \cdots \,, \\
k &\equiv i_1 = -\tfrac{3}{2}(5C_3 + 35C_5 + 126C_7 + 330C_9 + 715C_{11} + \cdots) \,, \\
l &\equiv j_1 = -\tfrac{1}{2}(35C_4 + 189C_6 + 594C_8 + 1430C_{10} + \cdots) \,.
\end{aligned} \tag{A9}$$

Deriving once again (A6), it is a simple matter to get

$$\sum_{J=2} C_J P_J{}^2(\cos\theta) = \sin^2\theta \sum_{n=0} [(2n+1)j_n - 2(n+1)j_{n+1} - 2(n+1)i_{n+1}\cos\theta]\sin^{2n}\theta \,. \tag{A10}$$

On the other hand, the expansions of $I_1(\theta)$ are

$$\sum_J B_J P_J{}^2(\cos\theta) = \sin^2\theta \sum_{n=0} (f_n + g_n \cos\theta)\sin^{2n}\theta \,, \tag{A11}$$

and thus

$$\begin{aligned}
f_n &= (2n+1)j_n - 2(n+1)j_{n+1} \,, \\
g_n &= -2(n+1)i_{n+1} \,,
\end{aligned} \tag{A12}$$

the coefficients i_n, j_n being given by (A8) with the substitution $C_J \to B_J$. The relations valid for every n follow as

$$\begin{aligned}
f_n &= \frac{(-)^n}{2^n n!} \frac{1}{(n+2)!} \sum_{k=n+1} \frac{(k+1)!}{(k-n-1)!} \frac{[2(k+n)+1]!!}{(2k-1)!!}(2k-1)B_{2k} \,, \\
g_n &= \frac{(-)^n}{2^n n!} \frac{1}{(n+2)!} \sum_{k=n+1} \frac{(k+1)!}{(k-n-1)!} \frac{[2(k+n)+3]!!}{(2k-1)!!} B_{2k+1} \,,
\end{aligned} \tag{A13}$$

the first of which are

$$\begin{aligned}
f &\equiv f_0 = 3(B_2 + 15B_4 + 70B_6 + 210B_8 + 495B_{10} + \cdots) \,, \\
g &\equiv g_0 = 3(5B_3 + 35B_5 + 126B_7 + 330B_9 + 715B_{11} + \cdots) \,, \\
h &\equiv f_1 = -\tfrac{3}{2}(35B_4 + 420B_6 + 2310B_8 + 8580B_{10} + \cdots) \,.
\end{aligned} \tag{A14}$$

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