PHYSICAL REVIEW C VOLUME 26, NUMBER 5 NOVEMBER 1982

Nuclear information from the proton spin observables in the (p,p') reactions at intermediate energies

E. Bleszynski,[a] M. Bleszynski,[a),(b] and C. A. Whitten, Jr.[a]
[a]*Department of Physics, University of California, Los Angeles, California 90024*
[b]*Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
(Received 17 May 1982)

A set of proton spin observables D_ξ, $\xi=0$, x, y, and z, is described which provides selective information on the tensor components of the collision matrix describing the (p,p') reaction. These D_ξ are related linearly to the experimentally determined components of the depolarization tensor. Simple relations are derived between the D_ξ and both the spin components of the nucleon-nucleon amplitudes and the nuclear multipole form factors. For unnatural parity transitions the observable D_x is related to the axial longitudinal form factor which does not enter in the (e,e') transition amplitudes for the one photon exchange mechanism.

[NUCLEAR REACTIONS New polarization parameters in (p,p') reactions.]

I. INTRODUCTION

With the recent developments in polarized proton beams and in high resolution spectrometers with focal plane polarimeters for intermediate energy (~ 1 GeV) proton-nucleus reaction studies, it has become possible to study experimentally elastic and inelastic scattering processes in which proton beams polarized in an arbitrary orientation are utilized and in which all the components of the polarization for the scattered protons are determined.[1] One of the motivations for the growing interest in these studies is that the theoretical description of the proton-nucleus interaction simplifies considerably at these energies since the projectile wavelength is small compared to the nuclear size. The interaction of the projectile with the target can therefore be described in terms of sequences of collisions with individual target nucleons. In particular, the Glauber multiple scattering theory,[2] as well as its extended version corrected for finite energy effects,[3] provides a transparent, parameter-free prescription for the construction of the transition amplitudes in terms of the (free) nucleon-nucleon (*NN*) amplitudes and the nuclear elastic or inelastic transition densities. Considerable efforts have also been made to develop the distorted-wave impulse approximation treatment in terms of potentials related to the free *NN* amplitudes.[4]

Until quite recently little attention has been given to the investigation of what new information on the reaction mechanism and on the structure of the nuclear states involved can be extracted from measurements of the spin observables in double scattering experiments at intermediate energies. The only theoretical studies have been concerned with polarization observables in the elastic scattering of protons from spin zero nuclei[5] and from deuterium[6]; and two related experiments have already been completed.[1,7]

In the experimental conditions where the incident and the scattered proton polarizations can be determined the maximum information on the scattering matrix describing the $(\vec{p},\vec{p}\,')$ reaction can be obtained by measuring all the Wolfenstein parameters.[8] However, although the Wolfenstein parameters are related in a direct way to the experimental measurements, they are not related in a transparent way to the (p,p') collision matrix. The purpose of this paper is to discuss a different parametrization of the experimental results, alternative to the conventional Wolfenstein parameters. We introduce a particular convenient set of observables D_0, D_x, D_y, and D_z which depend separately on the specific tensor components of the (p,p') collision matrix. These observables can be expressed in terms of linear combinations of the Wolfenstein parameters. As will be discussed below, the advantage of using this parametrization is that it displays clearly the sensitivity of the $(\vec{p},\vec{p}\,')$ reaction to the different ingredients in the reaction theory since the individual D_ξ are related to particular components of the NN amplitude and to particular nuclear multipole form factors. In particular, the observable D_x is related to the longitudinal part of the axial form factor which does not enter either into (e,e') transitions proceeding via the one photon exchange mechanism nor in (π,π') reactions.

In Sec. II the D_ξ observables are defined in terms of components of the general collision matrix and in terms of the experimental Wolfenstein parameters. In Sec. III expressions are derived for the D_ξ observables in the single collision approximation which display the dependence of the individual D_ξ on particular components of the NN amplitude and on particular nuclear form factors. Specific examples of the usefulness of the D_ξ observables are described and discussed in Sec. IV. Some of the results discussed here have already been reported in an abbreviated form.[9]

II. THE D_ξ OBSERVABLES

Let us consider the scattering process

$$p(\tfrac{1}{2})+A(J_i^{\pi_i})\to p(\tfrac{1}{2})+A^*(J_f^{\pi_f}) , \tag{2.1}$$

where p is a spin $\frac{1}{2}$ projectile (a proton in the examples we will be discussing in this paper) and A and A^* are the initial and final states of the target nucleus with spins (parities) $J_i(\pi_i)$ and $J_f(\pi_f)$, respectively. By using rotational invariance the general expression for the collision matrix of this reaction may be written as

$$F=F_0\sigma_0+F_x\sigma_x+F_y\sigma_y+F_z\sigma_z , \tag{2.2}$$

where $\sigma_0=1$ and σ_ξ, $\xi=x$, y, and z, are the Pauli spin matrices for the proton in the right-handed Cartesian frame where the x, y, and z axes are parallel to $(\vec{k}_f-\vec{k}_i)$, $\vec{k}_i\times\vec{k}_f$, and $\vec{k}_i+\vec{k}_f$, respectively, $\vec{k}_i$ and $\vec{k}_f$ being the initial and final momenta of the projectile in the center of mass (c.m.) frame. The F_ξ are $(2J_f+1)\times(2J_i+1)$ matrices connecting the spin spaces of the initial and final target states with elements

$$(F_\xi)_{M_fM_i}=\langle J_fM_f\,|\,\hat{F}_\xi\,|\,J_iM_i\rangle , \tag{2.3}$$

where $\hat{F}_\xi$ is a transition operator in the space of nuclear coordinates. Up to an arbitrary phase the complete determination of all the amplitudes can be achieved by performing a sufficient number of polarization measurements. Some of these measurements would require the determination of spin alignments for the nuclear target; and the requisite experiments are in general very difficult. Therefore we focus our attention here on the experimentally feasible scattering measurements, those in which one can analyze the spin projections of the incident and scattered proton, averaging over the magnetic substates of the target. In other words we analyze those reaction processes where a proton of known polarization scatters from an unpolarized target nucleus and the polarization of the scattered proton is measured.

In the situation described above, the components of the polarization vectors $\vec{P}^i$ and $\vec{P}^f$ for the respective incident and scattered protons are related through the equations:

$$[1+D_{0y}P_y^i]P_x^f=D_{xx}P_x^i+D_{xz}P_z^i , \tag{2.4a}$$

$$[1+D_{0y}P_y^i]P_y^f=D_{y0}+D_{yy}P_y^i , \tag{2.4b}$$

$$[1+D_{0y}P_y^i]P_z^f=D_{zx}P_x^i+D_{zz}P_z^i , \tag{2.4c}$$

where

$$D_{nm}(\vec{k}_i,\vec{k}_f)=\frac{\mathrm{Tr}(F\sigma_mF^\dagger\sigma_n)}{\mathrm{Tr}(FF^\dagger)} . \tag{2.5}$$

The trace in Eq. (2.5) is taken over the spin projections of both the projectile and the target. From the conservation of parity it follows that the only nonvanishing functions D_{nm} are the following: $D_{00}\equiv1$, D_{0y} (analyzing power), D_{y0} (polarization), D_{xx}, D_{yy}, D_{zz}, D_{zx}, and D_{xz}. The differential cross section, I_0, for the scattering of an unpolarized proton beam is given by:

$$I_0=\frac{\mathrm{Tr}FF^\dagger}{2(2J_i+1)} . \tag{2.6}$$

Now if we insert Eq. (2.2) into Eqs. (2.5) and (2.6) and evaluate explicitly the trace over the projections of the spin-$\frac{1}{2}$ projectile proton we obtain expressions for the D_{nm} and I_0 in terms of linear combinations of the functions

$$\mathrm{Tr}'(F_\xi F_\eta^\dagger) , \tag{2.7}$$

where the trace (primed symbol) is now taken over the spin projections of the target nucleus alone. In particular for I_0, D_{xx}, D_{yy}, and D_{zz} we then obtain:

$$I_0=\left[\frac{1}{(2J_i+1)}\right]\mathrm{Tr}'(F_0F_0^\dagger+F_xF_x^\dagger+F_yF_y^\dagger+F_zF_z^\dagger) , \tag{2.8a}$$

$$D_{xx}=\left[\frac{1}{(2J_i+1)I_0}\right]\mathrm{Tr}'(F_0F_0^\dagger+F_xF_x^\dagger-F_yF_y^\dagger-F_zF_z^\dagger) , \tag{2.8b}$$

$$D_{yy} = \left[\frac{1}{(2J_i+1)I_0} \right] \mathrm{Tr}'(F_0F_0^\dagger - F_xF_x^\dagger + F_yF_y^\dagger - F_zF_z^\dagger) , \tag{2.8c}$$

$$D_{zz} = \left[\frac{1}{(2J_i+1)I_0} \right] \mathrm{Tr}'(F_0F_0^\dagger - F_xF_x^\dagger - F_yF_y^\dagger + F_zF_z^\dagger) . \tag{2.8d}$$

Since the observables in Eqs. (2.8) are expressed in terms of functions [Eq. (2.7)] where $\xi = \eta$ it is both natural and useful to define the observables D_ξ by:

$$D_\xi = \left[\frac{1}{(2J_i+1)I_0} \right] \mathrm{Tr}'(F_\xi F_\xi^\dagger) = \left[\frac{1}{(2J_i+1)I_0} \right] \times \sum_{M_i,M_f} | \langle J_fM_f | \hat{F}_\xi | J_iM_i \rangle |^2 , \tag{2.9}$$

where $\xi = 0$, x, y, and z.

Using Eqs. (2.8) and (2.9) we can express the D_ξ in terms of D_{xx}, D_{yy}, and D_{zz}:

$$D_0 = \tfrac{1}{4}[1 + D_{xx} + D_{yy} + D_{zz}] , \tag{2.10a}$$

$$D_x = \tfrac{1}{4}[1 + D_{xx} - D_{yy} - D_{zz}] , \tag{2.10b}$$

$$D_y = \tfrac{1}{4}[1 - D_{xx} + D_{yy} - D_{zz}] , \tag{2.10c}$$

$$D_z = \tfrac{1}{4}[1 - D_{xx} - D_{yy} + D_{zz}] . \tag{2.10d}$$

From their definition [Eqs. (2.9)] it follows that the D_ξ are not independent:

$$D_0 + D_x + D_y + D_z = 1 ; \tag{2.11}$$

and that the D_ξ are non-negative:

$$0 \le D_\xi \le 1 . \tag{2.12}$$

In the usual experimental setup for $(\vec{p},\vec{p}\,')$ scattering the quantities D_{xx}, D_{yy}, and D_{zz} are not measured directly. The components of the polarization vector for the incident proton beam are expressed in terms of the directions $\vec{S}$, $\vec{N}$, and $\vec{L}$, where $\vec{L}$ is along the incident beam direction, $\vec{K}_i$, and $\vec{N}$ is along the direction $\vec{K}_i \times \vec{K}_f$. $\vec{K}_i$ and $\vec{K}_f$ are the laboratory momenta of the incident and scattered proton, respectively. The components of the polarization vector for the scattered proton are measured along the directions $\vec{S}\,'$, $\vec{N}\,'$, and $\vec{L}\,'$, where $\vec{L}\,'$ is along the scattered beam direction, $\vec{K}_f$, and $\vec{N}\,'$ is along the direction $\vec{K}_i \times \vec{K}_f$. (S,N,L) and (S',N',L') represent right-handed Cartesian frames. The geometrical relation between these frames and the (x,y,z) frame used in our discussion is shown in Fig. 1. For simplicity we use here the approximate relations $\theta = \theta_{\mathrm{lab}} = \theta_{\mathrm{c.m.}}$ which are strictly valid in the limits (projectile mass)/(target mass)$\to 0$, and (target excitation energy)/(projectile kinetic energy)$\to 0$.

In terms of the $(\vec{S},\vec{N},\vec{L})$ and $(\vec{S}\,',\vec{N}\,',\vec{L}\,')$ frames the relations between the components of the polarization vectors $\vec{P}$ and $\vec{P}\,'$ for the incident and scattered protons are given by:

$$[1 + D_{NO}P_N]P_{S'} = D_{SS'}P_S + D_{LS'}P_L , \tag{2.13a}$$

$$[1 + D_{NO}P_N]P_{N'} = D_{ON} + D_{NN'}P_N , \tag{2.13b}$$

$$[1 + D_{NO}P_N]P_{L'} = D_{SL'}P_S + D_{LL'}P_L . \tag{2.13c}$$

In the usual $(\vec{p},\vec{p}\,')$ experiment the measured depolarization parameters are $D_{SS'}(R)$, $D_{NN'}(D)$, $D_{LL'}(A')$, $D_{LS'}(A)$, and $D_{SL'}(R')$, where the symbols in parentheses are the conventional Wolfenstein parameters.[8] Equations (2.4) and (2.13) and the geometrical relations represented by Fig. 1 can be used to express the D_ξ defined by Eq. (2.9) in terms of the experimentally measured depolarization parameters:

$$D_0 = \tfrac{1}{4}[1 + D_{NN'} + (D_{SS'} + D_{LL'})\cos\theta - (D_{LS'} - D_{SL'})\sin\theta] , \tag{2.14a}$$

$$D_x = \tfrac{1}{4}[1 + D_{SS'} - D_{NN'} - D_{LL'}] , \tag{2.14b}$$

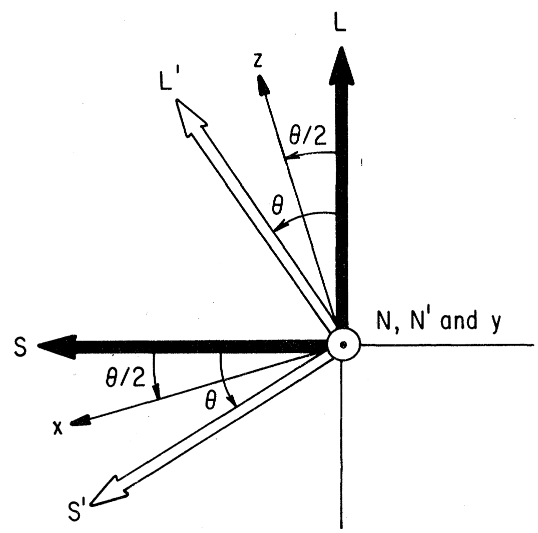

FIG. 1. Geometrical relation between the coordinate frames (S,N,L), (S',N',L'), and (x,y,z) for the case where (projectile mass)/(target mass)$\to 0$, or $\theta = \theta_{\mathrm{lab}} = \theta_{\mathrm{c.m.}}$. $N \equiv N' \equiv y$.

$$D_y=\tfrac{1}{4}[1+D_{NN'}-(D_{SS'}+D_{LL'})\cos\theta +(D_{LS'}-D_{SL'})\sin\theta] , \quad (2.14c)$$

$$D_z=\tfrac{1}{4}[1-D_{SS'}-D_{NN'}+D_{LL'}] . \quad (2.14d)$$

The selective dependence of the observables D_ξ on the components F_ξ of the collision matrix F can be very useful in the theoretical interpretation of the cross section and polarization data. Suppose, for example, that the reaction theory correctly predicts the three components F_0, F_y, and F_z, but fails to describe the F_x component. As a consequence the experimental cross section and Wolfenstein parameter data would be incorrectly predicted and it would be a difficult task to track down the error in the reaction theory. On the other hand, if one would compare the quantities $I_0^{\text{exp}}D_\xi^{\text{exp}}$ with the predictions of the reaction theory, then, in our hypothetical example, the theoretical difficulties could be easily traced to the miscalculation of F_x. Also, as we shall discuss, each D_ξ depends on different components of the NN amplitudes and on different nuclear multipole form factors which enter into the reaction theory calculation.

III. THE D_ξ OBSERVABLES IN THE SINGLE COLLISION APPROXIMATION

In order to gain some simple physical insight into the dependence of the parameters D_ξ on the NN amplitudes and the nuclear wave functions, it is instructive to derive expressions for the component matrices F_ξ of the general scattering matrix F [Eq. (2.2)] in the case where only the single collision term in the Glauber multiple scattering series is evaluated. These particular matrices will be denoted by $F_\xi^{(1)}$, $\xi=0$, x, y, and z. The transition amplitude between two nuclear states $|i\rangle$ and $|f\rangle$ can be written in the single collision approximation as:

$$\langle f|\hat{F}^{(1)}|i\rangle=A\langle f|e^{i\vec{q}\cdot\vec{r}}\hat{f}_{NN}(q)|i\rangle , \quad (3.1)$$

where $\hat{f}_{NN}(q)$ is the NN collision operator, $\vec{r}$ is the coordinate of a target nucleon (the nuclear wave functions are antisymmetrized), $\vec{q}$ is the momentum transferred, $\vec{k}_i-\vec{k}_f$, and A is the number of nucleons in the target nucleus. To derive the component matrices $F_0^{(1)}$, $F_x^{(1)}$, $F_y^{(1)}$, and $F_z^{(1)}$ of Eq. (2.2) the NN scattering operator is decomposed in terms of the projectile spin operators

$$\hat{f}_{NN}(q)=\hat{f}_0(q)\sigma_0+\hat{f}_x(q)\sigma_x +\hat{f}_y(q)\sigma_y+\hat{f}_z(q)\sigma_z , \quad (3.2)$$

where

$$\hat{f}_0(q)=\hat{\alpha}(q)\sigma_0^t+i\hat{\gamma}(q)\sigma_y^t , \quad (3.3a)$$

$$\hat{f}_x(q)=\hat{\delta}(q)\sigma_x^t , \quad (3.3b)$$

$$\hat{f}_y(q)=i\hat{\gamma}(q)\sigma_0^t+\hat{\beta}(q)\sigma_y^t , \quad (3.3c)$$

$$\hat{f}_z(q)=\hat{\epsilon}(q)\sigma_z^t . \quad (3.3d)$$

The σ_ξ^t are target nucleon spin operators; and the $\hat{\alpha}(q)$, $\hat{\beta}(q)$, $\hat{\gamma}(q)$, $\hat{\delta}(q)$, and $\hat{\epsilon}(q)$ are the conventional components of the NN collision matrix. These components can be further decomposed in terms of isoscalar and isovector contributions; for example,

$$\hat{\alpha}(q)=\alpha_0(q)1+\alpha_1(q)\vec{\tau}\cdot\vec{\tau}^{\,t} , \quad (3.4)$$

where $\vec{\tau}$ and $\vec{\tau}^{\,t}$ are the isospin operators for the projectile and target nucleon, respectively, with similar expressions for $\hat{\beta}(q)$, $\hat{\gamma}(q)$, $\hat{\delta}(q)$, and $\hat{\epsilon}(q)$. For simplicity we will suppress the isospin indices in the following discussion. By using Eq. (3.3) we can express the $\hat{F}_\xi^{(1)}$ operators as

$$\hat{F}_\xi^{(1)}(q)=A\int d^3r\, \hat{\rho}(\vec{r})e^{i\vec{q}\cdot\vec{r}}\hat{f}_\xi(q) , \quad \xi=0,\ x,\ y,\text{ and } z , \quad (3.5)$$

where $\hat{\rho}(\vec{r})$ is the single particle density operator.

At this stage it is convenient to perform a multipole expansion of the operators $\hat{F}_\xi^{(1)}$ in the manner analogous to the treatment of the operators entering into the Hamiltonians describing semileptonic and weak interactions in nuclei.[10,11] By using the expansions for the plane waves $e^{i\vec{q}\cdot\vec{r}}$, $\sigma_x^te^{i\vec{q}\cdot\vec{r}}$, $\sigma_y^te^{i\vec{q}\cdot\vec{r}}$, and $\sigma_z^te^{i\vec{q}\cdot\vec{r}}$ in the Cartesian frame of reference defined in Sec. II (see the Appendix), the following expressions are obtained for the Fourier transforms of the density, $\hat{\rho}(\vec{r})$, and spin density, $\hat{\rho}(\vec{r})\sigma^t$, operators entering in the formulas for the $\hat{F}_\xi^{(1)}(q)$:

$$\int d^3r\, e^{i\vec{q}\cdot\vec{r}}\hat{\rho}(\vec{r})\sigma_0^t=\sum_{L,M}c_{LM}^{(0)}\hat{\mathscr{M}}_{LM}(q) , \quad (3.6a)$$

$$\int d^3r e^{i\vec{q}\cdot\vec{r}}\hat{\rho}(\vec{r})\sigma_x^t=-\sum_{L,M}c_{LM}^{(0)}\hat{\mathscr{L}}_{LM}{}^5(q) , \quad (3.6b)$$

$$\int d^3r e^{i\vec{q}\cdot\vec{r}}\hat{\rho}(\vec{r})\sigma_y^t=-i\sum_{L,M}[c_{LM}^{(-)}\hat{T}_{LM}^{\text{mag}5}(q)+c_{LM}^{(+)}\hat{T}_{LM}^{\text{el}5}(q)] , \quad (3.6c)$$

$$\int d^3r e^{i\vec{q}\cdot\vec{r}}\hat{\rho}(\vec{r})\sigma_z^t = \sum_{L,M} [c_{LM}^{(+)}\hat{T}_{LM}^{\text{mag5}}(q)+c_{LM}^{(-)}\hat{T}_{LM}^{\text{el5}}(q)] \ . \tag{3.6d}$$

In the above formulas $\hat{\mathscr{M}}_{LM}$, $\hat{\mathscr{L}}_{LM}{}^5$, $\hat{T}_{LM}^{\text{mag5}}$, and $\hat{T}_{LM}^{\text{el5}}$ are the Coulomb, axial longitudinal, axial transverse magnetic, and axial transverse electric multipole operators.[10,11] These are:

$$\hat{\mathscr{M}}_{LM}(q)=\int d^3r\ \hat{\rho}(\vec{r})\Phi_{LM}(q,\vec{r}) \ , \tag{3.7a}$$

$$\hat{\mathscr{L}}_{LM}{}^5(q)=\frac{i}{q}\int d^3r\ \hat{\rho}(\vec{r})\{\nabla_{\vec{r}}\Phi_{LM}(q,\vec{r})\}\cdot\vec{\sigma}^t \ , \tag{3.7b}$$

$$\hat{T}_{LM}^{\text{mag5}}(q)=\frac{1}{i[L(L+1)]^{1/2}}\int d^3r\ \hat{\rho}(\vec{r})\{\vec{r}\times\nabla_{\vec{r}}\Phi_{LM}(q,\vec{r})\}\cdot\vec{\sigma}^t \ , \tag{3.7c}$$

$$\hat{T}_{LM}^{\text{el5}}(q)=\frac{1}{iq[L(L+1)]^{1/2}}\int d^3r\ \hat{\rho}(\vec{r})\{\nabla_{\vec{r}}\times(\vec{r}\times\nabla_{\vec{r}})\Phi_{LM}(q,\vec{r})\}\cdot\vec{\sigma}^t \ , \tag{3.7d}$$

where

$$\Phi_{LM}(q,\vec{r})=j_L(qr)Y_{LM}(\hat{r}) \ . \tag{3.8}$$

The constants $c_{LM}^{(\alpha)}$, $\alpha=0$, $+$, and $-$, in Eqs. (3.6) are given by:

$$c_{LM}^{(0)}=i^L[4\pi(2L+1)]^{1/2}d_{MO}^L(\pi/2) \ , \tag{3.9a}$$

$$c_{LM}^{(\pm)}=i^L[4\pi(2L+1)]^{1/2}\{d_{M1}^L(\pi/2)\pm d_{M-1}^L(\pi/2)\}/2 \ , \tag{3.9b}$$

where the $d_{MM'}^L$ are components of the matrices describing rotations about the y axis.

Since the nuclear states have definite parity and the operators $\hat{\mathscr{M}}$ and $\hat{T}^{\text{mag5}}$ have parity $(-1)^L$ while the parity of the operators $\hat{T}^{\text{el5}}$ and $\hat{\mathscr{L}}^5$ is $-(-)^L$, at most only a pair of these operators will enter into a nuclear transition matrix element with given L,M. By inserting Eqs. (3.6) into Eq. (3.5) we find that the natural parity transitions corresponding to the transfer L,M are described by the operators

$$\{\hat{F}_0^{(1)}(q)\}_{LM}=\alpha(q)Ac_{LM}^{(0)}\hat{\mathscr{M}}_{LM}(q) + \gamma(q)Ac_{LM}^{(-)}\hat{T}_{LM}^{\text{mag5}}(q) \ , \tag{3.10a}$$

$$\{\hat{F}_x^{(1)}(q)\}_{LM}=0 \ , \tag{3.10b}$$

$$\{\hat{F}_y^{(1)}(q)\}_{LM}=i\gamma(q)Ac_{LM}^{(0)}\hat{\mathscr{M}}_{LM}(q) - i\beta(q)Ac_{LM}^{(-)}\hat{T}_{LM}^{\text{mag5}}(q) \ , \tag{3.10c}$$

$$\{\hat{F}_z^{(1)}(q)\}_{LM}=\epsilon(q)Ac_{LM}^{(+)}\hat{T}_{LM}^{\text{mag5}}(q) \ , \tag{3.10d}$$

while the unnatural parity transitions corresponding to the transfer L,M are described by the operators

$$\{\hat{F}_0^{(1)}(q)\}_{LM}=\gamma(q)Ac_{LM}^{(+)}\hat{T}_{LM}^{\text{el5}}(q) \ , \tag{3.11a}$$

$$\{\hat{F}_x^{(1)}(q)\}_{LM}=-\delta(q)Ac_{LM}^{(0)}\hat{\mathscr{L}}_{LM}{}^5(q) \ , \tag{3.11b}$$

$$\{\hat{F}_y^{(1)}(q)\}_{LM}=-i\beta(q)Ac_{LM}^{(+)}\hat{T}_{LM}^{\text{el5}}(q) \ , \tag{3.11c}$$

$$\{\hat{F}_z^{(1)}(q)\}_{LM}=\epsilon(q)Ac_{LM}^{(-)}\hat{T}_{LM}^{\text{el5}}(q) \ . \tag{3.11d}$$

The matrix elements of the operators $\hat{F}_\xi^{(1)}$, $\xi=0$, x, y, and z, between two nuclear states can now be expressed in terms of the reduced matrix elements of the multipole operators of Eqs. (3.7). As an example let us calculate the transition amplitudes and the D_ξ functions for the transition

$$|0^+,0\rangle\to|J^\pi,T\rangle \tag{3.12}$$

in the single collision approximation. For the natural parity transitions we have

$$\langle J^\pi,M,T\,|\,\hat{F}_0^{(1)}\,|\,0^+,0,0\rangle=A\alpha_T(q)c_{JM}^{(0)}\langle J^\pi T||\hat{\mathscr{M}}_{JT}(q)||0^+0\rangle + A\gamma_T(q)c_{JM}^{(-)}\langle J^\pi T||\hat{T}_{JT}^{\text{mag5}}(q)||0^+0\rangle \ , \tag{3.13a}$$

$$\langle J^\pi,M,T\,|\,\hat{F}_x^{(1)}\,|\,0^+,0,0\rangle=0 \ , \tag{3.13b}$$

$$\langle J^\pi,M,T\,|\,\hat{F}_y^{(1)}\,|\,0^+,0,0\rangle=iA\gamma_T(q)c_{JM}^{(0)}\langle J^\pi T||\hat{\mathscr{M}}_{JT}(q)||0^+0\rangle - iA\beta_T(q)c_{JM}^{(-)}\langle J^\pi T||\hat{T}_{JT}^{\text{mag5}}(q)||0^+0\rangle \ , \tag{3.13c}$$

$$\langle J^\pi,M,T\,|\,\hat{F}_z^{(1)}\,|\,0^+,0,0\rangle=A\epsilon_T(q)c_{JM}^{(+)}\langle J^\pi T||\hat{T}_{JT}^{\text{mag5}}(q)||0^+0\rangle \ , \tag{3.13d}$$

while for the unnatural parity transitions we have

$$\langle J^\pi,M,T\,|\,\hat{F}_0^{(1)}\,|\,0^+0,0,\rangle = A\gamma_T(q)c_{JM}^{(+)}\langle J^\pi T\,||\,\hat{T}_{JT}^{\mathrm{el5}}(q)\,||\,0^+0\rangle \;, \tag{3.14a}$$

$$\langle J^\pi,M,T\,|\,\hat{F}_x^{(1)}\,|\,0^+,0,0\rangle = -A\delta_T(q)c_{JM}^{(0)}\langle J^\pi T\,||\,\hat{\mathscr{L}}_{JT}{}^5(q)\,||\,0^+0\rangle \;, \tag{3.14b}$$

$$\langle J^\pi,M,T\,|\,\hat{F}_y^{(1)}\,|\,0^+,0,0\rangle = -i\beta_T(q)Ac_{JM}^{(+)}\langle J^\pi T\,||\,\hat{T}_{JT}^{\mathrm{el5}}(q)\,||\,0^+0\rangle \;, \tag{3.14c}$$

$$\langle J^\pi,M,T\,|\,\hat{F}_z^{(1)}\,|\,0^+,0,0\rangle = \epsilon_T(q)Ac_{JM}^{(-)}\langle J^\pi T\,||\,\hat{T}_{JT}^{\mathrm{el5}}(q)\,||\,0^+0\rangle \;, \tag{3.14d}$$

where the symbol $\langle\,||\;\;||\,\rangle$ denotes the matrix elements reduced with respect to the isospin and spin projections. Now if we insert Eqs. (3.13) and (3.14) into Eqs. (2.8) and (2.9) and use the normalization properties of the $c_{JM}^{(\alpha)}$ coefficients

$$\sum_m |\,c_{JM}^{(0)}\,|^2 = 4\pi(2J+1) \;, \tag{3.15a}$$

$$\sum_m |\,c_{JM}^{(\pm)}\,|^2 = 2\pi(2J+1) \;, \tag{3.15b}$$

expressions may be obtained for the D_ξ observables in the single collision approximation. For the natural parity transitions

$$D_0^T(q) = \frac{|\,\alpha_T(q)\,|^2 + R_T^N(q)\,|\,\gamma_T(q)\,|^2}{I_T^{N'}(q)} \;, \tag{3.16a}$$

$$D_x^T(q) = 0 \;, \tag{3.16b}$$

$$D_y^T(q) = \frac{|\,\gamma_T(q)\,|^2 + R_T^N(q)\,|\,\beta_T(q)\,|^2}{I_T^{N'}(q)} \;, \tag{3.16c}$$

$$D_z^T(q) = \frac{R_T^N(q)\,|\,\epsilon_T(q)\,|^2}{I_T^{N'}(q)} \;, \tag{3.16d}$$

where

$$R_T^N(q) = \frac{1}{2}\left|\frac{\langle J^\pi T\,||\,\hat{T}_{JT}^{\mathrm{mag5}}(q)\,||\,0^+0\rangle}{\langle J^\pi T\,||\,\hat{\mathscr{M}}_{JT}(q)\,||\,0^+0\rangle}\right|^2 \;, \tag{3.17}$$

and

$$I_T^{N'}(q) = |\,\alpha_T(q)\,|^2 + |\,\gamma_T(q)\,|^2 + R_T^N(q) \times \{\,|\,\gamma_T(q)\,|^2 + |\,\beta_T(q)\,|^2 + |\,\epsilon_T(q)\,|^2\} \;, \tag{3.18}$$

while for the unnatural parity transitions:

$$D_0^T(q) = \frac{|\,\gamma_T(q)\,|^2}{I_T^{UN'}(q)} \;, \tag{3.19a}$$

$$D_x^T(q) = \frac{R_T^{UN}(q)\,|\,\delta_T(q)\,|^2}{I_T^{UN'}(q)} \;, \tag{3.19b}$$

$$D_y^T(q) = \frac{|\,\beta_T(q)\,|^2}{I_T^{UN'}(q)} \;, \tag{3.19c}$$

$$D_z^T(q) = \frac{|\,\epsilon_T(q)\,|^2}{I_T^{UN'}(q)} \;, \tag{3.19d}$$

where

$$R_T^N(q) = \frac{1}{2}\left|\frac{\langle J^\pi T\,||\,\hat{T}_{JT}^{\mathrm{mag5}}(q)\,||\,0^+0\rangle}{\langle J^\pi T\,||\,\hat{\mathscr{M}}_{JT}(q)\,||\,0^+0\rangle}\right|^2 \;, \tag{3.17}$$

and

$$I_T^{UN'}(q) = |\,\gamma_T(q)\,|^2 + |\,\beta_T(q)\,|^2 + |\,\epsilon_T(q)\,|^2 + R_T^{UN}(q)\,|\,\delta_T(q)\,|^2 \;. \tag{3.21}$$

Equations (3.16) and (3.19) demonstrate the sensitivity of the $D_\xi(q)$ observables to specific components of the NN amplitude and to specific nuclear form factors under the single collision approximation. In particular, for unnatural parity transitions a given $D_\xi(q)$ is dependent (to within an overall normalization factor) on only one component of the complete NN amplitude. Also, for unnatural parity transitions the dependence of the observable $D_x(q)$ on the axial longitudinal form factor should be especially noted. Because of the transverse nature of electromagnetic waves the axial longitudinal form factor does not enter either into (e,e') transition matrix elements in the one photon exchange mechanism as well as in (π,π') reactions. Thus the $(\vec{p},\vec{p}\,')$ reaction offers a rather unique means of studying nuclear transition densities corresponding to the matrix elements of the operator $\hat{\mathscr{L}}^5$.

IV. SPECIFIC EXAMPLES AND DISCUSSION

As an illustration of the usefulness of the D_ξ observables let us consider the (p,p') excitation of the $J^\pi = 1^+$ states in ^{12}C at 12.71 MeV ($T=0$) and 15.11 MeV ($T=1$), which can be well described in terms of the single particle-hole excitation mechanism. In the single scattering approximation it can be seen from Eqs. (3.14) that each of the elements in the matrices F_ξ (3×1 matrices in this case) is proportional to the product of a single isoscalar ($T=0$ excitation) or isovector ($T=1$ excitation) component of the NN amplitude and a single nuclear multipole form factor. The nonzero matrix elements $(F_\xi)_{M_f M_i}$ are the following:

$$(F_0^T)_{1,0}=(F_0^T)_{-1,0}$$
$$=i\sqrt{\pi}A\gamma_T(q)$$
$$\times\langle 1^+,T||\hat{T}^{\text{el}5}(q)||0^+,0\rangle\ , \quad (4.1\text{a})$$

$$(F_x^T)_{1,0}=-(F_x^T)_{-1,0}$$
$$=-i\sqrt{2\pi}A\delta_T(q)$$
$$\times\langle 1^+,T||\hat{\mathscr{L}}^5(q)||0^+,0\rangle\ , \quad (4.1\text{b})$$

$$(F_y^T)_{1,0}=(F_y^T)_{-1,0}$$
$$=i\sqrt{\pi}A\beta_T(q)$$
$$\times\langle 1^+,T||\hat{T}^{\text{el}5}(q)||0^+,0\rangle\ , \quad (4.1\text{c})$$

$$(F_z^T)_{0,0}=i\sqrt{2\pi}A\epsilon_T(q)$$
$$\times\langle 1^+,T||\hat{T}^{\text{el}5}(q)||0^+,0\rangle\ . \quad (4.1\text{d})$$

Thus from Eqs. (2.8), (2.9), and (4.1) we obtain

$$D_0^T(q)=\frac{|\gamma_T(q)|^2}{I'_T(q)}\ , \quad (4.2\text{a})$$

$$D_x^T(q)=\frac{R_T(q)\,|\delta_T(q)|^2}{I'_T(q)}\ , \quad (4.2\text{b})$$

$$D_y^T(q)=\frac{|\beta_T(q)|^2}{I'_T(q)}\ , \quad (4.2\text{c})$$

$$D_z^T(q)=\frac{|\epsilon_T(q)|^2}{I'_T(q)}\ , \quad (4.2\text{d})$$

where

$$I'_T=|\gamma_T(q)|^2+|\beta_T(q)|^2+|\epsilon_T(q)|^2$$
$$+R_T(q)\,|\delta_T(q)|^2\ , \quad (4.3)$$

$$R_T(q)=2\left|\frac{\langle 1^+,T||\hat{\mathscr{L}}^5(q)||0^+,0\rangle}{\langle 1^+,T||\hat{T}^{\text{el}5}(q)||0^+,0\rangle}\right|^2\ , \quad (4.4)$$

$$[R_T(q)\to 1 \text{ as } q\to 0]\ .$$

The ratio $R_T(q)$ can be calculated from the model of the nuclear structure involved; for example, by assuming that the 1^+ states represent pure $p_{3/2}\to p_{1/2}$ transitions from a closed $p_{3/2}$ shell ^{12}C ground state[12] or by using the wave functions of Cohen and Kurath.[13]

It should be mentioned that since in our example we assume the local form of the NN amplitude, we do not take into account the nostatic (exchange) interactions. These, in general, may lead to transitions which give negligible contributions at energies of our interest.[4]

In Fig. 2 we present the quantities $D_0(q)$, $D_x(q)$, $D_y(q)$, and $D_z(q)$ calculated for the (p,p') excitation of the 12.71 MeV ($T=0$) and 15.11 MeV ($T=1$) levels in ^{12}C at a bombarding energy of 500 MeV. The dashed lines represent the predictions of the single collision approximation [Eqs. (4.2)] while the solid lines represent the prediction of the full multiple scattering expansion in the Glauber theory. The NN amplitudes for these calculations were taken from phase shift analyses.[14] As can be seen from Fig. 2 it is an important feature of the D_ξ observables that they depend rather weakly on the multiple scattering corrections at small momentum transfers ($q\lesssim 0.6\ \text{fm}^{-1}$). Since Eqs. (4.2) show that, in the single scattering approximation, each of the observables D_0, D_x, D_y, and D_z is proportional to a single component (isoscalar or isovector) of the NN amplitude squared, this selectivity, taken together with the reasonable validity of the single scattering approximation at low momentum transfers, may be useful in resolving recently observed discrepancies between the data for polarization observables in proton-nucleus elastic scattering and the predictions of multiple scattering theory calculations based on free NN amplitudes.[1] As an experimental aside, it should be noted that it is in the region of low q where it is reasonably easy to measure the D_ξ parameters for the (p,p') excitation of the well known 1^+ states in ^{12}C. The cross section for the excitation of these 1^+ states are peaking at $q_{\min}(0°)$; and at small angles (low q) these states stand out above the background of broad natural parity transitions.

Another interesting aspect of studying the observables D_ξ is that their determination may be useful in resolving discrepancies between the experimental data and the reaction theory which can be enigmatic when only data on the unpolarized cross section are available. The unpolarized differential cross section I_0 may be written as

$$I_0=\left[\frac{d\sigma}{d\Omega}\right]_0+\left[\frac{d\sigma}{d\Omega}\right]_x+\left[\frac{d\sigma}{d\Omega}\right]_y+\left[\frac{d\sigma}{d\Omega}\right]_z\ , \quad (4.5)$$

where

$$\left[\frac{d\sigma}{d\Omega}\right]_\xi=D_\xi I_0=\text{Tr}'(F_\xi F_\xi^\dagger)\ . \quad (4.6)$$

Note that no interference terms between the F_ξ enter into the expression for the differential cross section. Now, if each of the contributions $(d\sigma/d\Omega)_\xi$ possesses a diffractive structure with maxima and minima positions occurring at dif-

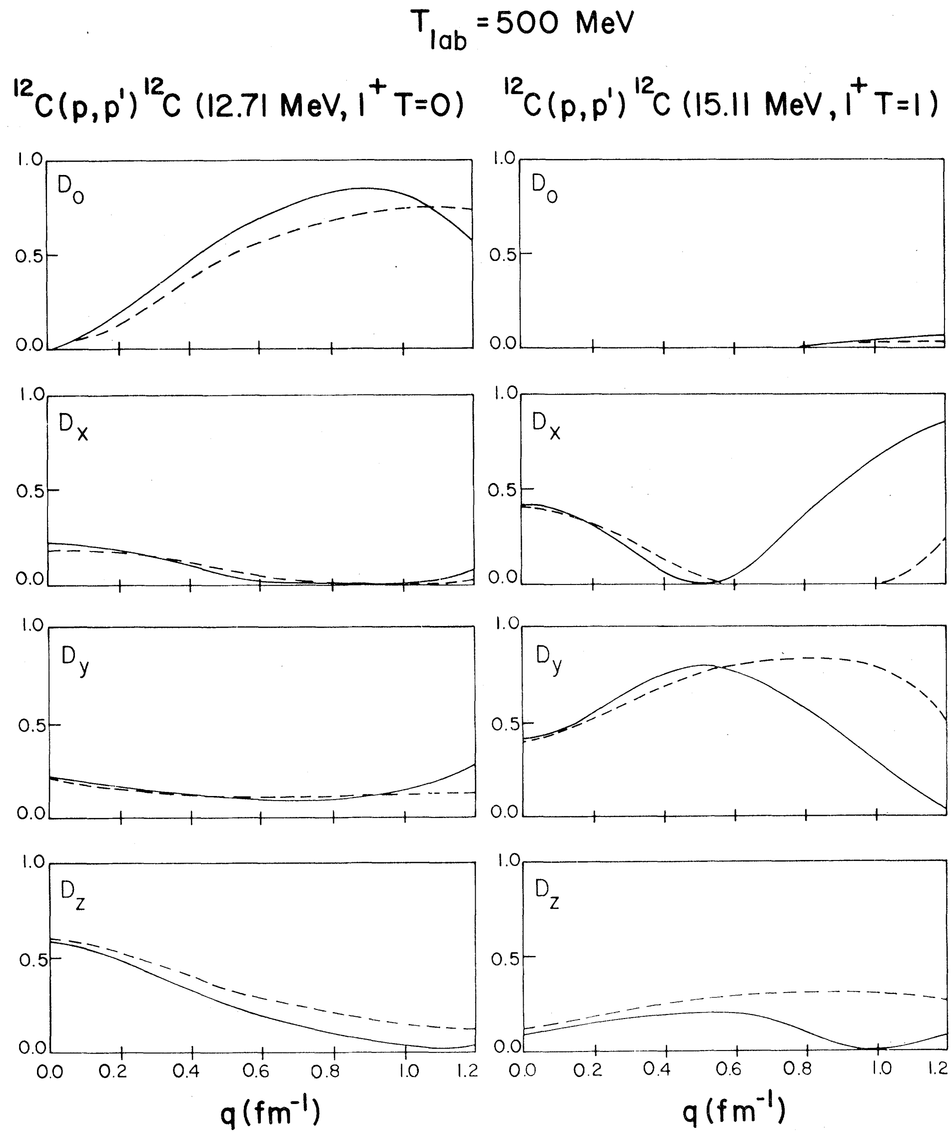

FIG. 2. The observables D_ξ, $\xi=0$, x, y, and z, calculated for the $^{12}C(p,p')$ reaction at an incident proton energy of 500 MeV for the excitations of the 12.71 MeV ($T=0$) and 15.11 MeV ($T=1$) 1^+ levels of ^{12}C. The dashed and solid curves represent Glauber theory predictions. The dashed curves correspond to the predictions of the single scattering approximation while the solid curves correspond to the full multiple scattering expansion. In these calculations the nuclear wave functions were those of Cohen and Kurath (Ref. 13), while the NN amplitudes were taken from the Arndt compilation (Ref. 14).

ferent values of q, the summed cross section in Eq. (4.5) may exhibit very little structure. In particular, for the excitation of the ^{12}C 1^+ states which have been considered above, the contribution due to F_x will have a different momentum dependence than the contributions due to F_0, F_y, and F_z, since F_x depends on the longitudinal axial form factor while F_0, F_y, and F_z depend on the transverse axial form factor [Eqs. (3.14)], and these two form factors have a different q dependence. The latter effect is illustrated in Fig. 3 where the predictions of a Glauber theory are compared with the experimental data[15,16] for the excitation of the 15.11 MeV $T=1$ 1^+ level in ^{12}C at 800 MeV. The Glauber calculation used the 800 MeV NN amplitudes of Arndt[14] and form factors calculated from the wave functions of Cohen and Kurath.[13] The solid curve represents the predicted total cross section while the dashed curves represent the individual contributions of the four terms in Eq. (4.5). Note that the kink in I_0 at

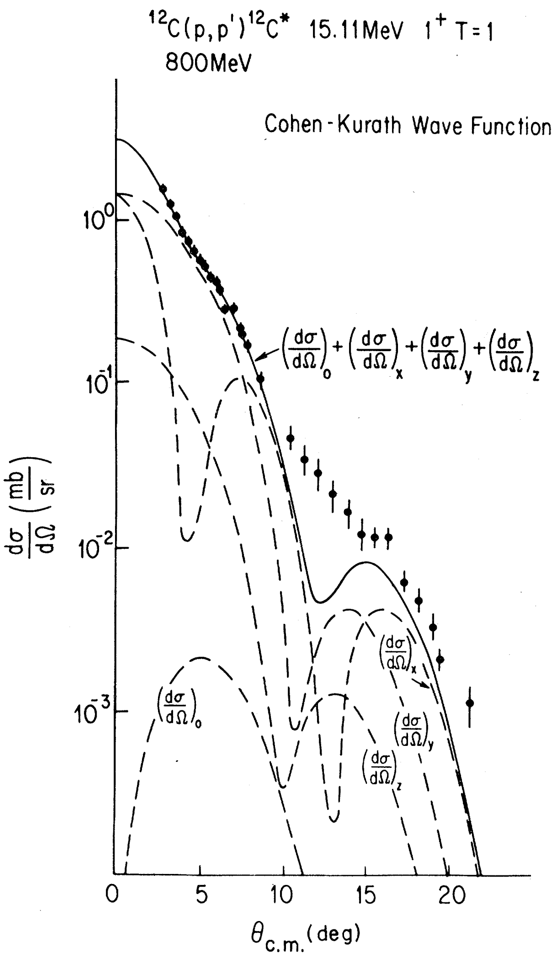

FIG. 3. Differential cross sections for the (p,p') excitation at 800 MeV incident energy of the 15.11 MeV $(T=1)$ 1^+ level of ^{12}C. The data are taken from Refs. 15 and 16. The dashed curves represent Glauber theory predictions for the contributions $(d\sigma/d\Omega)_\xi$, $\xi=0$, x, y, and z, described in the text, while the solid curve represents the sum (incoherent) of these individual contributions. In these calculations the nuclear wave functions were those of Cohen and Kurath (Ref. 13), while the NN amplitudes were taken from the Arndt compilation (Ref. 14).

$\theta_{c.m.}\cong 4°$ $(q=0.55\ \text{fm}^{-1})$ can be traced to the minimum in the contribution from $(d\sigma/d\Omega)_x$. The fact that the experimental data do not show the minimum at $\theta_{c.m.}\cong 12°$ $(q\cong 1.35\ \text{fm}^{-1})$ predicted by the Glauber theory could be due to the incorrect momentum transfer dependence of the axial longitudinal form factor, which, as has been mentioned above, cannot be measured in the (e,e') or $(\pi\pi')$ reactions. Thus the determination of the observable D_ξ should provide a rather sensitive test of the reaction theory, because it allows one to study specific components of the general collision matrix F. This interplay of the transverse and longitudinal form factors in the (p,p') differential cross section is somewhat related to the D-state effect in the unpolarized differential cross section for p-d elastic scattering, where the lack of a minimum in the angular distribution is related to the different momentum transfer behavior of the components of the elastic p-d collision matrix which depend upon the spherical and quadrupole deuteron form factors.[17]

For unnatural parity transitions the measurement of the observable D_x allows one to separate the contributions to the unpolarized differential cross section from the longitudinal and transverse parts of the spin transition density. Equation (4.5) can be written as

$$I_0=\left[\frac{d\sigma}{d\Omega}\right]_L+\left[\frac{d\sigma}{d\Omega}\right]_T, \tag{4.7}$$

where

$$\left[\frac{d\sigma}{d\Omega}\right]_L=I_0D_x$$

$$=\tfrac{1}{4}I_0(1-D_{NN'}+D_{SS'}-D_{LL'}), \tag{4.8}$$

and

$$\left[\frac{d\sigma}{d\Omega}\right]_T=I_0(1-D_x)$$

$$=\tfrac{1}{4}I_0(3+D_{NN'}-D_{SS'}+D_{LL'}). \tag{4.9}$$

This separation therefore requires four measurements: I_0, $D_{NN'}$, $D_{SS'}$, and $D_{LL'}$. The measurement of $(d\sigma/d\Omega)_L$ has particular interest since this quantity depends directly on the longitudinal part of the spin transition density, and this part does not contribute to (e,e') scattering in the one photon exchange reaction mechanism.

Finally, the extraction of the observables D_ξ may be interesting with regard to the question of the sensitivity of unnatural parity (p,p') excitation processes to pion condensation effects.[18,19] These effects, if present at all, should contribute mainly to the amplitude F_x and consequently to the observable D_x.[9]

In view of the above discussion the study of the proton polarization observables in the context of (p,p') reactions should deserve more attention in the future, both with regard to theory and experiment. We have suggested a new way of describing the experimental results in terms of the parameters D_0, D_x, D_y, and D_z which are related in a simple and useful way to the amplitudes describing the (p,p') reaction.

APPENDIX

In deriving Eqs. (3.6) we made use of expansions of the functions $e^{i\vec{q}\cdot\vec{r}}$, $xe^{i\vec{q}\cdot\vec{r}}$, $\hat{y}e^{i\vec{q}\cdot\vec{r}}$, and $\hat{z}e^{i\vec{q}\cdot\vec{r}}$, where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are unit vectors in the Cartesian frame of reference with the axes $x\,||(\vec{k}_f-\vec{k}_i)$, $y\,||(\vec{k}_i\times\vec{k}_f)$, and $z\,||(\vec{k}_i+\vec{k}_f)$. The general expansion of $e^{i\vec{q}\cdot\vec{r}}$ in terms of spherical harmonics is given by

$$e^{i\vec{q}\cdot\vec{r}}=4\pi\sum_{L=0}^{\infty}\sum_{M=-L}^{M=+L} i^L j_L(qr) \times Y^*_{LM}(\hat{q})Y_{LM}(\hat{r}) \,. \quad \text{(A1)}$$

In the frame of reference (x,y,z) we have

$$Y^*_{LM}(\hat{q})=[(2L+1)/4\pi]^{1/2}d^J_{MO}(\pi/2) \,, \quad \text{(A2)}$$

where d^J_{MO} is the Wigner d function for rotations about the y axis. The expansion of $e^{i\vec{q}\cdot\vec{r}}$ in Eq. (A1) then becomes

$$e^{i\vec{q}\cdot\vec{r}}=\sum_{L=0}^{\infty}\sum_{M=-L}^{M=+L} c^{(0)}_{LM}\Phi_{LM}(q,\vec{r}) \,, \quad \text{(A3)}$$

where $\Phi_{LM}(q,\vec{r})$ and $c^{(0)}_{LM}$ are defined by Eqs. (3.8) and (3.9), respectively. Expansions of the products of the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$, and $e^{i\vec{q}\cdot\vec{r}}$ can be derived in the analogous way as, for example, in Ref. 10 or Ref. 20. One obtains

$$\begin{aligned}
\hat{x}e^{i\vec{q}\cdot\vec{r}}&=\frac{1}{iq}\sum_{L,M}c^{(0)}_{LM}\nabla_{\vec{r}}\Phi(q,\vec{r}) \,,\\
\hat{y}e^{i\vec{q}\cdot\vec{r}}&=-\sum_{L,M}\frac{1}{[L(L+1)]^{1/2}}\left\{c^{(-)}_{LM}+\frac{1}{q}c^{(+)}_{LM}\nabla_{\vec{r}}\times\right\}(\vec{r}\times\nabla_{\vec{r}})\Phi_{LM}(q,\vec{r}) \,,\\
\hat{z}e^{i\vec{q}\cdot\vec{r}}&=\sum_{L,M}\frac{1}{i[L(L+1)]^{1/2}}\left\{c^{(+)}_{LM}+\frac{1}{q}c^{(-)}_{LM}\nabla_{\vec{r}}\times\right\}(\vec{r}\times\nabla_{\vec{r}})\Phi_{LM}(q,\vec{r}) \,,
\end{aligned} \quad \text{(A4)}$$

where the coefficients $c^{(+)}_{LM}$ and $c^{(-)}_{LM}$ are defined by Eq. (3.8). In Ref. 10 the z axis is taken along $\vec{q}$, while in our case $\vec{q}$ is along the negative x axis. Our choice for the quantization axis is more convenient in the actual calculations of the Glauber multiple scattering series.

[1]A. Rahbar *et al.*, Phys. Rev. Lett. 47, 1811 (1981).
[2]R. Glauber, in *Lectures in Theoretical Physics* (Interscience, New York, 1959), Vol. 1.
[3]M. Bleszynski and P. Osland, Phys. Lett. 84B, 157 (1979), and references therein.
[4]W. G. Love and M. A. Franey, Phys. Rev. C 24, 1073 (1981).
[5]R. J. Glauber and P. Osland, Phys. Lett. 80B, 401 (1979).
[6]M. Bleszynski, Phys. Lett. 92B, 91 (1980).
[7]A. Rahbar *et al.*, Bull. Am. Phys. Soc. 26, 1125 (1981).
[8]L. Wolfenstein, in *Annual Review of Nuclear Science* (Annual Reviews, Palo Alto, 1956), Vol. 6.
[9]E. Bleszynski, M. Bleszynski, and C. A. Whitten, Jr., in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe)*, Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, AIP Conf. Proc. No. 69, edited by G. G. Ohlson, R. E. Brown, N. Jarmie, M. W. McNaughton and G. M. Hale (AIP, New York, 1981), p. 556.
[10]J. D. Walecka, in *Muon Physics*, edited by V. Hughes and C. S. Wu (Academic, New York, 1973).
[11]T. W. Donnelly and W. C. Haxton, At. Data Nucl. Data Tables 23, 103 (1979).
[12]J. S. O'Connell, T. W. Donnelly, and J. D. Walecka, Phys. Rev. C 6, 719 (1972).
[13]S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
[14]R. Arndt, private communication.
[15]J. Moss *et al.*, Phys. Rev. Lett. 44, 1189 (1980).
[16]M. Haji-Saeid *et al.*, Phys. Rev. C 25, 3035 (1982).
[17]D. R. Harrington, Phys. Rev. Lett. 21, 1496 (1965).
[18]H. Toki and W. Weise, Phys. Rev. Lett. 42, 1034 (1979).
[19]J. Delorme, Nucl. Phys. A374, 541c (1982), and references therein.
[20]D. M. Brink and G. R. Satchler, *Angular Momentum* (Oxford University Press, Oxford, 1968).