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Extended Lee model and three-particle equations

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The extended Lee model describes the interaction of two fermions, V and N, with a scalar boson θ and its antiparticle $\bar{\theta}$ through the virtual processes $V \rightleftarrows N+\theta$ and $N \rightleftarrows V+\bar{\theta}$. Here it is shown that the amplitudes for the physical processes $V+\theta \to V+\theta$ and $V+\theta \to N+2\theta$ can be obtained in a reasonable approximation from the solution of an Amado-Lovelace type of three-particle equation. The analysis presented gives some insight into the problem of extending a crossing symmetric, Chew-Low type of T matrix off the energy shell.

[NUCLEAR REACTIONS Amado-Lovelace equations for V-θ sector of extended Lee model.]

I. INTRODUCTION

The Lee model[1] describes the interaction of two fermions, V and N, with a scalar boson θ through the virtual process $V \rightleftarrows N+\theta$. The model is tractable because of the conservation of charge and baryon number, and the lack of antiparticles in the theory. The baryon number is the number of V particles plus the number of N particles. Here we take the charges of N, V, and θ to be 1, 0, and -1, respectively. The model has been solved exactly in the N-θ sector[1] where only bare V and N-θ states contribute, and in the V-θ sector[2] which is spanned by bare V-θ and N-2θ states.

In Ref. 3 (hereafter referred to as F) it has been shown that the amplitudes for the processes $V+\theta \to V+\theta$, $V+\theta \to N+2\theta$, and $N+2\theta \to N+2\theta$ can be obtained from the solution of an Amado-Lovelace type of three-particle equation. The technique used to derive this equation depends in an essential way on the restricted nature of the states in each sector of the Lee model, and therefore cannot be used to treat field theories with particles and antiparticles present. Such a theory is the extension of the Lee model which has been studied in connection with the distinction between elementary and composite particles.[4] In this model the interaction takes place through the virtual processes $V \rightleftarrows N+\theta$ and $N \rightleftarrows V+\bar{\theta}$, where $\bar{\theta}$ is the antiparticle to θ. Because of the existence of the antiparticle, the model possesses crossing symmetry which is absent in the Lee model. Here we shall derive an Amado-Lovelace equation for V-θ scattering in this extended Lee model.[4]

The derivation is based on a dispersion relation obtained from an exact formal expression for the amplitude for the production process $V+\theta \to N+2\theta$. The dispersion relation is written in terms of the energy ω of one of the θ particles in the final state. The amplitude studied, which is a part of the full production amplitude, has a branch cut for $\omega \geq \mu$, where μ is the θ mass. The discontinuity across the low energy end of the cut ($\mu \leq \omega \leq M_V - M_N + 2\mu$) is related linearly to the amplitude itself. By assuming this discontinuity is valid for all $\omega \geq \mu$, a linear scattering integral equation is obtained. This technique is closely related to an approach used by Amado[5] to derive three-particle equations, and is of a very general nature.

The integral equation derived involves a somewhat surprising off-shell extension of the N-θ T matrix. The on-shell T matrix is of the form $g^2u^2(k)/h(\omega_k+i\epsilon)$, where g is a renormalized coupling constant, $u(k)$ is a cutoff function, and $h(z)$ is a real, analytic function of z which carries the right hand unitarity cut and the left hand crossing cut. An obvious off-shell extension of the T matrix is $gu(p)gu(q)/h(z)$, but this turns out to be incorrect. The denominator function $h(z)$ can be factored in the form $d(z)\,w(z)$, where $d(z)$ and $w(z)$ carry the right hand cut and left hand cut, respectively; and $d(z)$ has a zero at $z + M_N = M_V$ which gives rise to the V-particle pole in the N-θ T matrix. The off-shell T matrix that appears in the three-particle equation is $gf(p)gf(q)/d(z)$, where $f(q) = u(q)/w^{1/2}(\omega_q)$. Thus, the form factor $f(q)$ carries the crossing cut. This result is consistent with Myhrer and Thomas's[6] comments on extending the on-shell pion-nucleon T matrix off-shell for use in few particle theories. Another interesting result is that the form factor $f(p)$ can be obtained directly from the N-θ phase shift δ and inelasticity parameter η. The equations relating $f(p)$ to δ and η are of exactly the same form as those used by Londergan *et al.*[7] to determine the form factors in a separable potential model of the pion-nucleon T matrix. Thus, the present work provides a partial justification for their phenomenology.

The outline of the paper is as follows. In Sec. II the extended Lee model is given. The N-θ T matrix is analyzed in Sec. III. The dispersion theory derivation of the Amado-Lovelace equations for V-θ scattering is presented in Sec. III. A brief discussion is given in Sec. IV.

II. THE EXTENDED LEE MODEL

We take for the Hamiltonian the expression[4]

$$H= H_0^F+\sum_\alpha \int d^3k\; a_\alpha^\dagger(\vec{k})a_\alpha(\vec{k})\omega_k + \sum_\alpha \int d^3k[a_\alpha(\vec{k})J_\alpha(k)+a_\alpha^\dagger(\vec{k})J_\alpha^\dagger(k)]\;, \tag{1}$$

with $\alpha=\theta, \bar{\theta}$, and

$$H_0^F=M_V^{(0)}V^\dagger V+M_N^{(0)}N^\dagger N\;, \tag{2}$$

$$J_\theta(k)=g_0u(k)V^\dagger N=J_{\bar{\theta}}^\dagger(k)\;. \tag{3}$$

Here, $a_\alpha(\vec{k})$ and $a_\alpha^\dagger(\vec{k})$ are the annihilation and creation operators for mesons with three-momentum $\vec{k}$ and energy $\omega_k=(k^2+\mu^2)^{1/2}$, and satisfy the usual commutation rules for bosons. The operator H_0^F describes static, noninteracting fermions whose creation and annihilation operators, $V^\dagger$,$N^\dagger$ and V,N, respectively, obey the usual anticommutation rules. The bare masses $M_V^{(0)}$ and $M_N^{(0)}$ are renormalized to M_V and M_N by the interaction. The interaction term in H describes the processes

$$V \rightleftarrows N+\theta\;, \tag{4a}$$

$$N \rightleftarrows V+\bar{\theta}\;, \tag{4b}$$

with a bare coupling constant g_0 and a cutoff function $u(k)$ which depends only on the magnitude of the three momentum. In the Lee model only the virtual process (4a) occurs and there is no crossing symmetry, while here the additional process (4b) leads to crossing symmetry.

It is straightforward to verify that the following operators commute with H;

$$B=V^\dagger V+N^\dagger N\;,$$
$$Q=N^\dagger N-\int d^3k[\,a_\theta^\dagger(\vec{k})a_\theta(\vec{k}) - a_{\bar{\theta}}^\dagger(\vec{k})a_{\bar{\theta}}(\vec{k})]\;. \tag{5}$$

Clearly B is a baryon number operator and Q is a charge operator. In setting up the charge operator, we have assigned the charges 1, 0, -1, and 1 to the particles N, V, θ, and $\bar{\theta}$, respectively. This is consistent with the assignment made in F.

III. N-θ SCATTERING

We shall now examine the structure of the N-θ T matrix, as it will play an important role in the process $V+\theta \rightarrow N+2\theta$, for which we shall construct a three-particle theory in the next section. According to Eqs. (12), (7), and (5) of F, the N-θ T matrix is given by

$$\langle N\,|\,J_\theta^\dagger(k)\,|\,\vec{k}\theta N\rangle_\pm=g^2u^2(k)t(\omega_k\pm i\epsilon)\;, \tag{6}$$

where

$$t(z)=\langle N\,|\,j_\theta^\dagger \frac{1}{M_N+z-H} j_\theta + j_\theta \frac{1}{M_N-z-H} j_\theta^\dagger\,|\,N\rangle \tag{7}$$

and

$$j_\theta=\frac{J_\theta(p)}{gu(p)}\;. \tag{8}$$

The states $|N>$ and $|\vec{k}\theta N>_\pm$ are eigenstates of H with eigenvalues M_N and $M_N+\omega_k$, respectively. Throughout the rest of this work all states that we shall encounter will be physical states, i.e., eigenstates of H. The plus and minus signs in (6) indicate in and out states, respectively. The parameter g is a renormalized coupling constant which will be defined shortly. The N-$\bar{\theta}$ T matrix is given by

$$\langle N\,|\,J_{\bar{\theta}}^\dagger(k)\,|\,\vec{k}\bar{\theta}N\rangle_\pm=g^2u^2(k)\bar{t}(\omega_k\pm i\epsilon)\;, \tag{9}$$

where from (3) and (7)

$$\bar{t}(z)=t(-z)\;. \tag{10}$$

Relation (10) is an expression of the crossing symmetry in the model. Obviously, we also have the relation

$$t(z^*)=t^*(z)\ . \tag{11}$$

In order to determine the singularities of $t(z)$, we shall insert appropriate intermediate states in (7). It is straightforward to show that

$$[B,J_\theta(k)]=0\ ,$$
$$[Q,J_\theta(k)]=-J_\theta(k)\ , \tag{12}$$

from which it follows that

$$Qj_\theta\,|\,N\rangle=0\ ,$$
$$Qj_\theta^\dagger\,|\,N\rangle=2j_\theta^\dagger\,|\,N\rangle\ . \tag{13}$$

Thus we see that in the first term on the right side of (7) the states $|\,V\rangle$, $|\,N\theta\rangle$, $|\,V\theta\bar{\theta}\rangle,\ldots$ contribute, while in the second term we have $|\,N\bar{\theta}\rangle$, $|\,V\bar{\theta}\bar{\theta}\rangle,\ldots$. Using (3), (6), (8), and (9) we find

$$t(z)=\frac{1}{z-\Delta}+\int d^3q\ g^2u^2(q)\times\left[\frac{|\,t(\omega_q\pm i\epsilon)\,|^2}{z-\omega_q}-\frac{|\,\bar{t}(\omega_q\pm i\epsilon)\,|^2}{z+\omega_q}\right]+\ldots\ , \tag{14}$$

where we have chosen

$$\langle V\,|\,j_\theta\,|\,N\rangle=1=\langle N\,|\,j_\theta^\dagger\,|\,V\rangle\ , \tag{15}$$

and written

$$\Delta=M_V-M_N\ . \tag{16}$$

From (3), (8), and (15) we have

$$g=g_0\langle V\,|\,V^\dagger N\,|\,N\rangle\ , \tag{17}$$

which gives the relation between the renormalized coupling constant g and the bare coupling constant g_0. According to (11) and (14), $t(z)$ is a real analytic function of z except for a simple pole at $z=\Delta$, a right hand cut (RHC) beginning at $z=\mu$, and a left hand cut (LHC) beginning at $z=-\mu$.

The on-shell N-θ amplitude can be written in the form

$$g^2u^2(q)t(\omega_q\pm i\epsilon)=-\eta(q)\frac{e^{\pm i\delta(\omega_q)}\sin\delta(\omega_q)}{4\pi^2q\omega_q}\ ,$$
$$\omega_q\geq\mu\ , \tag{18}$$

where δ is a phase shift and η is an inelasticity parameter, which is equal to the ratio of the elastic to the total cross for N-θ scattering. By comparing the imaginary parts of (14) and (18), it is straightforward to show that $\eta(q)\ =\ 1$ for $\mu\leq\omega_q\leq\Delta+2\mu$. The on-shell N-$\bar{\theta}$ amplitude can also be written as in (18). We shall denote the corresponding phase shift and inelasticity parameter by $\bar{\delta}$ and $\bar{\eta}$, respectively. If we let

$$h(z)=1/t(z)\ , \tag{19}$$

it follows from (18) that

$$Im h(\omega_q+i\epsilon)=4\pi^2q\omega_qg^2u^2(q)/\eta(q)\ ,$$
$$\omega_q\geq\mu\ . \tag{20}$$

Using this relation, and one like it for the N-$\bar{\theta}$ amplitude, as well as (10), (14), and (19), as it is straightforward to show that

$$\frac{h(z)}{z-\Delta}=1+(z-\Delta)\int d^3q\ g^2u^2(q)\times\left[\frac{1}{\eta(q)(\omega_q-\Delta)^2(\omega_q-z)}+\frac{1}{\bar{\eta}(q)(\omega_q+\Delta)^2(\omega_q+z)}\right] \tag{21}$$

In deriving this it has been assumed that $h(z)$ has no CDD (Ref. 8) poles. It is not difficult to derive an expression for $h(z)$ in terms of the phases δ and $\bar{\delta}$. From (18), (19), and (10), we have

$$h(\omega_q-i\epsilon)/h(\omega_q+i\epsilon)=e^{2i\delta(\omega_q)}\ ,$$
$$\omega_q\geq\mu\ ,$$
$$h(-\omega_q+i\epsilon)/h(-\omega_q-i\epsilon)=e^{2i\bar{\delta}(\omega_q)}\ ,$$
$$\omega_q\geq\mu\ . \tag{22}$$

Using these relations and following the method used in Ref. 9, it is straightforward to show that

$$h(z)=(z-\Delta)\exp\left\{-\frac{(z-\Delta)}{\pi}\int_\mu^\infty d\omega\left[\frac{\delta(\omega)}{(\omega-\Delta)(\omega-z)}-\frac{\bar{\delta}(\omega)}{(\omega+\Delta)(\omega+z)}\right]\right\}\ . \tag{23}$$

It is easy to check that this expression for $h(z)$ satisfies (22) and is consistent with the structure of (21).

IV. THE THREE PARTICLE EQUATIONS

We now turn our attention to V-θ scattering. From the conservation of baryon number and charge it follows that the possible reactions are

$$\begin{aligned} V+\theta &\to V+\theta \\ &\to N+2\theta \\ &\to V+2\theta+\bar{\theta} \\ &\to \cdots \, . \end{aligned} \tag{24}$$

We shall assume that

$$E=M_V+\omega_k<M_V+3\mu \, , \tag{25}$$

so that at most we can have two mesons in the final state. Using Eq. (21) of F and Eqs. (8) and (15) of the present work, we have

$$\langle N\,|\,a_\theta(\vec{p})J_\theta^\dagger(q)\,|\,\vec{k}\theta V\rangle_+=gu(q)\delta^3(\vec{p}-\vec{k})+gu(p)gu(q)F(E+i\epsilon-M_N-\omega_p\;,E)\;, \tag{26}$$

where

$$F(z,E)=\langle N\,|\,j_\theta^\dagger\frac{1}{M_N+z-H}j_\theta^\dagger\,|\,\vec{k}\theta V\rangle_+ \, . \tag{27}$$

According to Eq. (19) of F, the amplitude for $V+\theta\to N+2\theta$ is obtained by adding to (26) the same expression with $\vec{p}$ and $\vec{q}$ interchanged, thus the complete amplitude is determined by $F(z,E)$.

We now proceed to determine the analytic structure of $F(z,E)$. From (13) it follows that the only intermediate states that contribute in (27) are those with zero charge, namely, $|\,V\rangle$, $|\,N\theta\rangle$, ... Using (15), we have

$$F(z,E)=\frac{\langle V\,|\,j_\theta^\dagger\,|\,\vec{k}\theta V\rangle_+}{z-\Delta}+\int d^3x\frac{\langle N\,|\,j_\theta^\dagger\,|\,\vec{x}\theta N\rangle_{\pm\,\pm}\langle\vec{x}\theta N\,|\,j_\theta^\dagger\,|\,\vec{k}\theta V\rangle_+}{z-\omega_x}+\cdots \, . \tag{28}$$

Combining Eq. (7) of F and Eqs. (8), (26), and (27) from here we find

$$_\pm\langle\vec{x}\theta N\,|\,j_\theta^\dagger\,|\,\vec{k}\theta V\rangle_+=\delta^3(\vec{x}-\vec{k})+gu(x)[F(\omega_x\mp i\epsilon,E)+F(E+i\epsilon-M_N-\omega_x,E)] \, . \tag{29}$$

We see that $F(z,E)$ has a simple pole at $z=\Delta$ and a RHC beginning at $z=\mu$. The residue at the pole, which is given by

$$\lim_{z\to\Delta}(z-\Delta)F(z)=\langle V\,|\,j_\theta^\dagger\,|\,\vec{k}\theta V\rangle_+ \, , \tag{30}$$

is essentially the V-θ elastic scattering amplitude [compare Eq. (6)], while according to (6), (28), and (29), the discontinuity across the RHC for $\mu\le z\le\Delta+2\mu$ is determined by the N-θ elastic scattering amplitude and $F(z,E)$ itself. Using (28), (29), (6), (8), (18), and (19), we find

$$\begin{aligned} &F(\omega_q+i\epsilon\;,E)-e^{2i\delta(\omega_q)}F(\omega_q-i\epsilon\;,E) \\ &\quad=-2\pi i\frac{g^2u^2(q)}{h(\omega_q+i\epsilon)}\left[\frac{\delta(\omega_q-\omega_k)}{gu(q)}+4\pi q\omega_q F(E+i\epsilon-M_N-\omega_q\;,E)\right]\;,\quad \mu\le\omega_q\le\Delta+2\mu \, . \end{aligned} \tag{31}$$

It is important to realize that this relation is exact, moreover, the values of ω_q indicated are those that arise when $\vec{p}$ and $\vec{q}$ in (26) are on-shell ($E=M_N+\omega_p+\omega_q$) and E satisfies (25). Now we let

$$G(z,E)=d(z)F(z,E)\;, \tag{32}$$

where we assume $d(z)$ is a real, analytic function except for a RHC beginning at $z=\mu$ with

$$\frac{d(\omega_q-i\epsilon)}{d(\omega_q+i\epsilon)}=e^{2i\delta(\omega_q)}\;,\quad \mu\le\omega_q\le\Delta+2\mu \, . \tag{33}$$

Also we assume that $d(z)$ has a simple zero at $z=\Delta$ and that

$$\lim_{z\to\Delta}\frac{d(z)}{z-\Delta}=1 \, . \tag{34}$$

Using (30)–(34) we find

$$G(\Delta, E) = \langle V | j_\theta^\dagger | \vec{k}\theta V \rangle_+ \,, \tag{35}$$

and

$$\mathrm{Im}G(\omega_q + i\epsilon, E) = -\pi g^2 f^2(q) \left[\frac{\delta(\omega_q - \omega_k)}{gu(q)} + 4\pi q \omega_q \frac{G(E + i\epsilon - M_N - \omega_q, E)}{d(E + i\epsilon - M_N - \omega_q)} \right], \quad \mu \le \omega_q \le \Delta + 2\mu \,, \tag{36}$$

where

$$f^2(q) = u^2(q) \frac{d(\omega_q + i\epsilon)}{h(\omega_q + i\epsilon)} = u^2(q) \left| \frac{d(\omega_q + i\epsilon)}{h(\omega_q + i\epsilon)} \right| . \tag{37}$$

In writing the last equality in (37) we have assumed that the phase of $d(\omega_q + i\epsilon)$ has been chosen equal to that of $h(\omega_q + i\epsilon)$. This is consistent with (22) and (33).

Using Cauchy's theorem and assuming

$$G(z,E) \to 0 \,, \quad |z| \to \infty \tag{38}$$

we can write

$$G(z,E) = \frac{1}{\pi} \int_\mu^\infty d\omega_q \frac{\mathrm{Im}G(\omega_q + i\epsilon, E)}{\omega_q - z} . \tag{39}$$

Using (27), (8), (3), and the anticommutation rules, we find

$$\lim_{|z| \to \infty} zF(z,E) = 0 \,, \tag{40}$$

thus according to (32), (38) will be true as long as $d(z)$ diverges no worse than z for large $|z|$. In order to proceed we must make some sort of approximation. The simplest thing to do is to assume (36) is valid for all $\omega_q \ge \mu$. If we do this and let

$$X(p,k; E + i\epsilon) = gf(p) G(E + i\epsilon - M_N - \omega_p, E) u(k)/f(k) \,, \tag{41}$$

we find an Amado-Lovelace type of equation, i.e.,

$$X(p,k;z) = Z(p,k;z) + \int d^3q \, Z(p,q;z) \frac{X(q,k;z)}{d(E + i\epsilon - M_N - \omega_q)} \,, \tag{42}$$

where

$$Z(p,q;z) = \frac{gf(p) gf(q)}{z - M_N - \omega_p - \omega_q} . \tag{43}$$

From (41), (25), (35), and (8) we have

$$X(k,k; E + i\epsilon) = \langle V | J_\theta^\dagger(k) | \vec{k}\theta V \rangle_+ \,, \tag{44}$$

which is the V-θ elastic scattering amplitude.

Equation (42) can be obtained from the AGS (Ref. 10) form of the three-particle equations for a system of two finite mass particles (the θ's) and one infinitely massive particle (the N) if relativistic energies are used for the θ's, it is assumed that there is no θ-θ interaction, and the N-θ T matrix is taken to be

$$T(p,q;z) = \frac{gf(p) gf(q)}{d(z)} . \tag{45}$$

Using (37), (19), and (6) we find that

$$T(k,k; \omega_k \pm i\epsilon) = \langle N | J_\theta^\dagger(k) | \vec{k}\theta N \rangle_\pm \,, \tag{46}$$

which is the on-shell N-θ T matrix. Thus (45) is an off-shell extension of the N-θ T matrix. It is worth noting that (45) can be written in the Kowalski-Noyes[11] form

$$T(p,q;\omega_k \pm i\epsilon)=\frac{f(p)}{f(k)}g^2u^2(k)t(\omega_k \pm i\epsilon)\frac{f(q)}{f(k)} , \tag{47}$$

which when combined with (18) leads to the off-shell unitarity relation

$$T(p,q;\omega_k+i\epsilon)-T(p,q;\omega_k-i\epsilon)=-2\pi i T(p,k;\omega_k \pm i\epsilon)\frac{4\pi k\omega_k}{\eta(k)}T(k,q;\omega_k \mp i\epsilon), \quad \omega_k \geq \mu . \tag{48}$$

This plus the fact that from (45) and (34) we have

$$T(p,q;z) \underset{z\to\Delta}{\longrightarrow} \frac{gf(p)gf(q)}{z-\Delta} , \tag{49}$$

guarantees[12] that the three-particle theory based on (42) satisfies three-particle unitarity relations if the on-shell amplitude for $V+\theta \to N+2\theta$ is taken to be

$$\frac{gf(q)}{d(\omega_q+i\epsilon)}X(p,k;E+i\epsilon)+\frac{gf(p)}{d(\omega_p+i\epsilon)}X(q,k;E+i\epsilon) ,E=M_N+\omega_p+\omega_q . \tag{50}$$

From (20) and (37) we find

$$\operatorname{Im} d(\omega_q+i\epsilon)=4\pi^2 q\omega_q g^2 f^2(q)/\eta(q), \quad \omega_q \geq \mu , \tag{51}$$

which when combined with (34) leads to

$$\frac{d(z)}{z-\Delta}=1+(z-\Delta)\int \frac{d^3q\, g^2 f^2(q)}{\eta(q)(\omega_q-\Delta)^2(\omega_q-z)} . \tag{52}$$

This is similar to (21) with the important difference that $d(z)$ has no LHC. The alternative expression

$$d(z)=(z-\Delta)\exp\left[-\frac{(z-\Delta)}{\pi}\int_\mu^\infty d\omega \frac{\delta(\omega)}{(\omega-\Delta)(\omega-z)}\right] \tag{53}$$

is similar to (23) and can be derived in the same way. Combining (37), (23), and (53), we find

$$f(q)=u(q)\exp\left[-\frac{(\omega_q-\Delta)}{2\pi}\int_\mu^\infty \frac{d\omega\bar{\delta}(\omega)}{(\omega+\Delta)(\omega+\omega_q)}\right] . \tag{54}$$

It is important to note that $f(q)=u(q)$ at $\omega_q=\Delta$ and that $f(q)$ has an additional LHC beginning at $\omega_q=-\mu$.

V. DISCUSSION

The equations derived here suggest results of a more general nature. First of all, when a T matrix such as (6) is used in a three-particle theory, the part of the denominator $h(z)$ which carries the LHC associated with the crossing symmetry should be absorbed into the form factor $u(k)$ as in (54). The representation (23) for $h(z)$ guarantees that this can be done. It is worth noting that there exists such a factored representation for the denominator function of the Chew-Low T matrix.[9] Second, (51) and (53) show that the modified form factor $f(q)$ can be obtained directly from a knowledge of the phase shift δ and inelasticity parameter η for the N-θ system. This is a partial justification for the use of phenomenological pion-nucleon T matrices which do not take the crossing symmetry explicitly into account. Finally, it is interesting to note that upon comparing Eqs. (42), (43), and (49) of F with Eqs. (42), (43), and (52) of the present work, it is found that the Amado-Lovelace equation for the Lee model goes over to the one presented here if the Lee model cutoff function is replaced by $f(q)$ and the N-θ inelasticity parameter η is set equal to one. This suggests that some systems, such as the pion-nucleon and pion-deuteron system, can be fruitfully analyzed using Lee model type field theories with cutoff functions or form factors which incorporate the crossing cut as in Eq. (54). Such field theories are much easier to analyze than those with crossing symmetry.

It seems clear that the techniques used here can be used to derive three-particle equations for the Chew-Low[13] and cloudy bag[14] models of the pion-nucleon system. While such models are unrealistic in that they neglect nucleon recoil, the three-particle equations should shed some light on the problem of extending the pion-nucleon T matrix off shell. The cloudy bag model is particularly interesting in that it includes the P_{33} resonance (the Δ) as part of the input. Lovelace's model[15] of the N-π-π system suggests that by so doing, it is possible to account for the large inelasticity in the P_{11} channel, as well as the change in sign of the P_{11} phase shift. These extensions of the work presented here will be the subject of future publications.

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