Multiple Coulomb excitation effects in heavy-ion compound and fusion cross sections

B. V. Carlson and M. S. Hussein
Instituto de Física, Universidade de São Paulo, São Paulo, Brasil
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We develop a simple model for the average S matrix that describes heavy ion direct processes in the presence of absorption due to compound nucleus formation. The fluctuation cross sections and the fusion cross section are then derived for deformed heavy ion systems where multiple Coulomb excitation is important. The modifications in the conventional expressions for these cross sections, due to multiple Coulomb excitation, are discussed. Applications are made to the systems $^{16}O + {}^{148,150,152}Sm$.

[NUCLEAR REACTIONS Coulomb excitation effect on heavy-ion compound and fusion processes are studied.]

I. INTRODUCTION

Heavy ion compound and fusion cross sections have usually been calculated assuming the complete separation from the fast, direct transitions. Of course the latter are partially accounted for through the use of appropriate absorptive potentials. It is by now, however, well established that the presence of directly coupled channels affects not only the values of the transmission coefficients needed in the calculation of the compound nucleus (fluctuation) and fusion cross sections, but, more importantly, the structure of the statistical theory (i.e., the Hauser-Feshbach theory). This is borne out by several investigations.[1]

An interesting example of directly coupled-channels effects on the compound nucleus and fusion cross sections of heavy ions is that of multiple Coulomb excitation. These effects have been very nicely demonstrated for the system $^{16}O + {}^{A}Sm$ ($A=148$, 150, 152, and 154) at sub-barrier energies by Stokstad *et al.*[2] Earlier discussion of these effects in α-induced reactions on deformed targets was given in Ref. 3.

For the purpose of simple analyses, several concepts have been introduced, e.g., static deformation,[4] dynamic deformation,[5] zero-point vibration,[6] etc. Clearly these concepts are physically motivated and represent to a large extent a simulation of the overall physics involved in a more complete, coupled channels description of the fusion process. In such a calculation one identifies the fusion cross section σ_F with the difference

$$\sigma_F \equiv \sigma_R - \sum_{I\neq 0} \sigma_I^D ,$$

where σ_R is the total reaction cross section in the entrance channel, and

$$\sum_{I\neq 0} \sigma_I^D$$

respresents the total *direct* reaction cross section.

In the present paper, we develop a theory for σ_F as well as the different components of σ_F, the fluctuation cross sections, which takes explicitly int› account multiple Coulomb excitation. It is found that the compound nucleus differential cross sections become less anisotropic as a result of Coulomb excitation. The oscillations seen in the singular distributions of transitions involving zero spin entrance and exit channels become more damped, and exhibit a large period. The fusion cross section at energies slightly below the Coulomb barrier is found to decrease by as much as 10% (in, e.g., $^{16}O + {}^{152}Sm$).

II. SALIENT RESULTS OF THE STATISTICAL THEORY

The general formula for the fluctuation cross section describing the transition $\alpha \to \beta$ for a given partial wave J is[1]

$$\sigma_{\alpha\beta}^{\rm fl}(J) = \frac{\pi}{{k_\alpha}^2}(2J+1)\frac{T_{\alpha\alpha}^J T_{\beta\beta}^J + T_{\alpha\beta}^J T_{\beta\alpha}^J}{{\rm Tr}\underline{T}^J} , \tag{1}$$

where T is Satchler's transmission matrix, given in terms of the average S matrix, $\bar{S}$, that describes the coupled direct channels part of the problem

$$\underline{T}=1-\underline{\bar{S}}^{\dagger}\underline{\bar{S}} . \quad (2)$$

Equation (2) is valid when the number of open channels, N, is large. Notice that unitarity is approximately satisfied in the sense

$$\sum_{\beta}\sigma_{\alpha\beta}^{\rm fl}(J)=\frac{\pi}{k_\alpha{}^2}(2J+1)\left[T_{\alpha\alpha}^J+\frac{(T^{J^2})_{\alpha\alpha}}{{\rm Tr}\underline{T}^J}\right]$$
$$=\frac{\pi}{k_\alpha{}^2}(2J+1)\left[T_{\alpha\alpha}^J+\mathcal{O}\left[\frac{1}{N}\right]\right] , \quad (3)$$

where the second term on the right hand side, of order $1/N$, measures the violation of unitarity in Eq. (1). However, Eq. (1) was originally obtained[1] by neglecting the same type of term as the one above. Therefore to be consistent, we shall neglect this term when calculating the total compound (i.e., fusion) cross section, σ_F, and write for channel α

$$\sigma_F^{(\alpha)}=\frac{\pi}{k_\alpha{}^2}\sum_{J=0}(2J+1)T_{\alpha\alpha}^J , \quad (4)$$

where J is the compound nucleus angular momentum (which is the same as the incident orbital angular momentum for the case of spinless projectile and target considered here).

Although we agree fully with the warning given by Mahaux and Weidenmüller[8] that the Hauser-Feshbach formula lacks a foundation in the case of heavy ion systems, we shall, however, use it as the basis of our theory. We remind the reader that the statistical, Hauser-Feshbach formula Eq. (1) (without the second term) has been widely used, and with success, in heavy ion compound reactions.[9]

It is clear from our Eqs. (1) and (2) that the basic quantity in our theory is the transmission matrix, $\underline{T}$. In order to construct this matrix one has to solve for the average S matrix, $\underline{\bar{S}}$, which requires a solution of the full coupled channels problem describing multiple Coulomb excitation *in the presence of compound nucleus absorption.* Although exact numerical solution of this problem is now feasible for heavy systems,[10] what we seek here is an approximate analytical solution for $\underline{\bar{S}}$ that would lead to a transparent expression for $\underline{T}$, and accordingly σ_F and the fluctuation cross sections, and the presence of multiple Coulomb excitation.

III. SIMPLE MODEL FOR THE AVERAGE S MATRIX

A very powerful method for solving iteratively the many-coupled-channels equations of heavy ion Coulomb excitation problem is the inward-outward integration scheme developed at Copenhagen.[11] We shall use this formulation to obtain an approximate expression for the matrix $\underline{\bar{S}}$. In the inward-outward method, the solutions, $\psi_\alpha(r)$, to the coupled radial Schrödinger equations

$$\left[\frac{d^2}{dr^2}+k_\alpha{}^2-\frac{l_\alpha(l_\alpha+1)}{r^2}-\frac{2\mu}{\hbar^2}U_\alpha(r)\right]\psi_\alpha(r)=\sum_\beta V_{\alpha\beta}(r)\psi_\beta(r) , \quad (5)$$

which are regular at the origin [$\psi_\alpha(0)=0$], are written in terms of r-dependent coefficients $\alpha(r)$ and $\alpha^{(+)}(r)$ of the regular and outgoing, $\varphi(r)$ and $h^{(+)}(r)$, parts of the homogeneous (uncoupled) optical wave function

$$\psi_\alpha(r)\frac{1}{k_\alpha^{1/2}}=a_\alpha(r)\varphi_\alpha(r)\frac{1}{k_\alpha^{1/2}}-a_\alpha^{(+)}(r)h_\alpha^{(+)}(r)\frac{1}{k_\alpha^{1/2}} . \quad (6)$$

Inserting Eq. (6) into Eq. (5) we then obtain the following two sets of coupled linear equations[11]:

$$\frac{d}{dr}a_\alpha(r)=\frac{1}{k_\alpha^{1/2}}\left[h_\alpha^{(+)}(r)\sum_\beta V_{\alpha\beta}(r)\varphi_\beta(r)\frac{1}{k_\beta^{1/2}}a_\beta(r)-h_\alpha^{(+)}(r)\sum_\beta V_{\alpha\beta}(r)h_\beta^{(+)}(r)\frac{1}{k_\beta^{1/2}}a_\beta^{(+)}(r)\right] , \quad (7)$$

$$\frac{d}{dr}a_\alpha^{(+)}(r)=\frac{1}{k_\alpha^{1/2}}\left[\varphi_\alpha(r)\sum_\beta V_{\alpha\beta}(r)h_\beta^{(+)}(r)\frac{1}{k_\beta^{1/2}}a_\beta(r)-\varphi_\alpha(r)\sum_\beta V_{\alpha\beta}(r)h_\beta^{(+)}(r)\frac{1}{k_\beta^{1/2}}a_\beta^{(+)}(r)\right] . \quad (8)$$

It should be clear that we have the freedom to consider a part of the channel-channel coupling in the generalized potential U. This is precisely what we shall do. The interaction V contains only the long-range Coulomb coupling, whereas U contains all short range (nuclear) couplings as well as the average effect of the coupling between the open-coupled-channel space and the compound nucleus (absorption due to fusion). We shall see that such a decomposition would be quite convenient for our purposes. The above implies that the S matrix resulting from solving the homogeneous equation (5), $\underline{\bar{S}}^{(0)}$, would be generally nondiagonal.

To simplify the discussion we shall use matrix notation in what follows. Outside the range of the short-range nuclear coupling the wave functions $\varphi(r)$ and $h^{(+)}(r)$ behave like

$$\varphi(r)\frac{1}{k^{1/2}}=\frac{i}{2}\left[H^{(-)}(r)\frac{1}{k^{1/2}}-H^{(+)}_{(r)}\frac{1}{k^{1/2}}\underline{S}^{(0)}\right]e^{i\sigma} , \tag{9}$$

$$h^{(+)}(r)\frac{1}{k^{1/2}}=e^{i\sigma}H^{(+)}(r)\frac{1}{k^{1/2}} , \tag{10}$$

where $H^{+(-)}$ is the outgoing (incoming) Coulomb wave function and σ is the Coulomb phase shift. Inserting Eqs. (9) and (10) into Eqs. (7) and (8) and dropping rapidly oscillating terms involving the products $H^{(+)}(r)H^{(+)}(r)$ and $H^{(-)}(r)H^{(-)}(r)$, we obtain[10]

$$\frac{d}{dr}e^{i\sigma}a(r)=\frac{i}{2}\frac{1}{k^{1/2}}H^{(+)}(r)V(r)H^{(-)}(r)\frac{1}{k^{1/2}}e^{i\sigma}a(r) , \tag{11}$$

$$\begin{aligned}\frac{d}{dr}e^{-i\sigma}a^{(+)}(r)=\frac{1}{2}\bigg[&\frac{1}{2}\frac{1}{k^{1/2}}H^{(+)}(r)V(r)H^{(-)}(r)\frac{1}{k^{1/2}}\underline{S}^{(0)}\\&+\frac{1}{2}\underline{S}^{(0)}\frac{1}{k^{1/2}}H^{(+)}(r)V(r)H^{(-)}(r)\frac{1}{k^{1/2}}\bigg]e^{i\sigma}a(r)\\&-\frac{i}{2}\frac{1}{k^{1/2}}H^{(-)}(r)V(r)H^{(+)}(r)\frac{1}{k^{1/2}}e^{-i\sigma}a^{(+)}(r) .\end{aligned} \tag{12}$$

Equations (11) and (12) may be further simplified by recognizing the fact that because of strong absorption, the long-range coupling potential, $V(r)$, modifies the wave function of the system at radial distances larger than the classical turning points. At these separation distances the following approximation is quite good.

$$H^{(+)}(r)V(r)H^{(-)}(r)\simeq 2F(r)V(r)F(r) , \tag{13}$$

where F represents a vector whose components are the regular Coulomb functions in the different channels. Corrections to Eq. (13) involve rapidly oscillating terms that would contribute very little when integrated. In what follows we use Eq. (13) for all values of r.

We introduce the following matrix:

$$\underline{C}(r)\equiv\frac{1}{k^{1/2}}\int_0^r F(r')V(r')F(r')dr'\frac{1}{k^{1/2}} . \tag{14}$$

We then have

$$\frac{d}{dr}e^{i\sigma}a(r)=i\left[\frac{d}{dr}\underline{C}(r)\right]e^{i\sigma}a(r) , \tag{15}$$

$$\frac{d}{dr}e^{-i\sigma}a^{(+)}(r)=\tfrac{1}{2}\left[\underline{S}^{(0)}\frac{d}{dr}\underline{C}(r)+\frac{d}{dr}\underline{C}(r)\underline{S}^{(0)}\right]e^{i\sigma}a(r)-\frac{i}{2}\left[\frac{d}{dr}\underline{C}(r)\right]e^{-i\sigma}a^{(+)}(r) . \tag{16}$$

Equations (15) and (16) have to be solved in conjunction with the boundary conditions.

$$a(\infty)=1 , \tag{17}$$

$$a^{(+)}(0)=0,\quad a^{(+)}(\infty)=\pi(\underline{t}-\underline{t}^{(0)}) , \tag{18}$$

where $\underline{t}$ is the total t matrix and $\underline{t}^{(0)}$ is the corresponding one for the homogeneous equation ($V=0$), i.e.,

$$\underline{t}^{(0)}=\frac{1}{2i}(1-e^{i\sigma}\underline{S}^{(0)}e^{i\sigma}) .$$

Of course the original equations (7) and (8) also satisfy the above boundary conditions.

Equations (15) and (16) with the conditions (17) and (18) can be solved analytically if we ignore ordering effects, namely if we set the commutator

$$\left[\left.\frac{d}{dr}\underline{C}\right|_r , \left.\frac{d}{dr}\underline{C}\right|_{r'}\right]=0 ,$$

which we believe to be related to the sudden approximation.[7] We thus find

$$a(r)=e^{-i\sigma}\exp[i[\underline{C}(r)-\underline{C}]]e^{-i\sigma}\ , \tag{19}$$

$$a^{(+)}(r)=\frac{e^{i\sigma}}{2i}[\bar{\underline{S}}^{(0)}e^{i\underline{C}(r)}-e^{-i\underline{C}(r)}\bar{\underline{S}}^{(0)}]\times e^{-i\underline{C}}e^{i\sigma}\ , \tag{20}$$

$$\underline{C}\equiv\underline{C}(r=\infty)\ .$$

Thus

$$a^{(+)}(\infty)=\pi[\underline{t}-\underline{t}^{(0)}]=\frac{e^{i\sigma}}{2i}[\bar{\underline{S}}^{(0)}-e^{-i\underline{C}}\bar{\underline{S}}^{(0)}e^{-i\underline{C}}]e^{i\sigma} \tag{21}$$

and finally

$$\pi\underline{t}=\frac{1-e^{i\sigma}e^{-i\underline{C}}\bar{\underline{S}}^{(0)}e^{-i\underline{C}}e^{i\sigma}}{2i}\ . \tag{22}$$

Therefore the total average S matrix can be identified through

$$\pi\underline{t}=\frac{1-\bar{\underline{S}}}{2i}\ ,\qquad \bar{\underline{S}}=e^{i\sigma}e^{-i\underline{C}}\bar{\underline{S}}^{(0)}e^{-i\underline{C}}e^{i\sigma}\ . \tag{23}$$

Equation (23) is the principal result of this section. It is interesting to observe that in the limit of pure Coulomb excitation, $\bar{\underline{S}}^{(0)}=1$, the $\bar{S}$ matrix becomes exactly the one obtained by Alder and Winther in the sudden limit,[7] namely

$$\bar{\underline{S}}^{\rm AW}=e^{i\sigma}e^{-2i\underline{C}}e^{i\sigma}\ . \tag{24}$$

Approximation (13) has been used previously in a more restricted sense and was found to give results quite close to the coupled channel calculations.[12] We call the above approximation the on-shell plus off-shell corrections method.[13]

We shall discuss the compound and fusion cross sections at energies higher than but close to the barrier. We thus feel comfortable in ignoring the short-range nuclear channel-channel coupling and take $\bar{\underline{S}}^{(0)}$ to be diagonal.

With $\bar{\underline{S}}^{(0)}$ diagonal, Eq. (23) written as

$$\bar{\underline{S}}^{(0)}=e^{i\underline{C}}e^{-i\sigma}\bar{\underline{S}}e^{i\sigma}e^{i\underline{C}} \tag{25}$$

supplies a nice example of the Engelbrecht-Weidenmüller (EW) transformation.[14] The matrix $U\equiv e^{i\underline{C}}$ diagonalizes $e^{-i\sigma}\bar{\underline{S}}e^{-i\sigma}$, in the sense that

$$\underline{U}^{T}e^{-i\sigma}\bar{\underline{S}}e^{-i\sigma}\underline{U}\equiv\bar{\underline{S}}^{(0)}\ ,$$

and also diagonalizes the transmission matrix given in Eq. (2) in the sense that

$$\bar{U}^{\dagger}\underline{T}\,\underline{U}=1-\underline{S}^{(0)\dagger}\underline{S}^{(0)}=\text{diagonal}\ .$$

However, since the matrix $e^{-i\underline{C}}$ corresponds to a physical process, namely the transition operator for pure Coulomb excitation at half the strength [see Eq. (24)], we do not need to deal explicitly with the EW transformation in our analysis, as we show below.

IV. THE TRANSMISSION MATRIX AND THE FLUCTUATION CROSS SECTIONS

In applying our results of the previous section, we shall assume that several collective channels (members of a rotational band of the deformed nucleus) are strongly coupled, thus giving rise to nondiagonal elements of the transmission matrix. We also assume the presence of many more weakly coupled channels. The totality of all the channels is assumed to be very large so that $\mathrm{Tr}\underline{T}\gg 1$.

The transmission matrix, $\underline{T}$, is obtained directly from the average S matrix through Eq. (2). Using our expression for $\bar{\underline{S}}$ of Eq. (23) we obtain the Hermitian matrix

$$\underline{T}=e^{-i\sigma}e^{+i\underline{C}}\underline{T}^{(0)}e^{-i\underline{C}}e^{i\sigma} \tag{26}$$

with

$$\underline{T}^{(0)}=1-\bar{\underline{S}}^{(0)\dagger}\bar{\underline{S}}^{(0)}\ . \tag{27}$$

If we further assume $\bar{\underline{S}}^{(0)}$ to be diagonal we obtain for the element $T_{\beta\alpha}$

$$T_{\beta\alpha}=\sum_{\gamma}(e^{+i\underline{C}})_{\beta\gamma}T_{\gamma}^{(0)}(e^{-i\underline{C}})_{\gamma\alpha}\times e^{i(\sigma_\alpha-\sigma_\beta)}\ . \tag{28}$$

The channel label γ implies $\{lI,J\}$ with l being the orbital angular momentum, I the intrinsic spin of the excited state, and J the total angular momentum of the channel which is conserved and also the angular momentum of the compound nucleus.

The diagonal elements, $T_{\alpha\alpha}$, have a very simple physical interpretation

$$T_{\alpha\alpha}=\sum_{\gamma}|(e^{-i\underline{C}})_{\alpha\gamma}|^{2}T_{\gamma}^{(0)}\ . \tag{29}$$

Equation (29) demonstrates the fact that the flux in the entrance channel is distributed among the strongly coupled channels before fusion takes place. Since the intermediate channels are not observed, the transitions from the entrance channel to these channels is described by the factors $|(e^{-i\underline{C}})_{\alpha\gamma}|^{2}$ which are the usual inelastic probabilities calculated at *half* the value of the coupling strength.

Using the fact that the operator $e^{-i\underline{C}}$ is unitary we can rewrite Eq. (29) as

$$T_{\alpha\alpha}=T_\alpha^{(0)}+\sum_\gamma |(e^{-i\underline{C}})_{\alpha\gamma}|^2[T_\gamma^{(0)}-T_\alpha^{(0)}]$$
$$\equiv T_\alpha^{(0)}+\Delta T_\alpha \ . \qquad (30)$$

At above-barrier energies, the second term on the right hand side of Eq. (30), which represents the coupled-channels effects on the compound nucleus transmission, is generally negative since the critical angular momentum associated with the bare entrance channel transmission coefficient $T_\alpha^{(0)}$ is larger than that of the inelastic channels $T_\gamma^{(0)}$. This is clearly seen in the particular case of two channels labeled 1 and 2,

$$T_{11}=T_1^{(0)}-|(e^{-i\underline{C}})_{12}|^2[T_1^{(0)}-T_2^{(0)}] \ , \qquad (31)$$

$$T_{22}=T_2^{(0)}+|(e^{-i\underline{C}})_{12}|^2[T_1^{(0)}-T_2^{(0)}] \ , \qquad (32)$$

and the nondiagonal elements

$$T_{12}=e^{-i(\sigma_1-\sigma_2)}[(e^{i\underline{C}})_{11}(e^{-i\underline{C}})_{12}[T_1^{(0)}-T_2^{(0)}]] \ , \qquad (33)$$

$$T_{21}=e^{i(\sigma_1-\sigma_2)}[(e^{i\underline{C}})_{21}(e^{-i\underline{C}})_{11}[T_1^{(0)}-T_2^{(0)}]] \ . \qquad (34)$$

Therefore the corrections to the compound nucleus transmission coefficients due to channel coupling are proportional *to the differences between bare transmission coefficients pertaining to different channels.* As functions of the angular momentum (J), these differences correspond to narrow windows centered close to the critical angular momentum for fusion, l_{cr}.

The partial cross sections, $\sigma_{\alpha\beta}^{(0)}(J)$ [Eq. (1)] that appear in the Hauser-Feshbach expression for the compound cross section are "windowlike" defined by a centroid, $J^{(0)}$ and a width $\Delta^{(0)}$.[15] As a consequence of this localization of $\sigma_{\alpha\beta}^{(0)}(J)$, the compound cross section is completely specified by three factors: an overall phase-space factor $(\sin\theta)^{-1}$, an oscillatory function given by $\sim\sin(2J^{(0)}\theta)$, and a damping function $F(\Delta^{(0)}\theta)$ which attains a maximum value of unity of $\theta=0$. At not too large angles, the above oscillations are quite conspicuous[15] (though becoming increasingly damped for nonzero final spin transitions).

Introducing the correction to $J_\alpha^{(0)}$ due to Coulomb excitation, Eq. (30), would imply changing the statistical window function, $\sigma_{\alpha\beta}^{(0)}(J)$, into (to first order in ΔT),

$$\sigma_{\alpha\beta}(J)\cong\sigma_{\alpha\beta}^{(0)}(J)-\frac{\pi}{k_\alpha{}^2}\frac{T_\alpha^{(0)}\Delta T_\beta+T_\beta^{(0)}\Delta T_\alpha}{\mathrm{Tr}\underline{T}^{(0)}}(2J+1) \ . \qquad (35)$$

If $l_{cr}^\alpha > l_{cr}^\beta$, the term $T_\beta^{(0)}\Delta T_\alpha$ would contribute very little as a consequence of the fact that ΔT_α peaks at a value of J larger than l_{cr}^β. Since

$$\frac{T_\alpha^{(0)}\Delta T_\beta}{\mathrm{Tr}\underline{T}^{(0)}}$$

peaks at a value of J larger than that of $\sigma_{\alpha\beta}^{(0)}(J)$, we conclude that the correction to the statistical window,[15] $\sigma_{\alpha\beta}^{(0)}(J)$, arising from Coulomb excitation, will result in an overall shift of its center of gravity to lower values of J. Furthermore, the effective width is increased, due to the increase in the diffusivity of the surface, with the overall shape of the window becoming more asymmetrical. These changes in the characteristics of the partial cross sections imply corresponding changes in the angular distributions: (a) a smaller period of angle oscillations (usually encountered in zero-spin transitions), (b) less damping with correspondingly larger coherence angles, and (c) a smaller overall magnitude. Owing to the shifting of the center of gravity of the statistical window towards smaller J, we also expect a smaller value of the total anisotropy,

$$R\equiv\left[\frac{\sigma_{\alpha\beta}(0)}{\sigma_{\alpha\beta}(\pi/2)}-1\right] .$$

Simple estimates of the changes in the angle period, P_θ, and the anisotropy can be made in the sharp cutoff and sudden limits, giving

$$\Delta P_\theta\simeq\frac{\sqrt{45}}{4}\pi\frac{\bar{x}_{0\to2}^\beta}{(J^{(0)})^2} \ , \qquad (36)$$

$$\Delta R\simeq-\frac{\sqrt{45}}{4}\pi\bar{x}_{0\to2}^\beta \ , \qquad (37)$$

where $\bar{x}_{0\to2}^\beta$ is the average quadrupole strength parameter in channel β (Ref. 7) (see below).

We should mention that the above esimates for ΔP_θ and ΔR could have been easily guessed by the mere consideration of the average collective angular momentum transferred in the Coulomb excitation process. However, we believe that the above estimates are only good for higher energies (see the later discussion on the fusion cross section) and are not quite valid for energies close to the barrier in the exit channel. For such cases Eqs. (35) and (30) for the corrections to $\sigma_{\alpha\beta}^{(0)}(J)$ should be fully incorporated into existing Hauser-Feshbach codes in order to obtain the changes in P and R. These calculations are in progress. We believe that the above considerations concerning the changes in the period of oscillation and more importantly, the anisotropy, are quite relevant to studies of high-spin states[9] in which the target and/or residual nucleons is deformed.

For cases involving *competition* between direct transitions dominated by Coulomb excitation and compound transitions, the nondiagonal elements of the transmission matrix, $T_{\alpha\beta}$, would contribute the second term of $\sigma^{\rm fl}_{\alpha\beta}(J)$ [Eq. (1)]. Although these cases involve small cross sections, we shall discuss them for completeness. To lowest order in the correction ΔT, we may keep just the first term of Eq. (1). For $\beta=\alpha$ (compound elastic), the correction to the HF formula arising from Coulomb excitation appears in the form of a *reduction* in the elastic enhancement factor (for strong absorption, this factor is ~ 2). Thus

$$\sigma^{\rm fl}_{\alpha\alpha}(J) \simeq 2\left[1-\frac{2\Delta T_\alpha}{T^{(0)}_\alpha}\right]\sigma^{(0)}_{\alpha\alpha}(J)\ . \qquad (38)$$

For inelastic transitions involving large energy losses, the fluctuation cross sections obtained from Eq. (35) are

$$\sigma^{\rm fl}_{\alpha\beta}(J) \simeq \left[1+\frac{\Delta T_\beta}{T^{(0)}_\beta}\right]\sigma^{(0)}_{\alpha\beta}(J)\ . \qquad (39)$$

Thus we obtain an *enhancement* over the conventional expression. Equations (38) and (39) are consistent. Owing to multiple Coulomb excitation, the channel-channel correlation present in the compound elastic cross section, which gives rise to the elastic enhancement factor of 2, is reduced. However, part of the lost strength should appear as an enhancement in the elastic cross sections Eq. (39).

Note that Eq. (39) is characteristic of heavy-ion reactions. Generally there should be another factor,

$$-\frac{\Delta T_\alpha}{T^{(0)}_\alpha}\ ,$$

inside the square bracket, which would cancel the factor

$$\frac{\Delta T_\beta}{T^{(0)}_\beta}$$

if the bare transmission coefficients $T^{(0)}_\alpha$ and $T^{(0)}_\beta$ are equal. This would happen at high energy and/or when a small energy loss is involved in the transition.

V. THE FUSION CROSS SECTION

We turn now to the fusion cross section in channel α (the entrance channel with the two nuclei in their ground states) defined in Eq. (4). Using our result for $T_{\alpha\alpha}$ given in Eq. (30) we obtain[16]

$$\sigma^{(\alpha)}_F=\frac{\pi}{k_\alpha^{\,2}}\sum_l(2l+1)T^{(0)l}_{l,0}-\frac{\pi}{k_\alpha^{\,2}}\sum_l(2l+1)\sum_{l'I\neq 0}|(e^{-i\underline{C}})_{l0,l'I}|^2[T^{(0)l}_{l,0}-T^{(0)l}_{l',I}]\ , \qquad (40)$$

where the total angular momentum of the compound nucleus, J, is set equal to the orbital angular momentum in the entrance channel, l, since the ground state spins of the two nuclei are assumed to be zero. Notice that l' could only have the values permitted by the selection rule[7]

$$l'+\lambda+l=\text{even}\ , \qquad (41)$$

where λ denotes the multipolarity of the transition.

At sub-barrier energies, the sum over l' in the second term on the right-hand side of Eq. (40) has to be evaluated very carefully since the difference

$$(T^{(0)l}_{l,0}-T^{(0)l}_{l'I})$$

is nonzero even for very small values of l. At higher energies, the contribution of this difference is centered about the critical angular momentum for fusion in the entrance channel, i.e., the angular momentum that specifies the value of

$$\frac{\pi}{k_\alpha^{\,2}}\sum_l(2l+1)T^{(0)l}_{l,0}\ ,$$

which in the sharp cutoff limit becomes

$$\frac{\pi}{k_\alpha^{\,2}}\sum_l(2l+1)T^{(0)l}_{l,0}\simeq\frac{\pi}{k_\alpha^{\,2}}(l^{(\alpha)}_{\rm cr}+1)^2\ , \qquad (42)$$

where

$$l^{(\alpha)}_{\rm cr}=\left[\frac{2\mu}{\hbar^2}R_C{}^2(E-E_C)\right]^{1/2}\ .$$

Since $l^{(\alpha)}_{\rm cr}\gg 1$ for heavy systems, even at energies slightly higher than the barrier, e.g., for $^{16}O+{}^{152}Sm$ at

$$\frac{E_{\rm c.m.}}{E_C}\sim 1.1\ \text{MeV},\quad l^{(\alpha)}_{\rm cr}\sim 25\ ,$$

we expect that an approximate evaluation of the l' sum involving consideration of the average value of

$T_{l,0}^{(0)l} - T_{l'I}^{(0)l}$, would be adequate. The correction to this approximation would be proportional to

$$\frac{x_{0\to 2}}{l_{\rm cr}^{\alpha}}$$

where $x_{0\to 2}$ is the quadrupole strength parameter,[7] which is a small quantity for the systems studied so far; for $^{16}O + {}^{152}Sm$ considered above,

$$\frac{x_{0\to 2}}{l_{\rm cr}^{\alpha}} \sim 0.2 \ .$$

Therefore we use for $T_{l0}^{(0)l}$ and $T_{l'I}^{(0)l}$ Hill-Wheeler forms, with I appearing in $T_{l'I}^{(0)l}$ only through the excitation energy, ϵ_I. Owing to the localization of $T_{l0}^{(0)l} - T_{l'I}^{(0)l}$ in l space and the rather slow variation of the reduced excitation probabilities within this l range, we may perform the l sum in closed form, obtaining for $\sigma_F^{(\alpha)}$ the following three-parameter expression

$$\sigma_F^{(\alpha)} \simeq \frac{\hbar\omega_c R_c{}^2}{2E} \times \sum_{I=0} P_I\left[\frac{\bar{x}_{0\to 2}^{(\alpha)}}{2}, \bar{\theta}(\bar{l}_I)\right] \times \ln\left[1+\exp\frac{2\pi}{\hbar\omega_c}(E-E_C-\epsilon_I)\right], \tag{43}$$

which is a generalization of Wong's formula.[4] In Eq. (43), ω_c is the frequency of the inverted parabola that approximates the fusion barrier and

$$P_I\left[\frac{\bar{x}_{0\to 2}^{(\alpha)}}{2}, \ \bar{\theta}(\bar{l}_I)\right]$$

are the Coulomb excitation probabilities at half the strength evaluated at an angle

$$\bar{\theta} = 2\tan^{-1}\frac{\bar{\eta}_I}{\bar{l}_I}$$

with

$$\frac{\bar{\eta}_I}{\bar{l}_I} = \frac{1}{2}\left[\frac{\eta_\alpha}{l_{\rm cr}^{\alpha}} + \frac{\eta_\gamma}{l_{\rm cr}^{\gamma}}\right].$$

The number of terms contributing to the I sum in Eq. (43) is determined by the value of the coupling strength $\bar{x}_{0\to 2}^{(\alpha)}$. For a pure quadrupole rotor, the sum may be truncated at

$$I_{\max} \sim \frac{\sqrt{45}}{4}\bar{x}_{0\to 2}^{(\alpha)} \ .$$

In obtaining Eq. (43) we have assumed that the frequency ω_c is the same in the different channels and that it varies slowly with l.

It is interesting to note that at high energies, $E > E_C + \epsilon_I$, the above expression for $\sigma_F^{(\alpha)}$ reduces to

$$\sigma_F^{(\alpha)} \simeq \pi R_C{}^2\left[1 - \frac{E_C}{E} - \frac{\Delta E}{E}\right], \tag{44}$$

where ΔE is the average energy loss of the projectile due to Coulomb excitation,

$$\Delta E = \sum_{l'I\neq 0} |(e^{-i\underline{C}})_{\bar{l}0,l'I}^{\bar{l}}|^2 \epsilon_I \ . \tag{45}$$

The quantity ΔE can be evaluated in closed form, in the sudden limit,[7] giving

$$\Delta E \cong \epsilon_{2+}\left[\frac{\bar{x}_{0\to 2}^{(\alpha)}}{2}\right]^2 R_z{}^2(\bar{\theta}(\bar{l})) = \frac{16\pi}{225}\frac{2\mu}{\hbar^2}\epsilon_{2+}\frac{B(E2)\uparrow}{e^2} \times \frac{R_2{}^2(\bar{\theta}(\bar{l}))E^3}{z_P{}^2 Z_T{}^4 e^4}, \tag{46}$$

where

$$R_2{}^2(\bar{\theta}) = \frac{9}{4}\frac{1}{\bar{l}^4}\left[1 - \frac{\tan^{-1}\bar{l}}{\bar{l}}\right]^2 + \frac{3}{4}\left[\frac{1}{l+\bar{l}^2}\right]^2, \quad \bar{l} \equiv \frac{\bar{l}_I}{\bar{\eta}_I} \ . \tag{47}$$

If one were to relax the sudden approximation and include nonzero energy loss, several complications arise in the evaluation of ΔE. For a full discussion we refer the reader to Ref. 17.

Clearly at energies close to the barrier one should use the more precise Eqs. (40) or (43). To exhibit the energy dependence of the correction to the fusion cross section, we have evaluated $\sigma_F^{(\alpha)}$, Eqs. (40) and (43) for the systems $^{16}O + {}^{148,150,152}Sm$. The bare transmission coefficients were calculated using a slightly modified approximation used recently by Dethier and Stancu[18] in their study of the fusion cross sections in light heavy-ion systems. The modification introduced cuts off the Coulomb "ear" that appears in the Dethier-Stancu approximation. The resulting transmission coefficients were found to approximate the WKB expression reasonably well. In our calculation of the coefficients we used for the ion-ion interaction the one recently employed by Esbensen.[6] This interaction is based on the proximity approximation and is quite similar to the Christensen-Winther empirical potential[19] (see the caption to Fig. 1). All pertinent physical parameters were extracted from the Nuclear Data Sheets. In our evaluation of the sum over I [Eq. (40)] we have considered the contribution of the 2^+ in the case of ^{148}Sm. In the cases of ^{150}Sm and ^{152}Sm we have included the 4^+ state with reorientation as well. The 4^+ state in ^{148}Sm was

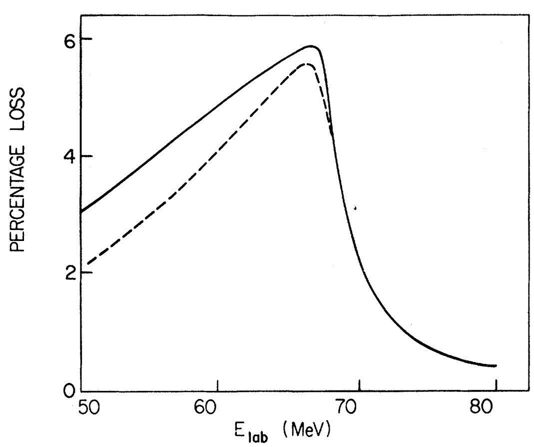

FIG. 1. The percentage correction (reduction) in the fusion cross section for the system ^{16}O + ^{148}Sm. The full curve is obtained from Eq. (40), the dashed one from Eq. (43). In the calculation, we have included the 2^+ state in ^{148}Sm ($\epsilon_{2^+}=0.522$ MeV). Other physical parameters were extracted from the Nuclear Data Sheets. The ion-ion potential employed is that of Ref. 6:

$$V_N(r)=-\frac{31.67R_PR_T}{R_P+R_T}\times\{1+\exp\times[(r-R_P-R_T-0.29)/0.63]\}^{-1}\ (\text{MeV})$$

with

$$R_{P,T}=1.233A_{P,T}{}^{1/3}-0.98A_{P,T}{}^{-1/3}\ (\text{fm})$$

and $A_{P,T}$ the projectile and target mass numbers. For the transmission coefficients appearing in Eq. (40) we used an approximation (Ref. 18) based on constructing a barrier composed of half a parabola on the inner side (obtained from the potential above) and a Coulomb potential on the outer side. To reduce the effect of the resulting Coulomb "ear," we have cut it off. The reduced Coulomb excitation probabilities that appear in Eq. (40) were evaluated using the explicit form of the coupling matrix $\underline{C}$ of Ref. 12.

found to contribute very little. Our results for the percentage correction (loss in fusion) in the system ^{16}O + ^{148}Sm are shown in Fig. 1. For comparison, we also show the results obtained with the generalized Wong formula, Eq. (43). It is clear from Fig. 1 that the percentage correction peaks at a value of $\sim 6\%$ at an energy slightly below the barrier. Similar results were found for the ^{16}O + ^{150}Sm and ^{16}O + ^{152}Sm systems with maximum percentage corrections of about 10%.

The rather large differences found between the results obtained with the generalized Wong formula, Eq. (43), and our more exact expression, Eq. (40), at low energies arise chiefly from the different approximations to the barrier used in these expressions. The inverted parabolic barrier used in the Wong expression yields a transmission coefficient which falls off more slowly with energy. The loss of energy due to excitation is then less important in inhibiting fusion and the percentage reduction of the fusion cross section is smaller. The neglect of the angular momentum dispersion (l dispersion) employed in the derivation of the generalized Wong expression was found to be a good approximation in the calculations performed. Comparisons were made using the transmission coefficient resulting from an inverted parabolic barrier in the more exact expression, Eq. (40). The generalized Wong formula was found to give a slightly higher percentage reduction at energies below the barrier and a slightly lower reduction above the barrier. In spite of its lack of importance in the cases studied, we believe that, in general, the dispersion in angular momentum should be treated carefully at sub-barrier energies.

In this case, it is possible that the dominant contribution to the correction comes from terms like

$$T_{l,0}^{(0)l}-T_{l-I,I}^{(0)l}\approx -T_{l-I,I}^{(0)l}\ ,$$

yielding an enhancement rather than a reduction of the fusion cross section. At these energies, an enhancement of the fusion cross section for deformed systems is expected.[2] In the cases studied in the present paper, however, the energy loss, rather than the l' dispersion, was found to be the dominant factor. The introduction of the dynamical deformation effects always led to a reduction of $\sigma_F^{(\alpha)}$. This result is in part consistent with the conclusions reached in, e.g., Ref. 5. The observed enhancement of $\sigma_F^{(\alpha)}$ must then be a result of the static effects of deformation. In our model these effects could be accounted for by performing equivalent spheres calculations of the bare transmission coefficients, $T_\gamma^{(0)}$. Such a calculation would presumably enhance the effect of the l' dispersion.

What is interesting about our formulae, Eqs. (40) and (43), is the clear separation between static deformation effects, which enter through $T_\gamma^{(0)}$, and the dynamic deformation effects related primarily to the present of the reduced transition probabilities.

In conclusion, we have developed a simple model which accounts for some of the effects of multiple Coulomb excitation of the heavy ion statistical, compound nucleus cross sections. More generally, we have supplied an indirect support for the validity of the generalized statistical theory of compound nucleus reactions in heavy ions. We have accomplished this in part by showing that within certain reasonable approximations, our formulae reduce to expressions which have very simple physical interpretations. A fuller account of the results presented in this paper, together with further developments, will be published later.

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[16]The total reaction cross section in channel α, $\sigma_R^{(\alpha)}$, is of course obtained from the elastic element of the average $\bar{S}$ matrix, namely

$$\sigma_R^{(\alpha)} = \frac{\pi}{k_\alpha{}^2} \sum_{l=0}^{\infty} (2l+1)[1 - |\bar{S}_{00}|^2]$$

$$= \sigma_F^{(\alpha)} + \frac{\pi}{k_\alpha{}^2} \sum_{l=0}^{\infty} (2l+1) \sum_{\gamma \neq 0} |\bar{S}_{0\gamma}|^2 \, .$$

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